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Study of Couette and Poiseuille flows of an Unsteady MHD Third Grade Fluid

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ABSTRACT

This article considered unsteady Magneto-hydrodynamic (MHD) flow of a generalized third grade fluid between two parallel plates. The flow is due to the plate oscillation and movement. Three special categories of flows namely, Couette, Poiseuille and Plug flows, are studied. The solutions for the non-linear Partial differential equations are obtained by using Homotopy Perturbation Method (HPM).
Finally, some features of the motion as well as the effect of the model parameters on the fluid motion have been plotted and discussed.
KEYWORDS: Unsteady Third grade fluid, MHD, Couette flow, Poiseuille flow, Plug flow, HPM.

I INTRODUCTION

The flow of non Newtonian electrically conducting fluid between two parallel plates in the presence of magnetic field has vast applications in various devices such as MHD power generators, MHD pumps, accelerators, polymer and petroleum industries. Islam et al. [1] discussed the solution of third grade fluid flows namely couette flow, Poiseuille flow and generalized couette flow by using OHAM. They also studied the heat transfer analysis. Hayat et al. [2] studied the MHD steady flow of oldroyd-6 constant fluid. The on linear equations of three different types of flows have been solved by using HAM. Attia [3] investigated the MHD non Newtonian unsteady couette and poisuille flows. The effect of Hall term and physical parameters are discussed for velocity and temperature distributions. Aiyesimi et al. [4-5] calculated the solution of MHD couette flow, Poiseuille flow and coquette Poiseuille flow problems of velocity and temperature distribution by using regular perturbation method. Siddiqui et al. [6] studied the third grade fluid flow between two parallel plates. The solution of different flows such that couette flow, Poiseuille flow and coquette Poiseuille flow problems of velocity distributions obtained by applying ADM and spectral method. Danish et al. [7] investigated the solutions for velocity field of Poisuille and coquette poisuille flow of third grade fluid. Haroon et al. [8] discussed the steady flow of the power law fluid between two parallel plates. The approximate solutions of momentum and energy equations have been obtained by using HPM. The effect of different physical parameters presented graphically.

Third grade fluid is a subclass of non-Newtonian fluid and its governing non-linear equation has effectively considered and treated in various literatures. Gul et al. [9-10] investigated the heat transfer analysis in electrically conducting thin film flow of third grade fluid on vertical belt. The lifting and drainage problems have been solved by using OHAM and ADM for both velocity and temperature fields. The results have been compared numerically and graphically. The effects of model parameter of velocity and temperature distributions have been discussed numerically and graphically. Ellahi et al. [11] studied examined the heat transfer analysis on third grade fluid. The numerical and analytical solutions have been obtained for velocity and temperature distributions. Ariel [12] discussed the steady flow of a third grade fluid through a porous flat channel. The non-linear boundary value problems have been solved by using different numerical methods. Makukula et al. [13] investigated the steady flow of third grade fluid through flat channel. Two methods successive linearization method (SLM) and improved spectral homotopy analysis method (ISHAM) were used to obtain the solutions. Aksoy and Pakdemili [14] studied the flow of third grade fluid between two parallel plates. The approximate solutions of momentum and energy equations have been obtained by using perturbation methods and compared the result. Shah et al. [15-16] discussed the unsteady flow of second grade fluid between wire and die. The partial differential equations of problem have been solved by using OHAM.

The unsteady magneto-hydrodynamics (MHD) thin film flows have been given significant attention in the history due to its large applications in the field of engineering, polymer industry and petroleum industries. Ali et al. [17] discussed the solution of electrically conducting fluid flow and heat transfer over porous stretching sheet. The governing non-linear partial differential equations of motion have been numerically solved by Method of Stretching Variables. The effects of physical parameters Magnetic parameter, Grashof number, Prandlt number and injection parameter S have been observed on velocity and temperature distributions. Yongqi

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and Wu [18] studied the unsteady flow of an incompressible fourth grade fluid in a uniform magnetic field. They compared the flow behavior of the fourth-grade with the Newtonian fluid. Alam et al. [19] discussed the magneto-hydrodynamic (MHD) thin-film flow of the Johnson–Segalman fluid on vertical plane. The problem have been solved analytically using the Adomian decomposition method (ADM) and discuss the effect of different physical parameters. Ajadi [20] studied the fluid flow in the presence of magnetic field and obtained the solutions for velocity and temperature distribution. Liao [21] investigated the MHD flow of non-Newtonian fluid over the stretching sheet. HAM method has been used to obtain the solution of problem. Gamal [22] examined the thin film flow of MHD micro polar fluid. Husain and Ahmad et al [23-24] investigated various numerical methods for the solution of combined effect of MHD and porosity. They have been observed the effect of different physical parameters on the velocity.

In this article we study the solution of unsteady thin film of a third grade fluid between two parallel plates under the influence of magneto hydrodynamics (MHD) using Homotopy Perturbation Method (HPM).He [25-26] discussed the fundamental introduction of HPM method. He applies the HPM method to the solution of wave equations and other non-linear boundary value problems. Mahmood and Khan [27] discussed the film flow of non- Newtonian fluid through porous inclined plane and HPM method is used to solve the problem. Ganji and Rafei [28] investigated the HPM method for the solution of Hirota Statsuma coupled partial differential equation. Lin [29] studied the solution of partial differential equation using HPM.

II BASIC EQUATION AND FORMULATION OF THEPLANE COUETTE FLOW PROBLEM

Two parallel and horizontal plates are considered such that the upper plate oscillating and moving with constant velocity$V$relative to the lower one. A uniform Magnetic field is applied transversely to the plates. The upper plate caries with itself a liquid layer of third grade fluid during its horizontal motion. The distance between plates is uniform and considered as$2h$.The coordinate system is chosen as in which the x-axis is taken perpendicular and y-axis is parallel to the plates. We are assuming that the flow is unsteady, laminar and incompressible.

