Numerical Solution of MHD of Boundary Layer Flow of Nanofluid Fluids Due to Porous Stretching Surface

M. A. El-Hakiem

Presently: Department of Mathematics, College of Science, Al-Zulfi, Majmaah University, Saudi Arabia.
Department of Mathematics, College of Science, Aswan, Aswan University, Egypt.

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ABSTRACT

In this study we have explored the numerical solution of MHD flow of an incompressible nanofluid towards a stretching surface. The obtained model of nonlinear partial differential equations is converted into a set of ordinary differential equations using an appropriate transformation. Shooting method is employed to solve the said system of boundary layer equations. Discussion of momentum, temperature and concentration profiles against emerging parameters like Prandtl number, Brownian motion parameter, thermophoresis parameter, Lewis number, stretching parameter, suction and injection parameter. Tabulated values of local Nusselt number and Sherwood number is an added feature with detailed discussion against different pertaining parameters. It is observed that temperature profile is higher for large values of Brownian motion parameter, thermophoresis parameter, and Suction and injection parameter. It is also found that for increasing values of magnetic field parameter and velocity ratio parameter, skin friction coefficient show increasing and decreasing behavior respectively.

KEYWORDS: MHD; incompressible nanofluid; boundary layer; shooting method; stretching sheet.

1. INTRODUCTION


In this article, we obtained numerical solution of MHD flow of an incompressible nanofluid towards a stretching surface. The results have been obtained by using an easy and straight forward numerical scheme. The effects of the physical parameters involved in the study have been observed on velocity, microrotation and concentration.

2. Mathematical Formulation

We consider a time independent two dimensional an incompressible nano viscous fluid flow over a stretching sheet. The external and stretching velocities \( u_e(x) = ax \) and \( u_w(x) = bx \) are considered respectively with \( a \) and \( b \) as positive constants. Effects of Newtonian heating are also considered here. Temperature and concentration at the surface are \( T_w \) and \( C_w \) whereas ambient temperature and concentration are \( T_\infty \) and \( C_\infty \) respectively. The governing boundary layer equations representing the system are:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B\tilde{u}}{\rho} \\
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= \frac{\partial^2 v}{\partial y^2} + \left[ \frac{D_B \frac{\partial c}{\partial y}}{\rho} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] \\
u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} &= \frac{D_B c}{\partial y^2} + \frac{D_T \partial c}{T_\infty \partial y^2}
\end{align*}
\]

with the boundary conditions
\[
\begin{align*}
u u &= \pm u_w(x) \quad v = u_w, \quad T = T_w, \quad C = C_w \text{ at } \ y = 0 \\
u u &= u_e(x) \quad T = T_\infty, \quad C = C_\infty \text{ at } \ y \to \infty
\end{align*}
\]

Here \( u \) and \( v \) denote the velocity components in the \( x \) and \( y \) directions respectively, \( T \) and \( C \) are the temperature and concentration. \( D_B, D_T, \) and \( \alpha \) are the Brownian diffusion coefficient, the thermophoretic diffusion coefficient, and the thermal diffusivity respectively. In order to find a similarity solution of equations [1-5], we use the following transformations:

\[
\eta = \sqrt{\frac{\alpha}{v}} y, \quad \Psi = \sqrt{\alpha \nu x f(\eta)}, \quad \theta(\eta) = \frac{T-T_\infty}{T_\infty}, \quad (\eta) = \frac{C-C_\infty}{C_\infty},
\]

with the velocity components
\[
\frac{\partial u}{\partial y} = \Psi, \quad \frac{\partial v}{\partial x} = -\Psi
\]

Equation (1) is identically satisfied while equations (2) to (5) are given by

\[
\begin{align*}
f'''' + f f'' - (f')^2 - Mf' &= 0 \\
\theta'' + Pr [f \theta' + Nb \theta \phi' + Nt (\theta')^2] &= 0 \\
\phi'' + Le \phi' + Nt \theta'' &= 0 \\
f(0) &= \pm f_w, \quad f'(0) = \varepsilon, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad \eta = 0 \\
f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \quad \eta \to \infty
\end{align*}
\]

where prime denotes differentiation with respect to \( \eta \) and \( Pr, Le, Nb, Nt, M \) and \( \varepsilon \) denoted Prandtl number, Lewis number, Brownian motion parameter, Thermophoresis parameter, Magnetic field parameter and ratio parameter, respectively. These parameters are defined as:

\[
Pr = \frac{\nu}{\beta B}, \quad Le = \frac{\nu}{R}, \quad Nb = \frac{D_B C_\infty}{\nu}, \quad Nt = \frac{D_T}{\nu}, \quad M = \frac{\sigma B\tilde{u}}{\mu a}, \quad \varepsilon = \frac{b}{a}
\]

The physical quantities as skin friction coefficient, Nusselt and Sherwood numbers are given by:

\[
\begin{align*}
c_f &= -\left[ \frac{\partial u}{\partial y} \right]_{y=0} = -f''(0), \quad N_u = \frac{x}{T_w-T_\infty} \left[ \frac{\partial T}{\partial y} \right]_{y=0} = -\frac{\theta'(0)}{\theta(0)}, \quad Sh = \frac{x}{C_w-C_\infty} \left[ \frac{\partial c}{\partial y} \right]_{y=0} = -\frac{\phi'(0)}{\phi(0)}
\end{align*}
\]

3 RESULTS AND DISCUSSION

The numerical solution of equations (8 − 10) with (11) has been found via shooting method to observe the effects of the parameters namely, \( Nb, Nt, \varepsilon, Pr, \) and \( Le \). A comparison of the values of surface temperature \( \theta(0) \), and heat transfer coefficient \( -\theta'(0) \) has been made to authenticate results obtained from the method used. Table I
depicts the values of skin friction \( \ddot{f}(0) \), heat transfer coefficient \( \dot{\theta}(0) \), and mas transfer coefficient \( \dot{\phi}(0) \) under the various values of \( M, \Pr, \Le, \fw, \varepsilon, \Nt, \text{ and } Nb \). The results for velocity, temperature, and concentration functions have been presented in graphical form.

