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Heat and Mass Transfer for MHD Mixed Convection Stagnation Point Flow over a Vertical Non-Linearly Stretching Surface

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ABSTRACT

Numerical study of heat and mass transfer for magnetohydrodynamic (MHD) mixed convection stagnation point flow over a vertical non-linearly stretching surface with prescribed heat flux is considered. The set of highly nonlinear partial differential equations involved in the mathematical model of the flow problem have been transformed to the corresponding ordinary differential form by employing the non-linear similarity transform. The resulting equations are then solved numerically by using shooting technique with Runge-Kutta fourth order method. Several computations have been made to observe the effects of suction parameter (S), magnetic parameter (M), velocity ratio parameter (ε), non- linearity parameter (m), Schmidt number (Sc), mixed convection parameter (λ) and Prandtl number (P_r). The distributions of fluid flow with heat and mass transfer are presented in tabular and graphical form. The results for skin friction coefficient, local Nusselt number and Sherwood number have also been computed.
KEYWORDS: MHD mixed convection, Porous surface, Heat and mass transfer, Stretching surface

1. INTRODUCTION

Heat and mass transfer in MHD flow finds applications in various engineering problems. The concept is used in wet-bulb thermometer, polymer solution and food processing. The formation of fog involves the simultaneous heat and mass transfer phenomenon. Bejan and Khair [1] studied the free convection boundary layer flow in a porous medium with to combined heat and mass transfer. Tien and Vafai [2] examined the free convection boundary layer flow in a porous medium owing to combined heat and mass transfer. Ravikumar [3] investigated the heat and mass transfer effects on MHD flow of viscous incompressible and electrically conducting fluid through a nonhomogeneous porous medium in the presence of heat source, oscillatory suction velocity. Takhar et al. [4] analyzed mixed convention of an incompressible viscous fluid in a porous medium past a vertical plate. Chamkha [5] studied unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. The study of combined heat and mass transfer in mixed convective MHD flow along a vertical plate in presence of heat source has been made by Zueco and Ahmed [6]. Chen [7] has studied heat and mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and concentration. Palani and Srikanth [8] have explained the mass transfer effects on MHD flow past a semi- infinite vertical plate. Babul et al. [9] examined the effects of radiation and heat source/sink on the steady two dimensional magnetohydrodynamic (MHD) boundary layer flow of heat and mass transfer past a shrinking sheet with wall mass suction. Gokhale et al. [10] studied effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi- infinite vertical plate with constant heat flux. Magyari and Keller [11] also focused on heat and mass transfer on boundary layer flow due to an exponentially continuous stretching sheet.

MHD stagnation point flows have applications in the aerodynamics, the cooling of an infinite metallic plate in a cooling bath, extrusion of plastic sheets, textile and paper industries, boundary-layer along material handling conveyers, and blood flow problems. Fluid flows over the tips of oil ships, submarines, aircrafts and rockets involve stagnation flow phenomenon [12]. Hiemenz [13] studied the stagnation flow problem and reduced the Navier–Stokes equations for the forced convection problem to an ordinary differential equation of third order via similarity transformation. Lok [14] considered steady two-dimensional magnetohydrodynamic (MHD) stagnation-point flow of a viscous and electrically conducting fluid over a permeable shrinking sheet. Two dimensional steady

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incompressible mixed convection non orthogonal stagnation flow towards a heated or cooled stretching vertical plate was considered by Yian et al. [15].

Ahmad [16] studied two dimensional stagnation point flow of an incompressible, electrically conducting fluid due to a shrinking sheet. Kumaran et al. [17] studied the problem of MHD boundary-layer flow of an electrically conducting fluid over a stretching and permeable sheet with injection/ suction through the sheet. Elbasheshy [18] studied heat transfer over an exponentially stretching continuous surface with suction, and also added dimension to the study of Ali [19] on exponentially continuous stretching surface. Sajjad et al. [20] studied MHD stagnation point flow of micropolar fluids towards a stretching sheet.

El- Hakiem et al. [21] discussed the effect of magnetic field and double dispersion on mixed convection heat and mass transfer in non-Darcy porous medium. Anjali Devi and Ganga [22] examined dissipation effects on nonlinear MHD flow over a stretching surface with prescribed heat flux. Ahmad et al. [23] found analytical solution of MHD flow over porous stretching sheet. Hussain and Ahmad [24] considered MHD flow and heat transfer for viscous fluids over a stretching sheet in the presence of a uniform magnetic field.

