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Analytical Solution of Magnetohydrodynamic flow of a Third Grade Fluid in Wire Coating Analysis

Zeeshan Khan[1], Saeed Islam[2], Haroon Ur Rashed[1], Hamid Jan[1], Arshad Khan[1]

[1]Sarhad University of Science and Information Technology, Peshawar, Pakistan
[2]Department of Mathematics,Abdul Wali Khan University,Mardan, KP, Pakistan

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ABSTRACT

The most important plastic resins used in wires and optical fibers are plastic polyvinyl chloride (PVC) and low-high density polyethylene (LDPE/HDPE), nylon and Polysulfone. In this paper, we investigate a theoretical model of the magneto-hydrodynamic flow of a third grade fluid for wire coating process inside a cylindrical coating die. The governor equations are modeled and then solved by utilizing by Optimal Homotopy Asymptotic Method (OHAM). The convergence of the series solution is established. The effect of different emerging parameters is discussed with several graphs.The results of this investigation1are verified by Adomian Decomposition Method (ADM). Furthermore, the obtained results are also compared with published work, as a special case of the problem where an admirable agreement with the current work is observed.
KEYWORDS: Metallic wire coating, MHD flow, Third grade fluid, ADM and OHAM solution.

1. INTRODUCTION

Many fluids, in industrial and domestic applications exhibit non-Newtonian behavior. The apparent viscosities of such fluids depend on the rate at which they are sheared and on their previous shear history. An understanding of non-Newtonian behavior is quite important. For instance for Chemical engineering such fluid can help in two aspects: one they can confer essential, desirable properties on the material and secondly, they can facilitate the observation of non-Newtonian behavior in the design of the process of plant and pipelines [1, 2]. Most of the fluids used in chemical industry, for manufacturing purpose, belong to the class of non-Newtonian fluids.Therefore, more attention has been given to this class of fluid. Usually, non-Newtonian fluids are compound, mixture slurries, pastes, plastics, gels, polymer solutions etc. [3, 4]. In this study non-Newtonian third grade fluids have been studied for their applicability. Third grade fluids have been studied by many researchers. Siddiqui et al. [5] studied the torsion flow of such fluid.The heat flux of such fluids in two parallel plates is discussed in [6]. Islam et al. [7] studied third grade fluid with heat transfer. Aksoy et al. [8] investigated the third grade fluid flow in parallel plates with a porous medium. Homotopy perturbation method was used for the thin film stream of a third grade of liquid onto a moving belt by Siddiqui et al. [9]. Thin film third grade fluid studied in [10]. Ellahi et al. [11] investigated MHD third grade fluid with a variable viscosity. The MHD thin film flow of a third grade fluid with temperature dependent viscosity was investigated by Gul et al. [12].

Material used for coating of wire also exhibits non-Newtonian behavior in nature. Different types of fluids are used for wire and fiber optics coating. The wire coating depends upon the temperature, geometry, fluid viscosity and polymer.It depends on the geometry of die, viscosity of fluid, the temperature of the wire and the molten polymer. Limited information is available in the literature regarding the wire coating problem.Shah et al. [13] investigated wire coating analysis with linearly varying temperature. Han and Rao [14] carried out an analysis on wire coating extrusion. Nom-Newtonian fluid model was used byAkter and Hashmi [15, 16] for wire coating. Siddiqui et al. [17] investigated wire coating extrusion in a pressurized type die. Fenner and Williams [18] investigated the coating flow in a pressurized die. Mitsoulis [19] studied the wire coating flow with heat transfer. Unsteady second grade fluid with oscillating boundary condition inside the wire coating die was investigated by Shah et al. [20]. Exact solution was obtained for unsteady second grad fluid for wire coating by Shah et al. [21]. The Oldroyd 8-constant fluid was used for wire coating analysis by Shah et al.[22]. Shah et al. [23] investigated the wire coating using third grade fluid flow along with heat transfer analysis. Recently Sajid et al. [24] used Sisko fluid for wire coating analysis by applying HAM. Recently Zeeshan et al. [25] used PhanThien Tanner fluid in double-layer optical fiber coating. The same author [26] investigated double-layer coating of optical glass fiber using wet-on-wet coating process using PTT fluids of different viscosities for the constant pressure gradient. Zeeshan et al. [27] investigated an approximate solution for optical fiber coating in a pressure type die using two immiscible Oldroyd 8-constant fluids using OHAM.