For incompressible fluid the basic equations are

$$\nabla.\mathbf{v} = 0, \tag{1}$$

$$\rho\frac{D\mathbf{v}}{Dt} = \nabla.\mathbf{T} + \rho g + \mathbf{J}\times\mathbf{B}, \tag{2}$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B}), \tag{3}$$

Where ρ is the fluid density,$\mathbf{v}$is the velocity vector of the fluid,gis gravity, $\mathbf{J}$ is the current density, σ is the electrical conductivity, $\mathbf{B} = (0, \mathrm{B}_0, 0)$is uniform magnatic field, $\mathbf{T}$ is the caushy stress tensor and the material time derivative$\frac{D(*)}{Dt} = \frac{\partial(*)}{\partial t} + (\mathbf{v}.\boldsymbol{\nabla})*$.

$$\mathbf{J}\times\mathbf{B} = \left[\sigma {\mathrm{B}_0}^2\mathbf{v}, 0,0\right], \tag{4}$$

The cauchy stress tensor $\mathbf{T}$ for second grade fluid is given by

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A_2} + \alpha_2\mathbf{A}_1^2 + \beta_1\mathbf{A}_3 + \beta_2(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A_2}\mathbf{A}_1) + \beta_3(tr\mathbf{A}_1^2)\mathbf{A}_1, \tag{5}$$

Here α_1 and α_2 are the material constants,$\mathbf{A}_1 and$ $\mathbf{A}_2$are the Rivlin-Ericksen tensorgiven by

$$\mathbf{A}_1 = \mathbf{I}, \quad \mathbf{A}_1 = grad\mathbf{v} + (grad\mathbf{v})^{\boldsymbol{T}}, \tag{6}$$

$$\mathbf{A}_n = \frac{D\mathbf{A}_{n-1}}{Dt} + \mathbf{A}_{n-1}\boldsymbol{L} + \boldsymbol{L}^{\boldsymbol{T}}\mathbf{A}_{n-1}, \quad n = 0,1,2,..., \tag{7}$$

The velocity field in its component form as

$$\mathbf{V} = (v(y,t), 0,,0) \tag{8}$$

Boundary conditions are:

$$\mathbf{v}(h,t) = V + Vcos\omega t\ , \qquad \mathbf{v}(-h,t) = 0 \tag{9}$$

Here $\omega$is used as frequency of the oscillating belt.

By using the above assumptions and Equations (8) then continuity equation (1) is satisfied identically and the momentum equation reduces to the form

$$\rho\frac{\partial\mathbf{v}}{\partial t} = \frac{\partial}{\partial y}T_{yx} - \sigma {B_0}^2\mathbf{v}, \tag{10}$$

The Cauchy stress component T_{xy} of the third order fluid is

$$T_{xy} = \mu\frac{\partial\mathbf{v}}{\partial y} + \alpha_1\frac{\partial}{\partial t}\left(\frac{\partial\mathbf{v}}{\partial y}\right) + \beta_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial\mathbf{v}}{\partial y}\right) + 2(\beta_2 + \beta_3)\left(\frac{\partial\mathbf{v}}{\partial y}\right)^3 = T_{yx}, \tag{11}$$

Putting equation (11) in (10)we get

$$\rho\frac{\partial\mathbf{v}}{\partial t} = \mu\frac{\partial^2\mathbf{v}}{\partial y^2} + \rho\alpha_1\frac{\partial}{\partial t}\left(\frac{\partial^2\mathbf{v}}{\partial y^2}\right) + 6\beta_3\left(\frac{\partial\mathbf{v}}{\partial y}\right)^2\left(\frac{\partial^2\mathbf{v}}{\partial y^2}\right) - \sigma B_0\mathbf{v}, \tag{12}$$

Introducing non-dimensional variables as

$$\tilde{v} = \frac{\mathbf{v}}{V}, \ \tilde{y} = \frac{y}{h}, \ \tilde{t} = \frac{t\mu}{\delta^2\rho}, \tag{13}$$

Where

$M = \frac{\delta^2\sigma\mathrm{B}_0}{\mu V}$is the magnetic parameter, $\beta = \frac{\beta_3 V^2}{\mu\delta^2}$ is the non-Newtonian parameter

$\alpha = \frac{\alpha_1}{\rho\delta^2}$is the non-Newtonian parameter, $\Omega = \frac{h^2}{\mu V}\frac{\partial p}{\partial y}$ pressure gradient parameter.
Using the above dimensionless variables in equation (12) and dropping bars we obtain

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) + 6\beta\left(\frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2}\right) - Mv, \tag{14}$$

And the boundary conditions are

$$v(1,t) = 1 + \cos\omega t\ , \quad v(-1,t) = 0\ , \tag{15}$$

III Basic Idea of HPM

The HPM method is a combination of the classical perturbation technique and homotopy technique. To illustrate the basic concept of HPM for solving the non linear partial differential equation we consider the following general equation

$$\mathcal{T}\big(v(y,t)\big) - \mathcal{G}(y,t) = 0, \mathcal{B}\big(v(y,t)\big) = 0, \tag{16}$$

Where $v(y,t)$ is the unknown function, $\mathcal{G}(y,t)$ is the known analytic function, $\mathcal{B}$ is the boundary operator and A is the general differential operator which is express in linear part $\mathcal{L}(v(y,t))$ and nonlinear part $\mathcal{N}(v(y,t))$ as

$$\mathcal{T}\big(v(y,t)\big) = \mathcal{L}(v(y,t)) + \mathcal{N}(v(y,t)), \tag{17}$$

Therefore equation (16) can be written as

$$\mathcal{L}(v(y,t)) + \mathcal{N}(v(y,t)) - \mathcal{G}(y,t) = 0, \tag{18}$$

Now according to the homotopic method define as

$$\mathcal{H}(v(y,t),\wp) = (1-\wp)\big[\mathcal{L}\big(v(y,t)\big) - \mathcal{L}\big(v_0(y,t)\big)\big] + \wp[\mathcal{T}(v(y,t) - \mathcal{G}(y,t))], \tag{19}$$

Or we can write equation (19) as

$$\mathcal{H}(v(y,t),\wp) = \mathcal{L}\big(v(y,t)\big) - \mathcal{L}\big(v_0(y,t)\big) + \wp\mathcal{L}\big(v_0(y,t)\big) + \wp[\mathcal{N}(v(y,t) - \mathcal{G}(y,t))], \tag{20}$$