In this article, we considered the problem of heat and mass transfer for magnetohydrodynamic (MHD) mixed convection stagnation point flow over a vertical non-linearly stretching surface with prescribed heat flux. The mathematical model of the problem in the form of ordinary differential equations is solved numerically. The results of physical quantities of interest have been computed and presented in tabular and graphical forms for two cases of the parameter ε ($\varepsilon < 1$ and $\varepsilon > 1$).

2. MATHEMATICAL MODEL

Consider viscous fluid flow near a stagnation point flow over a non-linear stretching surface. The flow is steady, incompressible and two- dimensional restricted in the region $y \geq 0$, where y is ordinate in the usual Cartesian plane. The free stream velocity is $U = cx^m$ and $u_w = ax^m$ is the sheet stretching velocity, m is the non-linearity constant and a, c are positive constants. A magnetic field of uniform strength is applied perpendicular to the surface y=0. The fluid temperature is T and mass concentration is C.

Under the above assumptions the equations of motion become as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \upsilon\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B^2(x)}{\rho}(U-u) + g\beta(T - T_\infty) \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right) \quad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\left(\frac{\partial^2 C}{\partial y^2}\right) \quad (4)$$

where $\underline{V} = V(u,v)$ is velocity, β is coefficient of thermal expansion, g is the acceleration due to gravity, ρ is density, $B(x)$ is transverse magnetic field υ, D and α are kinematic viscosity, molecular diffusivity of mass transfer and thermal diffusivity respectively.

The boundary conditions are:

$$u = u_w(x) + \frac{2-\sigma_0}{\sigma_0}\lambda_0\frac{\partial u}{\partial y}, \; v = v_w(x),$$

$$\frac{\partial T}{\partial y} = -\frac{q_w}{k}, \; \frac{\partial C}{\partial y} = -\frac{m_w}{D} \quad \text{at } y = 0,$$

$$u \to U(x), \; T \to T_\infty, \; C \to C_\infty \text{ as } y \to \infty \quad (5)$$

where λ_0 is the mean free path, σ_0 is the tangential accommodation coefficient, v_w is the suction/injection velocity, q_w is the surface heat flux, k is the thermal conductivity and m_w is concentration at surface.

Using the following similarity transformations:

$$u = cx^m f'(\eta),\ v = -\sqrt{c\upsilon}\,x^{(m-1)/2}\left[\frac{m+1}{2} f(\eta) + \frac{m-1}{2}\eta f'(\eta)\right],$$

$$T - T_\infty = \sqrt{\frac{\upsilon}{c}}\frac{q_0}{k} x^{2m-1}\theta(\eta), \qquad C - C_\infty = \sqrt{\frac{\upsilon}{c}}\frac{m_0}{D} x^{2m-1}\phi(\eta)$$

$$\eta = y\sqrt{\frac{a}{\upsilon}}x^{(m-1)/2} \qquad (6)$$

where the primes denote differentiation with respect to η.

$$B(x) = B_0 x^{(m-1)/2},$$

$$v_w = -\frac{\sqrt{c\upsilon}(m+1)}{2}x^{(m-1)/2}S,$$

$$q_w(x) = q_0 x^{(5m-3)/2},$$

$$m_w(x) = m_0 x^{(5m-3)/2}$$

where m_0, q_0, B_0 and S are constants. Also, S<0 stands for suction and S>0 for injection.

The equation of continuity (1) is identically satisfied.

Substituting (6) in to equations (3) and (4), we have repectively:

$$f''' + \frac{m+1}{2}ff'' + m(1 - f'^2) + M(1 - f') + \lambda\theta = 0 \quad (7)$$

$$\theta'' + \Pr\frac{m+1}{2}f\theta' - \Pr(2m-1)f'\theta = 0 \quad (8)$$

$$\phi'' + \frac{S_c(m+1)}{2}f\phi' - S_c(2m-1)f'\phi = 0 \quad (9)$$

The corresponding boundary conditions are:

$$f(0) = S, \quad f'(0) = -1,$$
$$\theta'(0) = -1, \phi'(0) = -1 \qquad (10)$$
$$f'(\infty) \to 0, \quad \theta(\infty) \to 0, \phi(\infty) \to 0$$

Where the magnetic parameter $M = \frac{\sigma B_0^2}{\rho c}$, the mixed convection parameter $\lambda = \frac{g\beta q_0}{k c^{5/2}}\sqrt{\upsilon}$, Prandtl number $\Pr = \frac{\upsilon}{\alpha}$ and Schmidt number $S_c = \frac{\upsilon}{D}$.