Nowadays magneto-hydrodynamic (MHD) system are used effectively in many applications such as power generators, pumps, accelerators, wire coating, electrostatic filters, and droplet purifiers. Thus, in order to achieve

***Corresponding Author:**Zeeshan Khan, Sarhad University of Science and Information Technology, Peshawar, Pakistan.

the desired characteristics of the final product, the use of electrically conducting fluids subject to a magnetic field has gained great attention. Recently, Sajid et al. [28] studied MHD Oldroyd 8-constant fluid for wire coating.

In scrutiny of the above incentive, in the present study, weanalyze the wire coating analysis using MHD flow of a Third grade fluid.To the best of our knowledge, no previous investigation has been reported to develop the governing equations for MHD flow of a third grade fluid in the wire coating analysis. Well known mathematical techniques, namely OHAM and ADM are used for a series solution. The OHAM [20, 22, 23, 27, 33-35] is a steadfast method which has been broadly used by the researchers to solve nonlinear problems. Additionally the results are also verified by ADM [29-32]. Furthermore, the obtained results are also compared with preceding published related literature, as a special case of the problem and admirable agreement is observed in this case also.

2. Modelingof the Problem

Take an Elasto-hydrodynamic coating system in which the continuum enters between the leakage control units that is attached to the melting chamber. The continuum after crossing the melting chamber enters the plasto-hydrodynamic pressure unit. Here, the hydrodynami pressure helps to deposit a coating on the wire. The bull block after wounding a coated wire is driven by a variable speed motor. Figs. 1 and 2 show the physical model of the problem. Here, R_w is the radius of the wire and Lis the unit length (or length die).

The continuity and momentum equations for incompressible third grade fluid are [5-12, 23]

$$\nabla.u = 0, \tag{1}$$

$$\rho\frac{Du}{Dt} = \nabla.\mathbf{T} + \mathbf{J}\times\mathbf{B}, \tag{2}$$

where ρ is the fluid density, $\frac{D}{Dt}$ the material derivative, $\mathbf{T}$ the Cauchy stress tensor, $\mathbf{J}$ the current density, $\mathbf{B}$ the total magnetic field and u is the velocity vector.

$$\mathbf{T} = -\mathrm{p}\mathbf{I} + \mathbf{S}, \tag{3}$$

where $p\mathbf{I}$ denotes spherical stress and extra stress tensor $\mathbf{S}$ is given by

$$\mathbf{S} = \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1 + \tau_1\mathbf{A}_2 + \tau_2\left(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1\right) + \tau_3\left(tr\mathbf{A}_2\right)\mathbf{A}_1, \tag{4}$$

in which μ, is the coefficient of the viscosity of the fluid, $\alpha_1, \alpha_2, \tau_1, \tau_2, \tau_3$ are the material constant and A_1, A_2, A_3 are line kinematic tensors defined by

$$\mathbf{A}_1 = \mathbf{L}^T + \mathbf{L}, \mathbf{A}_n = \mathbf{A}_{n-1}\mathbf{L}^T + \mathbf{L}\mathbf{A}_{n-1} + \frac{D\mathbf{A}_{n-1}}{Dt}, n = 2,3 \tag{5}$$

where T denotes transpose of the matrix.