Here $\wp \in [0,1]$ is the embedding parameter and $v_0(y,t)$ is the initial approximation of equation (15) satisfying the boundary condition. Now from equation (19) and (20) we have

$$\mathcal{H}(v(y,t),0) = \mathcal{L}\big(v(y,t)\big) - \mathcal{L}\big(v_0(y,t)\big) = 0, \tag{21}$$

$$\mathcal{H}(v(y,t),1) = \mathcal{T}\big(v(y,t) - \mathcal{G}(y,t)\big) = 0, \tag{22}$$

By the variation of $\wp$ from $0\ to\ 1$, $v(y,t,\wp)$ change from $v_0(y,t)$ to $v(y,t)$ which is called Deformation. $\mathcal{L}\big(v(y,t)\big) - \mathcal{L}\big(v_0(y,t)\big)and\mathcal{T}\big(v(y,t) - \mathcal{G}(y,t)\big)$ are called Homotropic.
The approximate solution of equation (15) can be expressed as a series of the power of$p$ as

$$v(y,t) = v_0(y,t) + \wp v_1(y,t) + \wp^2 v_2(y,t) + \cdots, \tag{23}$$

Setting$\wp = 1$, then the approximate solution of equation (16) becomes

$$v(y,t) = \lim_{\wp\to 1} v(x,y) = v_0(y,t) + v_1(y,t) + v_2(y,t) + \cdots, \tag{24}$$

IV THE HPM SOLUTION OF THEPLANE COUETTE FLOW PROBLEM

We write equation (14) in standard form of OHAM and study zero, first and second component problems

Zero component problem:

$$\frac{\partial^2 v_0(y,t)}{\partial y^2} = 0 \tag{25}$$

Solution of zero component problem using boundary condition in equation (15) is

$$v_0(y,t) = \mathrm{Cos}[\tfrac{t\omega}{2}]^2 + \mathrm{Cos}[\tfrac{t\omega}{2}]^2 y, \tag{26}$$

First component problem:

$$\frac{\partial^2 v_1(x,t)}{\partial x^2} = -\mathrm{M}v_0 - \left(\frac{\partial v_0}{\partial \mathrm{t}}\right) + 2\frac{\partial^2 v_0}{\partial y^2} + 6\beta\left(\frac{\partial v_0}{\partial y}\right)^2\left(\frac{\partial^2 v_0}{\partial y^2}\right) + \alpha\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial^2 v_0}{\partial y^2}\right), \tag{27}$$

Solution of first component problem using boundary condition in equation (16) is

$$v_1(x,t) = \left[\mathrm{Cos}[\tfrac{t\omega}{2}]^2 + \mathrm{Cos}[\tfrac{t\omega}{2}]\mathrm{Sin}[\tfrac{t\omega}{2}]\right]\left[\left(\frac{\omega-M}{3}\right)\mathrm{y} + \left(\frac{\omega-M}{2}\right)\mathrm{y}^2 + \left(\frac{\omega-M}{6}\right)\mathrm{y}^3\right], \tag{28}$$

Second component problem:

$$\frac{\partial^2 v_1(x,t)}{\partial x^2} = -\mathrm{M}v_1 - \left(\frac{\partial v_1}{\partial \mathrm{t}}\right) + +12\beta\left(\frac{\partial v_0}{\partial \mathrm{y}}\right)\left(\frac{\partial v_1}{\partial \mathrm{y}}\right)\left(\frac{\partial^2 v_0}{\partial y^2}\right) + 2\frac{\partial^2 v_1}{\partial y^2} + 6\beta\left(\frac{\partial v_0}{\partial y}\right)^2\left(\frac{\partial^2 v_1}{\partial y^2}\right) + \alpha\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial^2 v_1}{\partial y^2}\right), \tag{29}$$

Inserting boundary conditions in equation (15) the second component solution obtained as

$$v_2(x,t) = \cos(\omega t)\left[\frac{m^2}{48}(y^4-1) + \frac{45}{32}M\beta(1-y^2) + \frac{\omega^2}{48}(1-y^4) + \frac{\alpha\omega^2}{4}(y^2-1) + \frac{15}{32}M\beta(y-y^3) + \frac{\alpha\omega^2}{12}(y^3-y) + \left(\frac{M^2-\omega^2}{240}\right)y^5 + \left(\frac{\omega^2-M^2}{720}\right)y + \left(\frac{M^2-\omega^2}{36}\right)y^3\right] + \cos(2\omega t)\left[\frac{9}{16}M\beta(1-y^2) - \frac{3}{16}M\beta(y-y^3)\right] + \cos(3\omega t)\left[\frac{3}{32}M\beta(y-y^3) + \frac{1}{32}M\beta(y^2-y^4) +\right] + \sin(\omega t)\left[\frac{1}{24}M\omega(1-y^4) + \frac{1}{4}M\alpha\omega(y^2-1) + \frac{15}{32}\omega\beta(y^2-1) + \frac{1}{12}M\omega\alpha(y+y^3) + \left(\frac{23}{360}M\omega\right)y - \left(\frac{1}{18}M\omega\right)y^3 - \left(\frac{1}{120}M\omega\right)y^5\right] + \sin(2\omega t)\left[\frac{3}{8}\beta\omega(y^2-1) + \frac{1}{8}\beta\omega(y^3-y)\right] + \sin(3\omega t)\left[\frac{3}{32}M\omega(y^2-1) + \frac{1}{32}\beta\omega(y^3-y)\right], \tag{30}$$