3. RESULTS AND DISCUSION

The equations (7) to (9) along with boundary conditions (10) have been solved numerically by using coding in FORTRAN 90. The numerical scheme involves shooting technique and Runge- Kutta fourth order method. In order to look into the physical nature of the problem, several computations have been carried out for different values of the effective parameters of interest. The results have been obtained for skin friction coefficient $f''(0)$, local Nusselt number $1/\theta(0)$ and Sherwood number $\varphi'(0)$ under the effect of non linearity parameter m. Tables 1 to 4 and Tables 5 to 8 respectively contain the results of these physical quanties of interest for $\varepsilon < 1$ and $\varepsilon > 1$. These results are computed for the values of λ= -2, -1, 0, 1, 2 from top to down in the tables. It is noticed that the values of $f''(0)$ and 1/ θ(0) increase with λ but $\varphi(0)$ decreases. Also $f''(0)$ is positive for $\varepsilon < 1$ and negative for $\varepsilon > 1$.

The results for non dimensional velocity f', temperature function θ and mass concentration φ have been presented in graphical form. Fig.1 demonstrates the velocity component f' under the influence of parameter λ for the two cases $\varepsilon < 1$ and $\varepsilon > 1$.

The velocity field depicts boundary layer structure. When the stretching velocity exceeds the free stream velocity ε > 1, the velocity shows an inverted boundary layer structure. However, boundary layer thickness increases with ε for ε > 1(ε =2) but it decreases for ε > 1(ε =0.1).

Fig.2 and fig.3 respectively show that temperature function *θ(η)* and the mass concentration *φ(η)* decrease with increase in the values of λ. Fig.4 depicts the effect of non- linearity parameter *m* on the velocity component f'. The boundary layer thickness decreases with increase in the values of *m*. The temperature distribution and mass concentration reduces with increase in the values of *m* as shown respectively in fig.5 and fig.6.

The fig.7, fig.8 and fig.9 respectively presents the velocity f', temperature *θ(η)* and mass concentration *φ(η)* under the the influence of parameter *S*. It is observed the boundary layer thickness of velocity function decreases with increase in the values of *S*, while temperature distribution and mass concentration reduces.

The effects of magnetic parameter *M* on velocity, temperature and mass concentration are noticed and presented in the fig.10, fig.11 and fig.12 respectively. The increase in magnetic field reduces the boundary layer thickness for velocity. Also, the temperature and mass concentration reduces with increase in magnetic field strength.

The fig.13 and fig.14 respectiely show the results for θ' and ϕ' and for different values of *m.*

Table # 1

m=0.6 ϵ=0.1		
$f''(0)$	1/ θ(0)	$\varphi(0)$
0.76725	0.95507	1.78979
1.26241	0.97691	1.77963
1.71682	0.99607	1.7707
2.14	1.0132	1.76271
2.53826	1.02876	1.75544

Table # 2

m=0.8 ϵ=0.1		
$f''(0)$	1/ θ(0)	$\varphi(0)$
1.1089	1.09722	1.72142
1.50189	1.12058	1.70985
1.86580	1.14125	1.6996
2.20669	1.15987	1.69036
2.5289	1.17685	1.68193

Table # 3

m=1.0 ϵ=0.1		
$f''(0)$	1/ θ(0)	$\varphi(0)$
1.39017	1.23747	1.65358
1.71017	1.25943	1.64249
2.00987	1.27921	1.6325
2.29328	1.29727	1.62339
2.56309	1.31393	1.61500

Table # 4

m=2.0 ϵ=0.1		
$f''(0)$	1/ θ(0)	$\varphi(0)$
2.37282	1.86533	1.36824
2.52703	1.87799	1.36252
2.67726	1.89013	1.35707
2.82386	1.90178	1.35185
2.96711	1.913	1.34685

Table # 5

m=0.6 ϵ=2.0		
$f''(0)$	1/ θ(0)	$\varphi(0)$
-2.86688	1.22818	1.67829
-2.57269	1.23801	1.67384
-2.2879	1.24731	1.66964
-2.01152	1.25615	1.66564
-1.7427	1.26457	1.66182