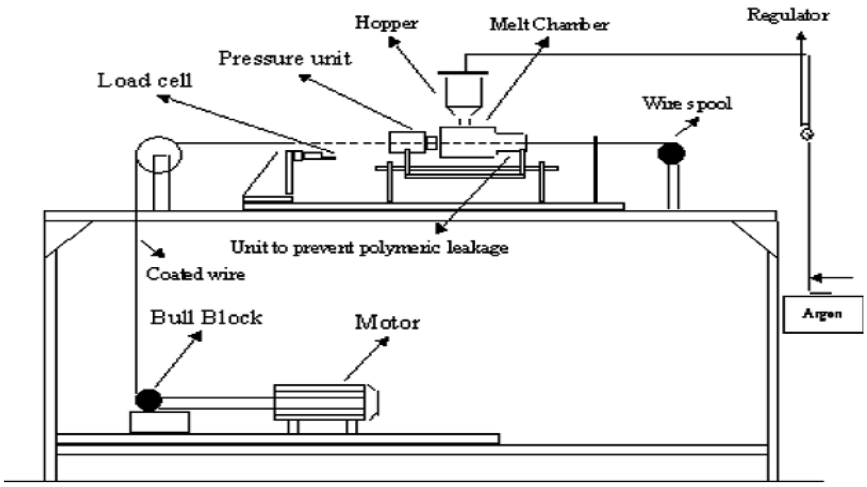

Figure1. Typical wire coating line [24].

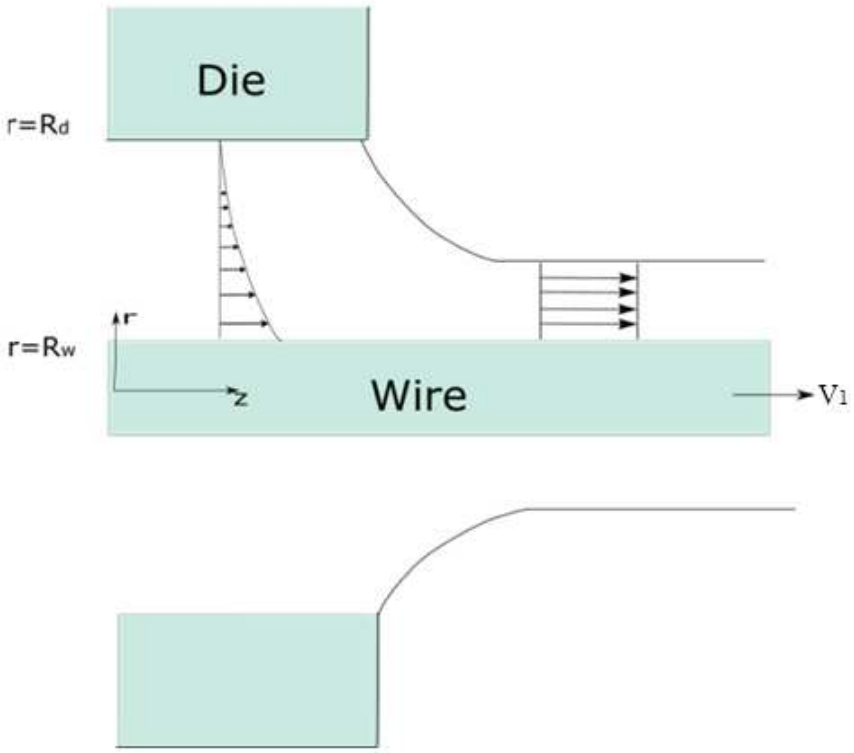

Figure. 2. Wire coating in pressure type coating die.

The flow is assumed to be linear, steady and no slippage occurs between the boundaries.
The velocity fieldis

$$u = \left[0, 0, w(r)\right], \mathbf{S} = \mathbf{S}(r). \tag{6}$$

For electrically conducting fluid, the Maxwell's equations are

$$\nabla . \mathbf{E} = 0, \nabla \times \mathbf{B} = \mu \mathbf{J}, \nabla \times \mathbf{E} = 0, \tag{7}$$

and the Ohm's law is

$$\mathbf{J} = \sigma\left(\mathbf{E} + w \times \mathbf{B}\right). \tag{8}$$

In the above equations $\mathbf{J}, \mu, \mathbf{E}, \sigma$ are the current density, magnetic permeability, electric field, electric conductivity, respectively and $\widehat{\mathbf{B}}$ is the total magnetic field so that $\mathbf{B} = \mathbf{B}_0 + b, b$ is the induced magnetic field. The magnetic field $\mathbf{B}$ normal to the velocity field w and the induced magnetic field is negligible compared with the imposed magnetic field so that the magnetic Reynolds number is small. Based on these considerations, particularly on small magnetic Reynolds number, the magneto-hydrodynamic force becomes