Now the general solution is

$$v(y,t) = v_0(y,t) + v_1(y,t) + v_2(y,t), \tag{31}$$

$$v(y,t) = \Big[\mathrm{Cos}[\tfrac{t\omega}{2}]^2 + \tfrac{1}{8}\mathrm{Cos}[t\omega]\left(\tfrac{1}{6}\omega^2 - \tfrac{1}{6}M^2 + \tfrac{45}{4}M\beta - 2\alpha\omega^2\right) + \tfrac{9}{16}M\beta\mathrm{Cos}[2t\omega] + \tfrac{3}{32}M\beta\mathrm{Cos}[3t\omega] + \tfrac{\omega}{8}\mathrm{Sin}[t\omega]\left(\tfrac{M}{6} - 2M\alpha - \tfrac{15\beta}{4}\right) - \tfrac{3}{8}\beta\omega\mathrm{Sin}[2t\omega] - \tfrac{3}{32}\beta\omega\mathrm{Sin}[3t\omega] - \tfrac{M^2}{48} + \tfrac{15M\beta}{16}\Big]\mathrm{y} + \Big[\mathrm{Cos}[\tfrac{t\omega}{2}]^2\left(1 - \tfrac{1}{3}M\right) + \mathrm{Cos}[t\omega]\left(\tfrac{15}{32}M\beta - \tfrac{23}{720}M^2 + \tfrac{23}{720}\omega^2 - \tfrac{1}{12}\alpha\omega^2\right) + \tfrac{1}{16}M\beta\left(3\mathrm{Cos}[2t\omega] + \tfrac{1}{2}\mathrm{Cos}[3t\omega]\right) + \tfrac{1}{3}\omega\mathrm{Sin}[\tfrac{t\omega}{2}]\left(\mathrm{Cos}[\tfrac{t\omega}{2}] + \tfrac{23}{120}M - \tfrac{1}{4}M\alpha - \tfrac{5}{8}\beta\right) - \tfrac{1}{8}\beta\omega\left(\mathrm{Sin}[2t\omega] - \tfrac{1}{4}\mathrm{Sin}[3t\omega]\right) - \tfrac{23M^2}{720} + \tfrac{5M\beta}{16}\Big]y^2 + \Big[\tfrac{1}{4}\mathrm{Cos}[t\omega]\left(\alpha\omega^2 - \tfrac{45}{8}M\beta\right) - \tfrac{1}{2}M\mathrm{Cos}[\tfrac{t\omega}{2}]^2 - \tfrac{3}{16}M\beta\left(3\mathrm{Cos}[2t\omega] - \tfrac{1}{2}\mathrm{Cos}[3t\omega]\right) + \tfrac{1}{2}\omega\mathrm{Sin}[\tfrac{t\omega}{2}]\left(\mathrm{Cos}[\tfrac{t\omega}{2}] + \tfrac{1}{2}M\alpha + \tfrac{15}{166}\beta\right) + \tfrac{3}{8}\beta\omega\left(\mathrm{Sin}[2t\omega] + \tfrac{3}{4}\mathrm{Sin}[3t\omega]\right) - \tfrac{15M\beta}{16}\Big]y^3 + \Big[\tfrac{M^2}{36} - \tfrac{5M\beta}{16} - \tfrac{1}{6}M\mathrm{Cos}[\tfrac{t\omega}{2}]^2 + \tfrac{1}{4}\mathrm{Cos}[t\omega]\left(\tfrac{1}{9}M^2 - \tfrac{15}{8}M\beta - \tfrac{1}{9}\omega^2 + \tfrac{1}{3}\alpha\omega^2\right) - \tfrac{1}{16}M\beta\left(3\mathrm{Cos}[2t\omega] - \tfrac{1}{2}\mathrm{Cos}[3t\omega]\right) + \tfrac{1}{6}\omega\mathrm{Sin}[\tfrac{t\omega}{2}]\left(\mathrm{Cos}[\tfrac{t\omega}{2}] - \tfrac{1}{3}M + \tfrac{1}{2}M\alpha + \tfrac{5}{6}\beta\right) + \tfrac{1}{8}\beta\omega\left(\mathrm{Sin}[2t\omega] + \tfrac{1}{4}\mathrm{Sin}[3t\omega]\right)\Big]y^4 + \Big[\tfrac{M^2}{48} + \tfrac{1}{48}\mathrm{Cos}[t\omega](M^2 - \omega^2) - \tfrac{1}{24}M\omega\mathrm{Sin}[t\omega]\Big]y^5 + \tfrac{1}{120}\Big[\tfrac{M^2}{2} + \tfrac{1}{2}\mathrm{Cos}[t\omega](M^2 - \omega^2) - M\omega\mathrm{Sin}[t\omega]\Big]y^6 \quad (32)$$

V Formulation of the plane Poiseuille flow

Inpoiseuille flow both plates are stationary and the flow between the plates is maintained due to the pressure gradient. Equation (2) for plane poiseuille flow with boundary conditions is

$$\rho\frac{\partial \mathbf{v}}{\partial t} = \mu\frac{\partial^2 \mathbf{v}}{\partial y^2} + \alpha_1\frac{\partial}{\partial t}\left(\frac{\partial^2 \mathbf{v}}{\partial y^2}\right) + 6\beta_3\left(\frac{\partial \mathbf{v}}{\partial y}\right)^2\left(\frac{\partial^2 \mathbf{v}}{\partial y^2}\right) - \sigma B_0\mathbf{v} - \frac{\partial p}{\partial y}, \quad (33)$$

$$\mathbf{v}(h,t) = V cos\omega t\ , \qquad \mathbf{v}(-h,t) = 0, \quad (34)$$

Using the dimensionless parameters define in equation (8) we obtain

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) + 6\beta\left(\frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2}\right) - Mv - \Omega, \quad (35)$$

$$v(1,t) = cos\omega t, \quad v(-1,t) = 0, \quad (36)$$

Zero component problem:

$$\frac{\partial^2 v_0(y,t)}{\partial y^2} = \Omega \quad (37)$$

Solution of zero component problems using boundary condition in equation (34) is

$$v_0(y,t) = \tfrac{1}{2}[\mathrm{Cos}[t\omega] - \Omega + \mathrm{Cos}[t\omega]\mathrm{y} + \Omega y^2], \quad (38)$$

First component problem: $\frac{\partial^2 v_1(y,t)}{\partial y^2} = -2\Omega - Mv_0 - \frac{\partial v_0}{\partial t} + 2\frac{\partial^2 v_0}{\partial y^2} + 6\beta\left(\frac{\partial v_0}{\partial y}\right)^2\left(\frac{\partial^2 v_0}{\partial y^2}\right) + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v_0}{\partial y^2}\right), \quad (39)$