Fig.1 demonstrates the inverted parabolic shape of velocity curves that decrease in magnitude with increase in the value of magnetic parameter \( M \). It is because the Lorentz force decelerates the fluid flow. Fig.2 and fig.3 depict a slight increase in the values of \( \theta(\eta) \) and \( \phi(\eta) \) with increase in the values of \( M \). The prandtl number \( \Pr \) decreases the temperature distribution as shown in fig.4 but it increases concentration function slightly as depicted in fig.5. The brownian motion parameter \( \B_p \) has increasing effect on \( \theta(\eta) \) but it has decreasing effect on \( \phi(\eta) \) as shown in fig.6 and fig.7 respectively. Fig.8 and fig.9 respectively demonstrate that Lewis number \( \Le \) increases \( \theta(\eta) \) but it decreases \( \phi(\eta) \) as shown in fig.10 and fig.11 respectively. Fig.12 shows that the suction parameter \( f_w \) decreases the fluid velocity near the surface and it increases the velocity away from the surface but reciprocal effect is observed in fig.16 for injection. Fig.17 and fig.18 respectively demonstrate that \( \theta(\eta) \) and \( \phi(\eta) \) decrease with increase in suction but Fig.19 and Fig. 20 respectively depict that \( \theta(\eta) \) and \( \phi(\eta) \) increases with increase in injection.

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Fig. 1: Effect of Magnetic field on the velocity profile $f'(\eta)$

Fig. 2: Effect of Magnetic field on the temperature profile $\theta(\eta)$

Fig. 3: Effect of Magnetic field on the concentration profiles $\phi(\eta)$
Fig. 4: Effect of Prandtl number \( Pr \) on the temperature profile \( \theta(\eta) \)

Fig. 5: Effect of Prandtl number \( Pr \) on the concentration profile \( \phi(\eta) \)

Fig. 6: Effect of Brownian motion parameter \( Nb \) on the temperature profile \( \theta(\eta) \)
Fig. 7: Effect of Brownian motion parameter $Nb$ on the concentration profile $\phi(\eta)$

M = 0.2, Pr = 5.0, Le = 1.0, f_w = 0.0, $\varepsilon$ = 1.0,

$Nb = 0.1, 0.2, 0.4.$

Fig. 8: Effect of thermophoresis parameter $Nt$ on the temperature profile $\theta(\eta)$

M = 0.2, Pr = 5.0, Le = 1.0, f_w = 0.0, $\varepsilon$ = 1.0,

$Nt = 0.01, 0.02, 0.04, 0.06,$

Fig. 9: Effect of thermophoresis parameter $Nt$ on the concentration profile $\phi(\eta)$

M = 0.2, Pr = 5.0, Le = 1.0, f_w = 0.0, $\varepsilon$ = 1.0,

$Nt = 0.01, 0.02, 0.04, 0.06,$
Fig. 10: Effect of Lewis number $Le$ on the temperature profile $\theta(\eta)$

Fig. 11: Effect of Lewis number $Le$ on the concentration profile $\phi(\eta)$
Fig. 12: Effect of ratio parameter $\varepsilon$ on the velocity profile $f'(\eta)$

Fig. 13: Effect of ratio parameter $\varepsilon$ on the temperature profile $\theta(\eta)$
Fig. 14: Effect of ratio parameter $\varepsilon$ on the concentration profile $\phi(\eta)$

Fig. 15: Effect of parameter $fw$ on the velocity profile $f'(\eta)$
Fig. 16: Effect of the parameter $f_w$ on the temperature profile $\theta(\eta)$

Fig. 17: Effect of the parameter $f_w$ on the concentration profile $\phi(\eta)$
Fig. 18: Effect of the parameter $f_w$ on the velocity profile $f'(\eta)$

Fig. 19: Effect of the parameter $f_w$ on the temperature profile $\theta(\eta)$
4. Conclusion

The numerical solution of MHD flow of an incompressible nanofluid towards a stretching surface has been investigated to observe the effects of the physical parameters of the study on fluid flow and heat transfer phenomena. The main results of this work are summarized below:

- The velocity $f'(\eta)$ decreases in magnitude with increase in the value of magnetic parameter $M$.
- The values of $\theta(\eta)$ and $\phi(\eta)$ increase slightly with increase in the values of $M$.
- The prandtl number $Pr$ decreases the temperature distribution but it increases concentration function slightly.
- The Brownian motion parameter $B_r$ has increasing effect on $\theta(\eta)$ but it has decreasing effect on $\phi(\eta)$.
- The thermophoresis parameter $N_t$ has slightly decreasing effect on $\theta(\eta)$ but significant increasing effect on $\phi(\eta)$.
- Lewis number $Le$ increases $\theta(\eta)$ but it decreases $\phi(\eta)$.
- The velocity ratio parameter $\varepsilon$ increases the fluid velocity $f'(\eta)$ but it decreases $\theta(\eta)$ and $\phi(\eta)$.
- The suction parameter $f_w$ decreases the fluid velocity near the surface and it increases the velocity away from the surface but reciprocal effect is observed for injection.

The functions $\theta(\eta)$ and $\phi(\eta)$ decrease with increase in suction but these quantities increases with increase in injection.

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