$$\mathbf{J} \times \mathbf{B} = -\sigma B_0^2 w. \tag{9}$$

In view of Eqs. (1-6) and (9), assuming that there is no pressure gradient along axial direction, the momentum Eq. (2) reduce to the following form

$$2\left(\tau_2 + \tau_3\right)\frac{d}{dr}\left(r\left(\frac{dw}{dr}\right)^3\right) + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) - \sigma B_0^2 w = 0, \tag{10}$$

Boundary conditions are

$$w = V_1 \text{ at } r = R_w, w = V_2 \text{ at } r = R_d, \tag{11}$$

where, R_d is the radius of the die and V_2 is the velocity of the gas surrounding the coated wire.

The average velocity is

$$w_{ave} = \frac{2}{R_d^2 - R_w^2} \int_{R_w}^{R_d} rw(r)\, dr \tag{12}$$

At some control surface downstream, the volume flow rate is

$$Q = \pi V_1 \left(R_c{}^2 - R_w^2 \right). \tag{13}$$

whereR_c is the radius of coated wire.

The volume flow rate is

$$Q = \int_{R_w}^{R_d} rw(r)\, dr. \tag{14}$$

The thickness of the coated wire can be obtained from Eq. (13) and (14) as:

$$R_c = \left[R_w^2 + \frac{2}{V_1} \int_{R_w}^{R_d} rw(r)\, dr \right]^{1/2}. \tag{15}$$

The force on the total wire is

$$S_{rz}\big|_{r=R_w} = \mu \frac{dw}{dr} + 2\tau_0 \left(\frac{dw}{dr} \right)^3 \Bigg|_{r=R_w}. \tag{16}$$

The force on the surface of the total wire is

$$F_w = 2\pi R_w L S_{rz}\big|_{r=R_w}. \tag{17}$$

Introducing the dimensionless parameters

$$r^* = \frac{r}{R_w}, w^* = \frac{w}{V_1}, \tau_0 = \tau_2 + \tau_3, \frac{R_d}{R_w} = \delta > 1, \frac{V_2}{V_1} = U, \beta = \frac{\tau_0}{\mu \left(\frac{R_w^2}{V_1^2} \right)}, M^2 = \frac{\sigma B_0^2 R_w}{\mu}. \tag{18}$$

in view of Eq. (18), the Eqs. (10-17) after dropping the asterisks become

$$r \frac{d^2 w}{dr^2} + \frac{dw}{dr} + 2\beta \left(3r \frac{d^2 w}{dr^2} \left(\frac{dw}{dr} \right)^2 + \left(\frac{dw}{dr} \right)^3 \right) - M^2 w = 0, \tag{19}$$

$$w(1) = 1, w(\delta) = U, \tag{20}$$

$$q_{avg} = \frac{w_{ave} \left(R_d^2 - R_w^2 \right)}{2 R_w V_1} = \int_1^{\delta} rw(r)\, dr, \tag{21}$$

$$q = \frac{Q}{2\pi R_w^2 V_1} = \int_1^{\delta} rw(r)\, dr, \tag{22}$$

$$R_c = \frac{h}{R_w} = \left[1 + 2 \int_1^{\delta} rw(r)\, dr \right]^{1/2}, \tag{23}$$

$$\Upsilon\big|_{r=1} = \frac{S_{rz}R_w}{\mu V_1}\bigg|_{r=1} = \frac{dw}{dr} + 2\beta\left(\frac{dw}{dr}\right)^3\bigg|_{r=1}, \tag{24}$$

$$F = \frac{F_w}{2\pi\mu L V_1} = \frac{dw}{dr} + 2\beta\left(\frac{dw}{dr}\right)\bigg|_{r=1}^3, \tag{25}$$

3. Analysis of Adomain Decomposition Method (ADM)

The Adomian Decomposition Method is used to decompose the unknown function $w(r)$ into a sum of an infinite number of components defined by the decomposition series.

$$w(r) = \sum_n^{\infty} w_n(r). \tag{26}$$

The decomposition method is used to find the components w_0, w_1, w_2 ..., separately. The determination of these components can be obtained through simple integrals.