Table # 6

m=0.8 ϵ=2.0		
$f''(0)$	1/ θ(0)	$\varphi(0)$
-2.93575	1.48497	1.56205
-2.71886	1.49403	1.55777
-2.50732	1.50270	1.55368
-2.30066	1.51102	1.54975
-2.0985	1.51902	1.54597

Table # 7

m=1.0 ϵ=2.0		
$f''(0)$	1/ θ(0)	$\varphi'(0)$
-3.05289	1.71611	1.46391
-3.88247	1.72404	1.4602
-3.71529	1.7317	1.45661
-3.55113	1.73911	1.45315
-3.38981	1.7463	1.4498

Table # 8

m=2.0 ϵ=2.0		
$f''(0)$	1/ θ(0)	$\varphi'(0)$
-3.80186	2.64903	1.1377
-3.72204	2.6533	1.1361
-3.64278	2.65751	1.13453
-3.56407	2.66167	1.13298
-3.4859	2.66578	1.13145

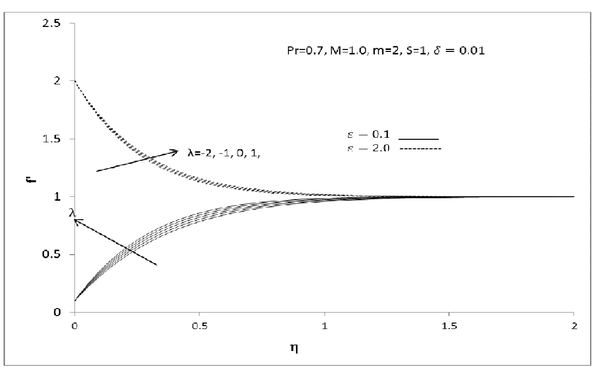

Fig.1: Graph of f' for different values of λ.

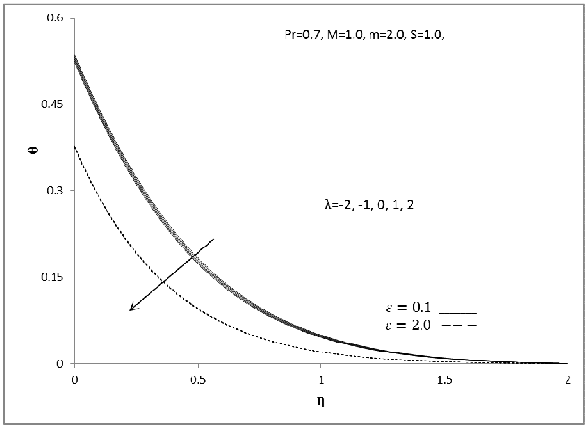

Fig.2: Graph of θ for different values of λ.

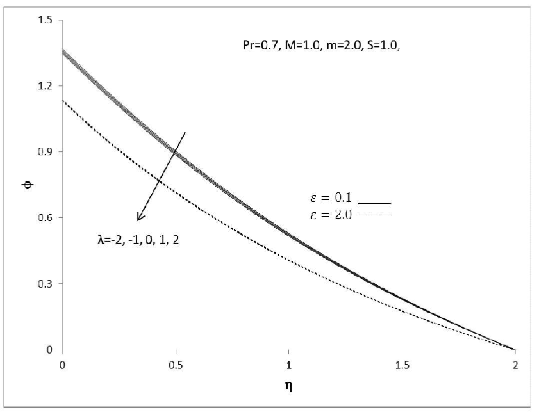

Fig.3: Graph of φ for different values of λ.