Solution of first component problem using boundary condition in equation (34) is

$$v_1(y,t) = \Big[\tfrac{3}{8}\beta\Omega\mathrm{Cos}[2t\omega] - \mathrm{Cos}[t\omega]\left(\tfrac{1}{6}M + \beta\Omega^2\right) + \tfrac{1}{6}\omega\mathrm{Sin}[t\omega] - \Omega + \tfrac{5M\Omega}{24} + \tfrac{3\beta\Omega}{8} + \tfrac{\beta\Omega^3}{2}\Big]\mathrm{y} + \Big[-\Omega + \tfrac{M\Omega}{4} + \tfrac{3\beta\Omega}{8} - \tfrac{1}{4}M\mathrm{Cos}[t\omega] + \tfrac{3}{8}\beta\Omega\mathrm{Cos}[2t\omega] + \tfrac{1}{4}\omega\mathrm{Sin}[t\omega]\Big]y^2 + \Big[\mathrm{Cos}[t\omega]\left(\tfrac{1}{12}M - \beta\Omega^2\right) + \tfrac{1}{12}\omega\mathrm{Sin}[t\omega]\Big]y^2 - \Big[\tfrac{M\Omega}{24} - \tfrac{\beta\Omega^3}{2}\Big]y^2, \quad (40)$$

Second component problem:

$$\frac{\partial^2 v_2(y,t)}{\partial y^2} = -Mv_1 - \frac{\partial v_1}{\partial t} + 12\beta\frac{\partial v_0}{\partial y}\frac{\partial v_1}{\partial y}\left(\frac{\partial^2 v_0}{\partial y^2}\right) + 2\left(\frac{\partial^2 v_1}{\partial y^2}\right) + 6\beta\left(\frac{\partial v_0}{\partial y}\right)^2\left(\frac{\partial^2 v_1}{\partial y^2}\right) + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v_0}{\partial y^2}\right), \quad (41)$$

Solution of second component problem is too large. Derivations are given up to first order and graphical solutions are given up to second order.

VI Formulation of the plane Couette Poiseuille flow

In plane Couette Poiseuille flow the motion of fluid is depend upon on the motion of the upper plate and external pressure gradient. So the momentum equation (2) become

$$\rho\frac{\partial \mathbf{v}}{\partial t} = \mu\frac{\partial^2 \mathbf{v}}{\partial y^2} + \rho\alpha_1\frac{\partial}{\partial t}\left(\frac{\partial^2 \mathbf{v}}{\partial y^2}\right) + 6\beta_3\left(\frac{\partial \mathbf{v}}{\partial y}\right)^2\left(\frac{\partial^2 \mathbf{v}}{\partial y^2}\right) - \sigma B_0\mathbf{v} - \frac{\partial p}{\partial y}, \quad (42)$$

With boundary conditions

$$\mathbf{v}(h,t) = V + V cos\omega t\ , \qquad \mathbf{v}(-h,t) = 0, \quad (43)$$

Using the dimensionless parameters define in equation (32) we obtain

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) + 6\beta\left(\frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2}\right) - Mv - \Omega, \quad (44)$$

$$v(1,t) = 1 + cos\omega t, \quad v(-1,t) = 0, \quad (45)$$

Zero component problem:

$$\frac{\partial^2 v_0(y,t)}{\partial y^2} = \Omega \quad (46)$$

Solution of zero component problem using boundary condition in equation (43) is

$$v_0(y,t) = \tfrac{1}{2}[1 - \Omega + \mathrm{Cos}[t\omega] + (1 + \mathrm{Cos}[t\omega])\mathrm{y} + \Omega y^2], \quad (47)$$

First component problem:

$$\frac{\partial^2 v_1(y,t)}{\partial y^2} = -2\Omega - Mv_0 - \frac{\partial v_0}{\partial t} + 2\frac{\partial^2 v_0}{\partial y^2} + 6\beta\left(\frac{\partial v_0}{\partial y}\right)^2\left(\frac{\partial^2 v_0}{\partial y^2}\right) + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v_0}{\partial y^2}\right), \quad (48)$$

Solution of first component problem using boundary condition in equation (43) is

$$v_1(y,t) = -\frac{M}{6} + 0.012\Omega + \frac{5M\Omega}{24} - 0.9\Omega^2 + 0.45\Omega^3 - \frac{1}{6}M\mathrm{Cos}[t\omega] + 1.349\Omega\mathrm{Cos}[t\omega] - 0.9\Omega^2\mathrm{Cos}[t\omega] + 0.33\Omega\mathrm{Cos}[2t\omega] + \frac{1}{6}\omega\mathrm{Sin}[t\omega], -\frac{M}{4} + 0.012\Omega + \frac{M\Omega}{4} - \frac{1}{4}M\mathrm{Cos}[t\omega] + 1.34\Omega\mathrm{Cos}[t\omega] + 0.33\Omega\mathrm{Cos}[2t\omega] + \frac{1}{4}\omega\mathrm{Sin}[t\omega], -\frac{M}{12} + 0.9\Omega^2 - \frac{1}{12}M\mathrm{Cos}[t\omega] + 0.9\Omega^2\mathrm{Cos}[t\omega] + \frac{1}{12}\omega\mathrm{Sin}[t\omega], -\frac{M\Omega}{24} + 0.45\Omega^3, \quad (49)$$

Second component problem:

$$v_2(y,t) = -Mv_1 - \frac{\partial v_1}{\partial t} + 12\beta\frac{\partial v_0}{\partial y}\frac{\partial v_1}{\partial y}\left(\frac{\partial^2 v_0}{\partial y^2}\right) + 2\left(\frac{\partial^2 v_1}{\partial y^2}\right) + 6\beta\left(\frac{\partial v_0}{\partial y}\right)^2\left(\frac{\partial^2 v_1}{\partial y^2}\right) + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v_0}{\partial y^2}\right), \quad (50)$$

Solution of second component problem is too large. Derivation is given up to first order and graphical solutions are given up to second order.