To give a clear overview of ADM, we consider the linear differential equation in an operator1 orm as:

$$L_r w(r) + R\ w(r) + N\ w(r) = g(r), \tag{27}$$

$$L_r w(r) = g(r) - R\ w(r) - N\ w(r), \tag{28}$$

where $L_r = \dfrac{\partial^2}{\partial r^2}$ is linear in the differential equation and is easily invertible, $g(r)$ is a source term, $R\ w(r)$ is a remainder linear operator and $N\ w(r,t)$ is a nonlinear analytical term expandable in Adomian Polynomials.

Applying the inverse operator L_r^{-1} on to both sides of Eq. (28).

$$L_r^{-1}L_r w(r) = L_r^{-1}g(r) - -L_r^{-1}R\ w(r) - L_r^{-1}N\ w(r), \tag{29}$$

$$w(r) = f(r) - L_r^{-1}R\ w(r) - L_r^{-1}N\ w(r), \tag{30}$$

here, the function $f(r)$ represents the terms arising from $L_r^{-1}g(r)$ after using the given conditions Eq. (20). $L_r^{-1} = \iint(.)\,\mathrm{drdr}$ is used as an inverse operator for the second order differential equation. Similarly for higher order differential equation L_r^{-1} and L_r, depend on the order of differential equation.

Adomian Decomposition Method defines the series solution $w(r)$ as,

$$\sum_n^{\infty} w_n(r) = f(r) - L_r^{-1}R\sum_n^{\infty} w_n(r) - L_r^{-1}N\sum_n^{\infty} w_n(r). \tag{31}$$

The nonlinear term $N\sum_n^{\infty} w_n(r)$ can be expressed in term of Adomian Polynomials as

$$N\sum_n^{\infty} w_n(r) = \sum_n^{\infty} A_n, \tag{32}$$

From Eq. (31) and (32), we have

$$w_0 + w_1 + w_2 + w_3 + w_4 \ldots\ldots = f(r) - L_r^{-1}R(w_0 + w_1 + w_2 + w_3 + w_4 \ldots) - L_r^{-1}N(A_0 + A_1 + \ldots). \tag{33}$$

To determine the components w_0, w_1, w_2, w_3, w_4 ..., it is important to note that ADM suggests that the function $f(r)$ actually described the zero component w_0 which is obtained by using the boundary conditions Eq. (20).

The formal recursive relation is defined as:

$$w_0(r) = f(r), \tag{33}$$

$$w_1(r) = -L_r^{-1}R\left[w_0(r)\right] - L_r^{-1}(A_0), \tag{35}$$

$$w_2(r) = -L_r^{-1}R\left[w_1(r)\right] - L_r^{-1}(A_1), \tag{36}$$

$$w_3(r) = -L_r^{-1}R\left[w_2(r)\right] - L_r^{-1}(A_2). \tag{37}$$

And so on.

4. Analysis of Optimal Homotopy Asymptotic Method (OHAM)

OHAM has been applied successfully applied by many researchers for solving nonlinear differential equations in different areas and in particularly in fluid mechanics. One special area of application of this method is to solve equations arising when non-Newtonian fluids are studied.

For better understanding we consider

$$\mathrm{A}(w(r)) + G(r) = 0,\ \ r \in \Lambda,\ \mathrm{B}(w,\ \frac{dw}{dr}) = 0,\ r \in \Im, \tag{38}$$

where A is the differential operator and B is a boundary operator, $u_{(i)}(r)$ is the unknown function, r denote the spatial independent variable, $\Im$ is the boundary of the domain Λ and $G(r)$ is the unknown analytical function. The operator A can be written as

$$\mathrm{A} = L + N, \tag{39}$$

Where L and N are the linear and nonlinear operators respectively.