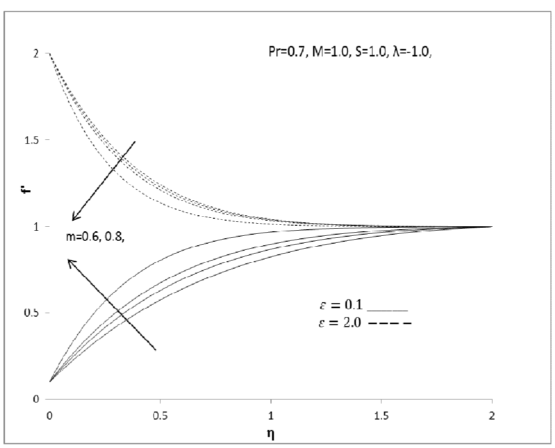

Fig.4: Graph of f' for different values of m

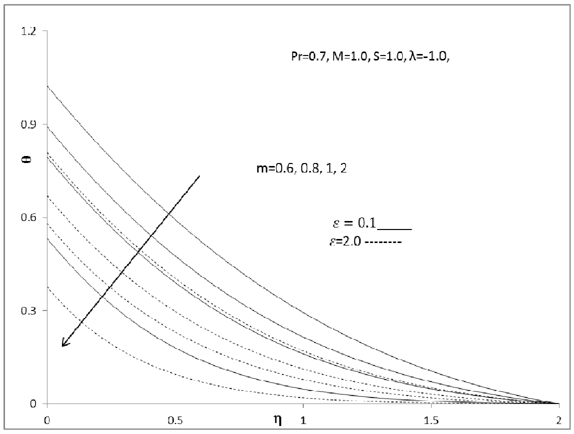

Fig.5: Graph of θ for different values of *m*.

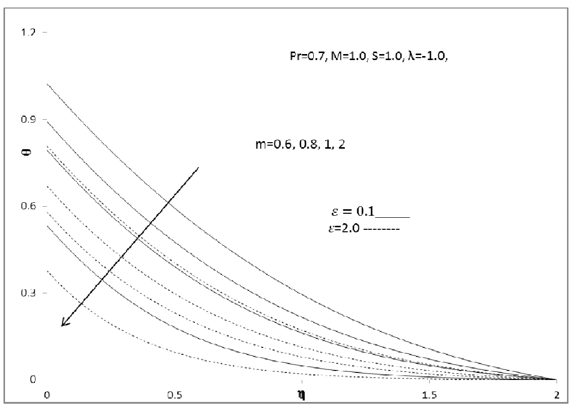

Fig.6: Graph of φ for different values of *m*

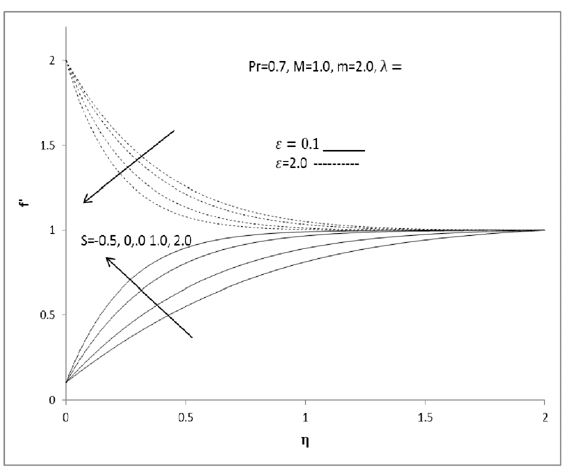

Fig.7: Graph of f' for different values of *S*.

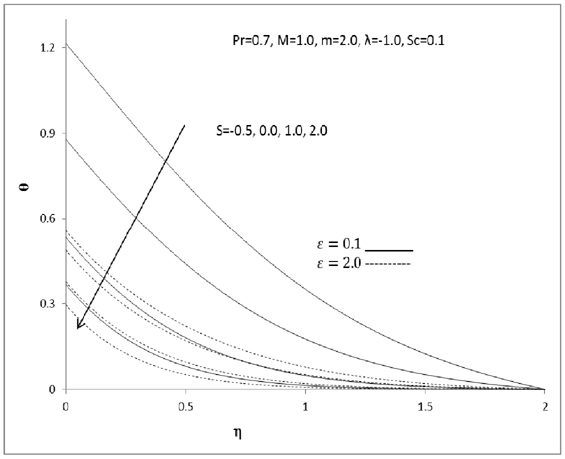

Fig.8: Graph of θ for different values of *S*.

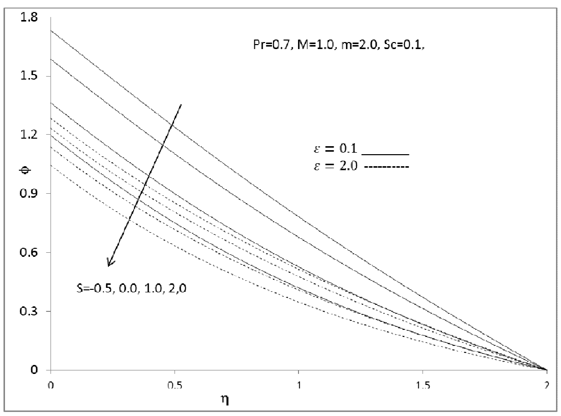

Fig.9: Graph of φ for different values of *S*.