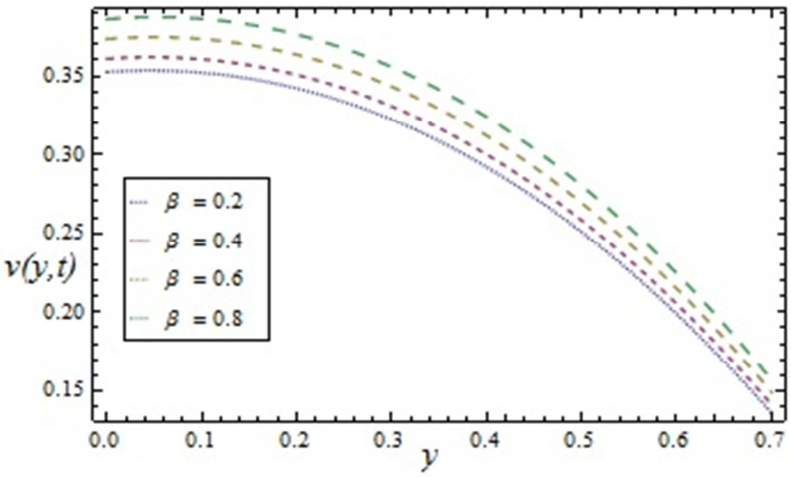

Fig 1: Effect of the non Newtonian parameter on the plane couette velocity profile when$M = 0.5, \Omega = 0.6, t = 4, \alpha = 0.3, \omega = 0.2$.

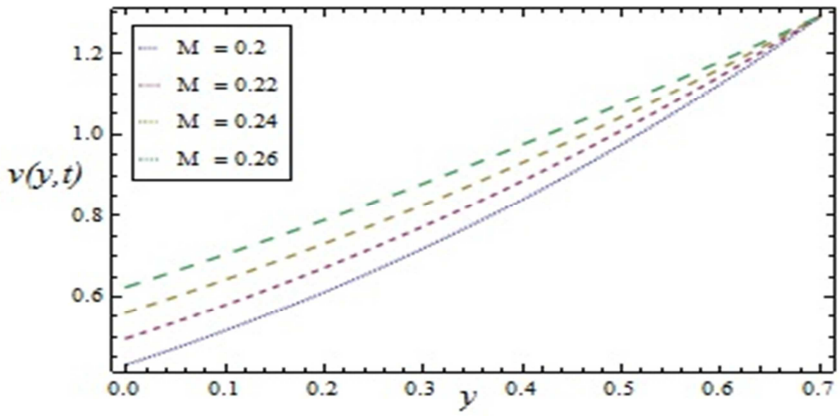

Fig 2: Effect of the Magnetic parameter M on the plane couette velocity profile when$\beta = 0.5, \Omega = 0.6, t = 1, \alpha = 0.3, \omega = 0.2$.

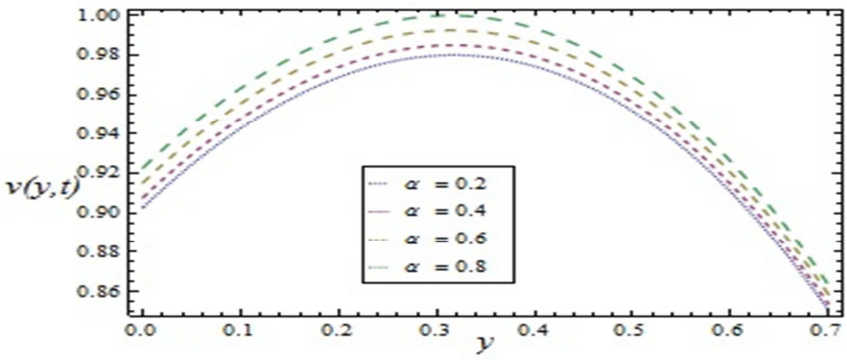

Fig 3: Effect of αon theplane couette velocity profile when $M = 0.4, \Omega = 0.4, t = 5, \beta = 0.3, \omega = 0.22$.

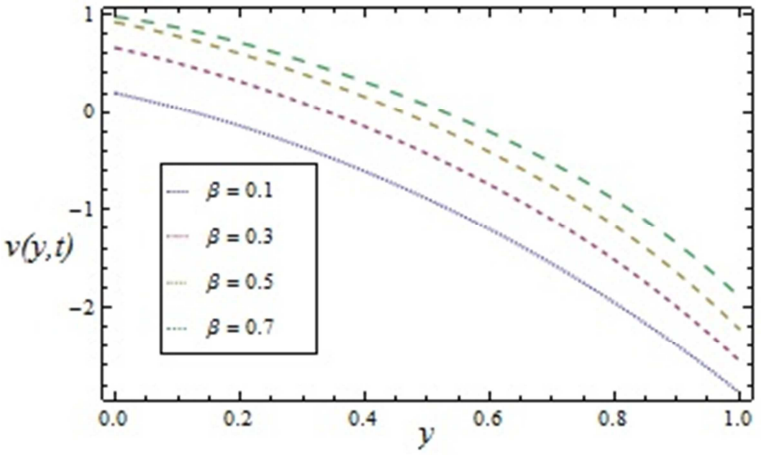

Fig 4: Effect of the non Newtonian parameterβon the plane Poiseuille velocity profile when$\alpha = 1, \Omega = 1, M = 0.5, \omega = 2, t = 2.$

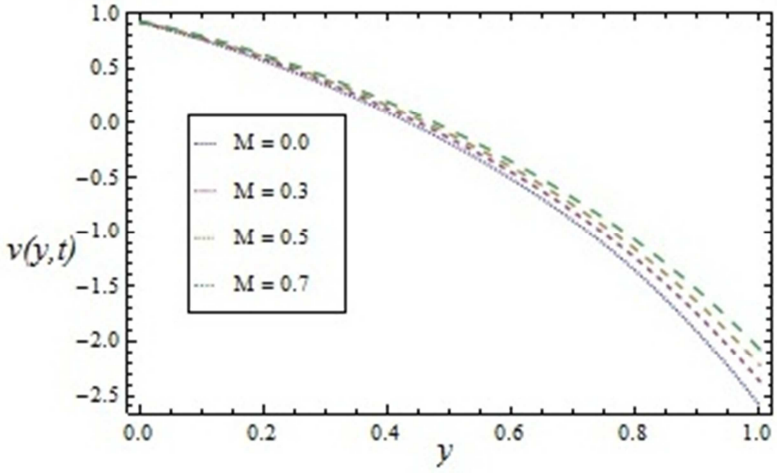

Fig 5: Effect of the magnetic parameter M on theplane Poiseuille velocity profile when$\alpha = 0.1, \Omega = 1, \beta = 0.5, \omega = 2, t = 2.$