We construct a homotopy $\varnothing(r,p) : \Lambda \times [0,1] \to R$ which satisfies

$$[1-p]\left[L(\varnothing(r,p)) + G(r)\right] - H(p)\begin{bmatrix} L\left[w(r)\right] + \\ N\left[w(r)\right] + G(r) \end{bmatrix} = 0,\ \mathrm{B}\left(w(r,p), \frac{\partial w(r,p)}{\partial r}\right) = 0. \tag{40}$$

Where $r \in R$ and $p \in [0,1]$ is an embedding parameter, $H(p)$ is a non-zero auxiliary function and $\varnothing(r,p)$ is an unknown function. For $p = 0$, the homotopy given in Eq. (40) only recover the linear solution i.e.,

$$L\left((\varnothing(r,0))\right) + G(r) = 0,\ \ B(w_0, \frac{dw_0}{dr}). \tag{41}$$

For $p = 1$, we recuperate the nonlinear boundary value problem and this solution approach to the exact solution such as $\varnothing(r,1) = w(r)$. So we can say that the solution $\varphi(r,p)$ approaches to exact solution as p approaches 0 to 1.

The auxiliary function $H(p)$ isselected as

$$H(p) = pC_1 + p^2C_2 + p^3C_3 + \ldots \tag{42}$$

$C_1, C_2, C_3, \ldots$, are auxiliary constants to be resolute such that to reduce solution inaccuracy.

For estimated solution, $\varnothing(r,p)$ is expanding with respect to p by using Taylor series[31-34].

$$\varnothing(r,p,C_i) = w_0(r) + \sum_{k=1}^{\infty} w_k(r,C_i)\,p^k, \tag{43}$$

By using Eqs. (42) and (43) into equation (40), and comparing the coefficient of like power of p, we obtain various order problems, where the zeroth order problem is given in Eq. (41) and the first and second order problems are as follows:

$$L(w_1(r)) + G(r) = C_1N_0(w_0(r)),\ \mathrm{B}(w_1, \frac{dw_1}{dr}) = 0,$$

(44) $$L(w_2(r)) - L(w_1(r)) = C_2N_0(w_0(r)) + C_1[L(w_1(r)) + N_1(w_1(r))],\ \mathrm{B}(w_2, \frac{dw_2}{dr}) = 0. \tag{45}$$

In general

$$L(w_k(r)) - L(w_{k-1}(r)) = C_k N_0(w_0(r)) + \sum_{i=1}^{k-1} C_i [L(w_{k-1}(r)) + N_{k-1}(w_0(r), w_1(r), ..., w_{k-1}(r))]'$$

$$\mathrm{B}(w_k, \frac{dw_k}{dr}) = 0,\ k = 2,3,... \tag{46}$$

Here $N_{k-1}(w_0(r), w_1(r), ..., w_{k-1}(r))$ is the coefficient of p^{k-1} in the extension of $N(\varnothing(r, p))$.

$$N(\varnothing(r, p)) = N_0(w_0(r)) + \sum_{k-i=1}^{\infty} N_{k-i}(w_0, w_1, ..., w_{k-i}). \tag{47}$$

The junction of Eq. (43) depends upon the auxiliary constants and the order of the problem.

If it converges at $p = 1,$ one has:

$$w(r, C_i) = w_0(r) + \sum_{k=1}^{\infty} w_k((r, C_i)),\ ; i = 1,2,3,...,m. \tag{48}$$

Using Eq. (48) into Eq. (38), the expression for the residual in the following is obtained as:

$$R(r, C_i) = L(w(r, C_i) + G(r) + N(w(r, C_i)), i = 1,2,\ldots,m, \tag{49}$$

Many methods such as Ritz Method, Galerkin'sMethod, Collocation method and Least Square method are used to find the auxiliary constants.