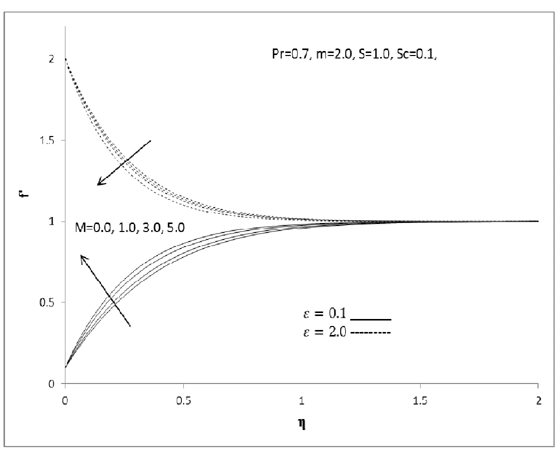

Fig.10: Graph of f' for different values of *M*.

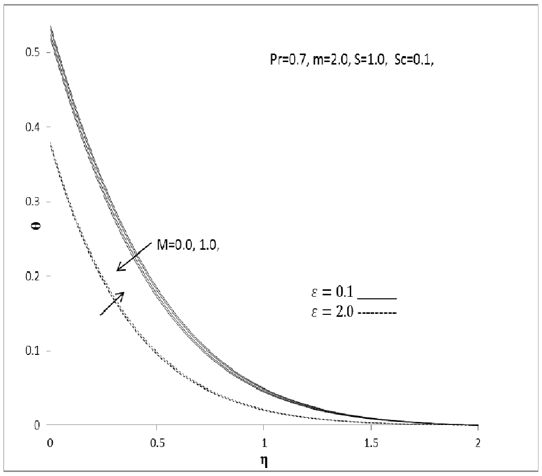

Fig.11: Graph of θ for different values of *M*.

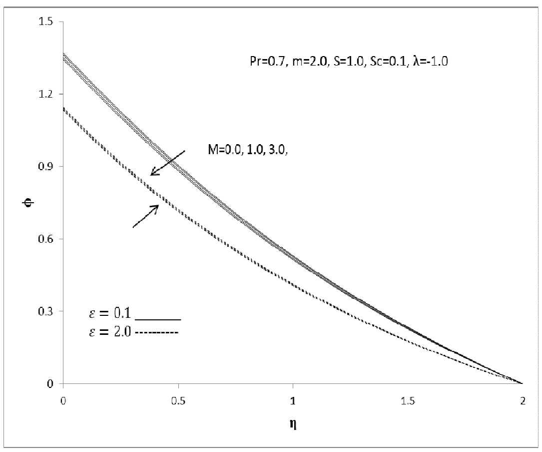

Fig.12: Graph of φ for different values of *M*.

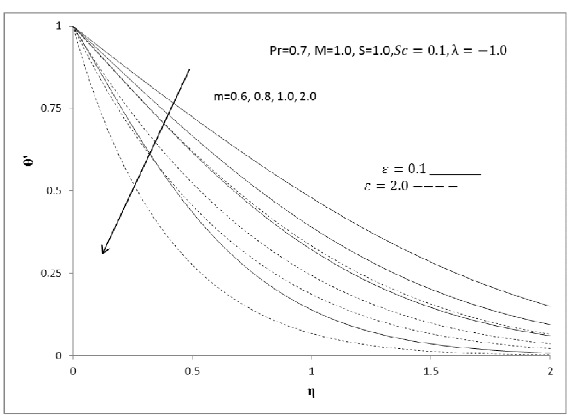

Fig.13: Graph of - θ' for different values of *m*

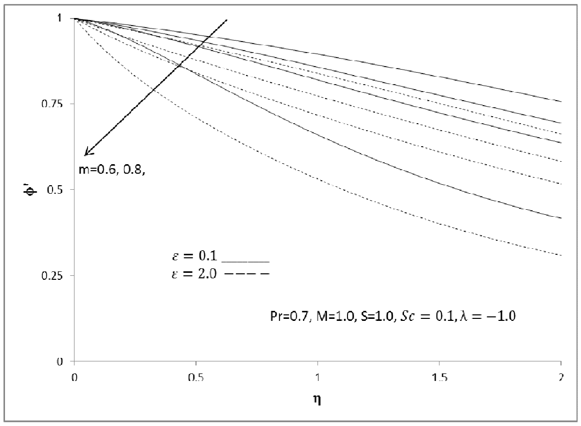

Fig.14: Graph of - ϕ' for different values of *m*

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