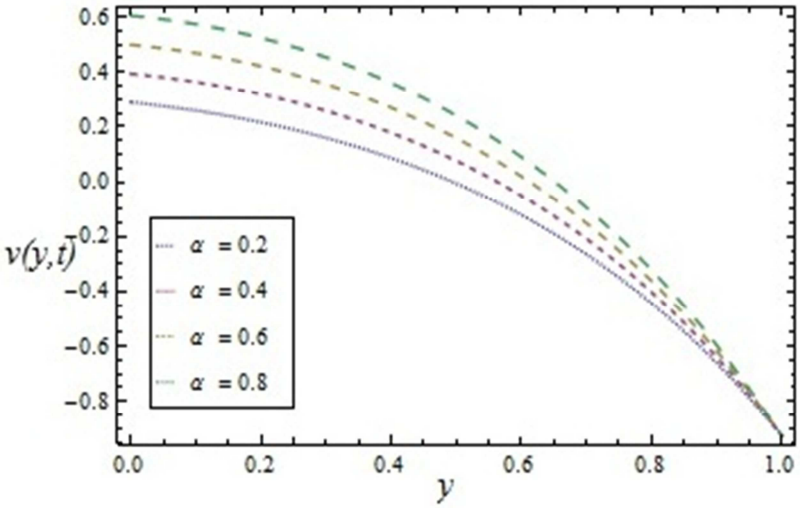

Fig 6: Effect of α on theplane Poiseuille velocity profile when$\beta = 0.5, \Omega = 0.6, t = 5, M = 3, \omega = 0.1$

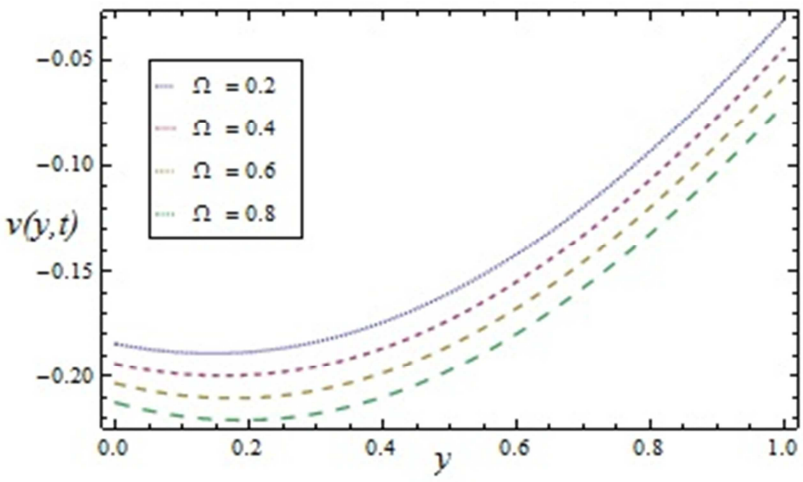

Fig 7: Effect of Ωon theplane Poiseuille velocity profile when$\beta = 0.5, \alpha = 0.6, t = 5, M = 3, \omega = 0.3$

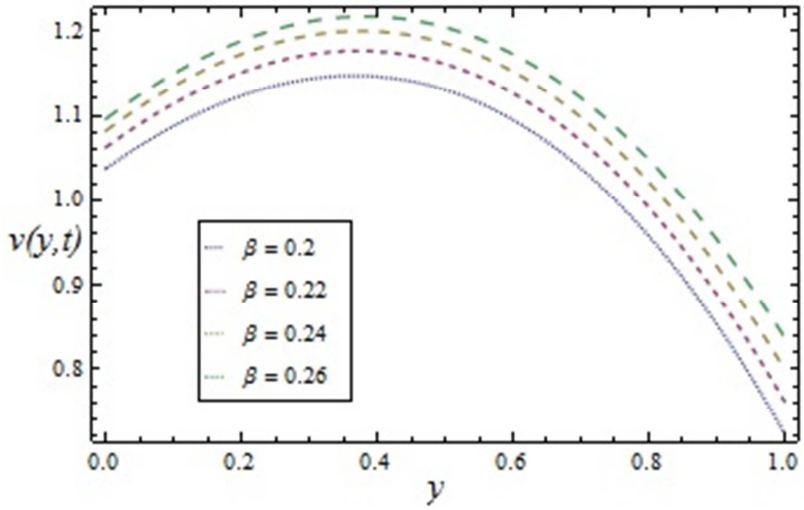

Fig 8: Effect of the non Newtonian parameterβ on theplane coquette Poiseuille velocity profile when$\alpha = 0.2, \Omega = 0.5, M = 0.5, \omega = 0.2, t = 5$

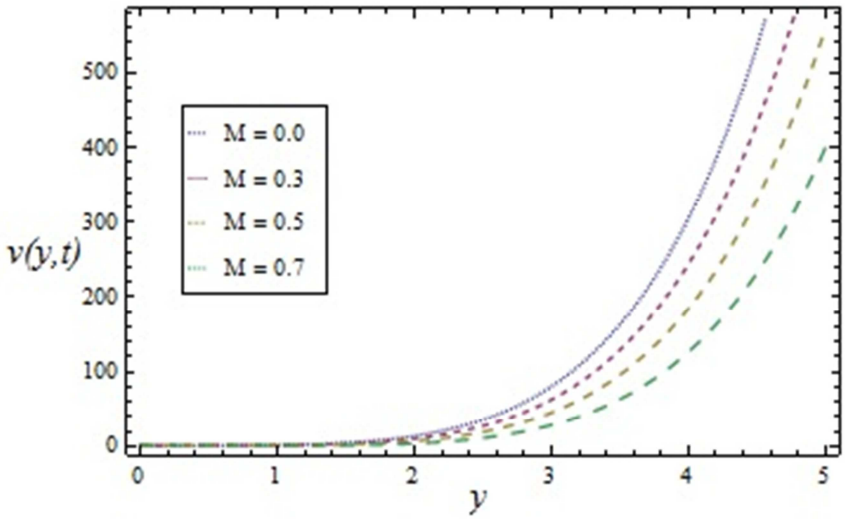

Fig 9: Effect of the magnetic parameter M on theplane CouettePoiseuille velocity profile when$\alpha = 0.2, \Omega = 0.5, \beta = 0.5, \omega = 0.2, t = 5$