Here we use the least squares method to find the auxiliary constant:

$$\mathrm{J}(C_i) = \int_a^b R^2(r, C_i)\, dr,\ \ ; i = 1,2,3,...,m, \tag{50}$$

$$\frac{\partial \mathrm{J}}{\partial C_i} = 0,\ ; i = 1,2,3,...,m, \tag{51}$$

here a, b (taking from domain) are constant that locate auxiliary constants which minimize the residual. Many researchers fruitfully implemented this method for solving highly non-linear boundary value problems of physics and engineering and gained pleasing outcome. As the number of the auxiliary constant increase the solution errors, reduce and a consequence the solution of the problem converges to the exact solution.

5. Solution of the problem

Here we applied optimal homotopy asymptotic method (OHAM) toEqs. (19, 20).

From Eq. (19), we have

$$L[\varphi(r, p)] = r\frac{d^2w}{dr^2} + \frac{dw}{dr}, \tag{52}$$

$$N[\varphi(r, p)] = 2\beta\left(3r\frac{d^2w}{dr^2}\left(\frac{dw}{dr}\right)^2 + \left(\frac{dw}{dr}\right)^3\right) - M^2 w, g(r) = 0, \tag{53}$$

where L, N and, g are linear, nonlinear operators, and source term respectively.

The boundary conditions are

$$\varphi(1, p) = 1 \text{ and } \varphi(\delta, p) = U. \tag{54}$$

By using Eqs. (52)-(54) and (42) into equation (40), and comparing the coefficient of like power of p, we will obtain various order problems, whereas the zero, first and second order problems with appropriate boundary conditions are as follows:

Zeroth-order problem with boundary conditions

$$p^0 : r\frac{d^2w_0}{dr^2} + \frac{dw_0}{dr} = 0, \tag{55}$$

$$w_0(1) = 1, w_0(\delta) = U, \tag{56}$$

First-order problem with boundary conditions

$$p^1 : r\frac{d^2w_1}{dr^2}+\frac{dw_1}{dr}+M^2rC_1-(C_1+1)\frac{dw_0}{dr}-2\beta C_1\left(\frac{dw_0}{dr}\right)^3$$
$$-r(C_1+1)\frac{d^2w_0}{dr^2}-6r\beta C_1\frac{d^2w_0}{dr^2}\left(\frac{dw_0}{dr}\right)^2=0, \tag{57}$$

$$w_1(1)=0, w_1(\delta)=0, \tag{58}$$

Second-order problem with boundary conditions

$$p^2 : r\frac{d^2w_2}{dr^2}+\frac{dw_2}{dr}+M^2rC_2-C_2\frac{dw_0}{dr}-2\beta C_2\left(\frac{dw_0}{dr}\right)^3-\frac{dw_1}{dr}(1+C_1)-r\frac{d^2w_0}{dr^2}(1+C_2)s-6r\beta C_1\frac{dw_1}{dr}$$
$$\left(\frac{dw_0}{dr}\right)^2-6r\beta C_2\frac{d^2w_0}{dr^2}\left(\frac{dw_0}{dr}\right)^2-12r\beta C_1\frac{d^2w_0}{dr^2}\frac{dw_0}{dr}\frac{dw_1}{dr}-r\frac{d^2w_1}{dr^2}-rC_1\frac{d^2w_1}{dr^2}-6r\beta C_1\frac{d^2w_1}{dr^2}\left(\frac{dw_0}{dr}\right)^2=0, \tag{59}$$

$$w_2(1)=0, w_2(\delta)=0. \tag{60}$$

The corresponding solutions of equations (55)-(60) are:

$$w_0=1+\frac{lnr}{ln\delta}(U-1), \tag{61}$$

$$\mathrm{w}_1=r^2\Lambda_1+\frac{1}{r^2}\Lambda_2+\Lambda_3 lnr, \tag{62}$$

$$w_2=r^2\Lambda_4+\frac{1}{r^2}\Lambda_5+\frac{1}{r^4}\Lambda_6+\Lambda_7 lnr, \tag{63}$$

where $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_5, \Lambda_6,$ and Λ_7 are constants which hold auxiliary constants C_1 and C_2 are given in appendix.