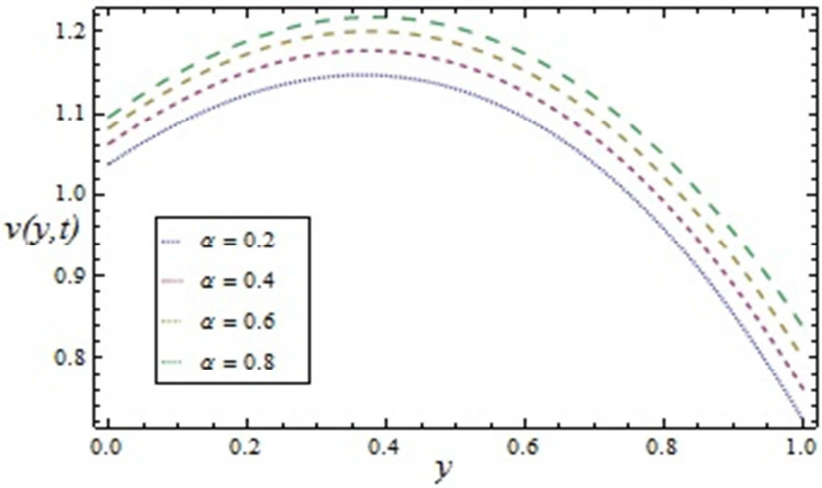

Fig 10: Effect of α on the plane Couette Poiseuille velocity profile when $\Omega = 0.3, M = 0.5, \beta = 0.5, \omega = 0.1, t = 5.$

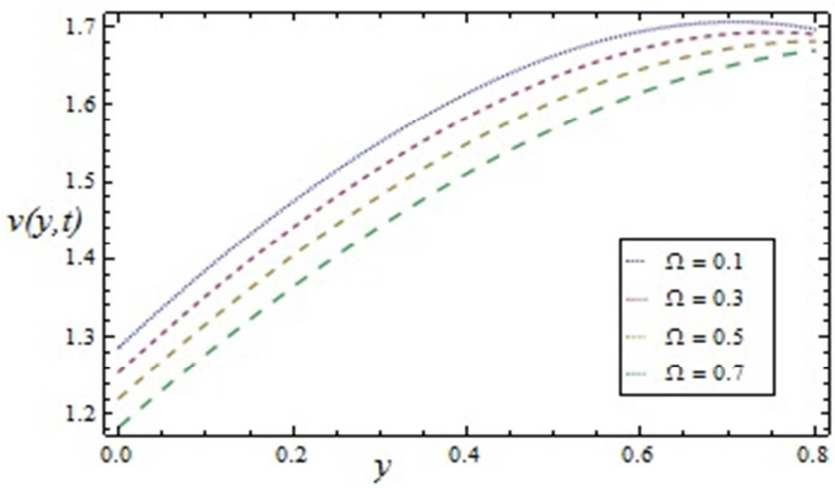

Fig 11: Effect of Ω on the plane Couette Poiseuille velocity profile when$\alpha = 0.3, M = 0.5, \beta = 0.5, \omega = 0.1, t = 5$

VII RESULTS AND DISCUSSION

In this section we shall proceed to discuss three unsteady flow problems namely, Couette flow, Poiseuille flow and Couette-Poiseuille flow of MHD third grade fluid between two parallel plates. From the flows different non-linear partial differential equations of velocity profile with specific boundary conditions are obtained and solved by using HPM method. In figures 1-11 we discussed and present graphically the effects of model parameters on velocity profile of Couette flow, Poiseuille flow and Couette-Poiseuille flow. Figures 1-3 shows the variation of velocity field for couette flow. Fig 1 is plotted to observe the effect of non-Newtonian parameter β. It has been observed that velocity increase by increasing the value of β. Fig 2 shows the effect of Magnetic parameter Mon velocity profile. The velocity field decrease by increasing M. Fig 3 shows the effect of α on velocity profile and velocity increase by increasing the value of α. Figures 4-7 shows the variation of velocity field for Poiseuille flow. Fig 4 is plotted to examine the effect of non-Newtonian parameter β. It has been observed that velocity increase by increasing the value of β. Fig 5 shows the effect of Magnetic parameter Mon velocity profile. The velocity field decrease by increasing M. Fig 6 is plotted to shows the effect of α on velocity profile and velocity increase by increasing the value of α.Fig 7 is plotted to shows the effect of pressure gradient Ω on velocity profile and it is observed that velocity increase by increasing the value ofΩ. Similarlyfigures 8-11 are plotted to shows the variation of velocity field forCouettePoiseuille flow. Fig 8 is plotted to examine the effect of non-Newtonian parameter β. It has been observed that velocity increase by increasing the value of β. Fig 9 shows the effect of Magnetic parameter Mon velocity profile. The velocity field decrease by increasing M. Fig 10 is plotted to observe the effect of α on velocity profile and velocity increase by increasing the value of α.Fig 11 is

plotted to examine the effect of pressure gradient Ω on velocity profile and it is observed that velocity increase by increasing the value ofΩ.

VIII CONCLUSION

In this paper, the solution of velocity profile of Couette flow, Poiseuille flow and Couette-Poiseuille flow of an MHD third grade fluid between two parallel plates has been discussed. The governing non-linear partial differential equations have been solved analytically by using HPM. The effect of model parameters, non-Newtonian parameter β ,magnetic parameter$M$, non Newtonian effect $\alpha$and pressure gradient parameter Ω involved in the problem are discussed and plotted graphically to examine the effect of these parameters on velocity profile. It is concluded that velocity increases as the non- Newtonian parameter increases, the velocity decreases as the magnetic parameter$M$increases.

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