Now, since the second order approximation is:

$$\mathrm{w}=\mathrm{w}_0(r)+\mathrm{w}_1(r)+\mathrm{w}_2(r). \tag{64}$$

The second order approximate solution is given by

$$w=r^2(\Lambda_1+\Lambda_4)+\frac{1}{r^2}(\Lambda_2+\Lambda_5)+\frac{1}{r^4}\Lambda_6+(\Lambda_3+\Lambda_7)lnr. \tag{65}$$

5. RESULTS AND DISCUSSION

The theoretical model of a third grade fluid for wire coating analysis inside an annular die is considered. The fluid is electrically conducted in the presence of uniform magnetic field. The analytical solution for the occurring nonlinear differential equation in this model is obtained. The series form of the solution is found using OHAM.

The convergence of the method is also necessary to check the reliability of the methodology. The convergence of the method is given in tables by assigning numerical values to the physical parameters of interest. The convergence of method can also be observed from figure 3. Additionally, the results are also compared with previously published relevant literature [23] of the same flow problem of a third grade fluid as depicted in Fig.4.

The influence of the non-Newtonian parameter of third grade fluid β, the velocity ratio U, the wire velocity V_1 and the radii ratio δ on the velocity and thickness of the coated wire are studied. In order to get a clear insight into physical problem, the velocity profile and thickness of coated wire has been discussed by

assigning the numerical values to the non-dimensional parameters encountered in figures 5-9. Figure 5 shows the variation of the magnetic parameter M on the non-dimensional velocity profile. Here, we vary the magnetic parameter M i.e., $M = 0.5, 1.0, 1.5, 2.0$ with fixed values for $\beta = 0.2, \delta = 2, U = 0.2.$ This figure reveals that increasing M reduces the speed of the flow inside the polymer coating. This is due to the fact that the introduction of transverse magnetic field has tendency to develop a drag force that resists the flow. Figure 6 and 7 show the behavior of non-dimensional velocity with the variation of non-Newtonian parameters β and the velocity ratio U respectively. Figure 6 has been obtained by varying the dimensionless parameter β i.e., $\beta = 0.2, 0.6, 1.2, 1.6$ and keeping all other parameters fixed. It is clear from figure 6 that the behavior of increasing β (keeping M fixed) on the velocity is quite similar to that of M (shown in Figure 5). Also, the boundary layer thickness is decreased by increasing β. Figure 7 shows that the velocity profile increases by increasing the velocity ratio U. Figure 8 and 9 show the variation of the radii ratio δ and the non-Newtonian parameter β_1 on the thickness of the coated wire against the velocity of the wire V_1 respectively. Figure 8 presents the impact of the increase in the radii ratio δ (ratio of the die radius to the radius of the wire) along with increasing wire drawing speed, on the thickness of coated wire. Here, we observed that by increasing the radii ratio (specially the coating die radius) significantly affects the thickness of the coated1 ire. It is found that the change in wire drawing speed is reflected less sensitively on the change in thickness of coated wire, especially when the diameter of the coating die is small. In Fig. 9 we varied the non-dimensional parameter β i.e., $\beta = 0.1, 0.2, 0.3, 0.4$ and fixed the values of $U = 0.2, M = 0.1.$ It demonstrates that the thickness of the coated wire decrease with the increasing values of β, however, the thickness of the coated wire increases along with increasing the radii ratio δ. Here, we can observe that the thickness of the coated wire can be maintained at required level by adjusting the non-diemnsional parameter β, the radii ratio δ, and the wire drawing speed V_1.

Table 1.Convergence of the method for $\alpha = 0.3, \beta = 0.2, M_i = 0.1, \delta = 2.$

r	1st Order	2nd Order
1	0	0
1.1	7.51E-14	7.93E-16
1.2	2.77E-12	2.21E-14
1.3	1.73E-11	1.11E-13
1.4	5.02E-11	2.46E-13
1.5	9.34E-11	3.12E-13
1.6	1.28E-10	2.43E-13
1.7	1.39E-10	1.15E-13
1.8	1.23E-10	1.40E-14
1.9	-7.50E-11	1.97E-14
2	1.95E-11	2.26E-13

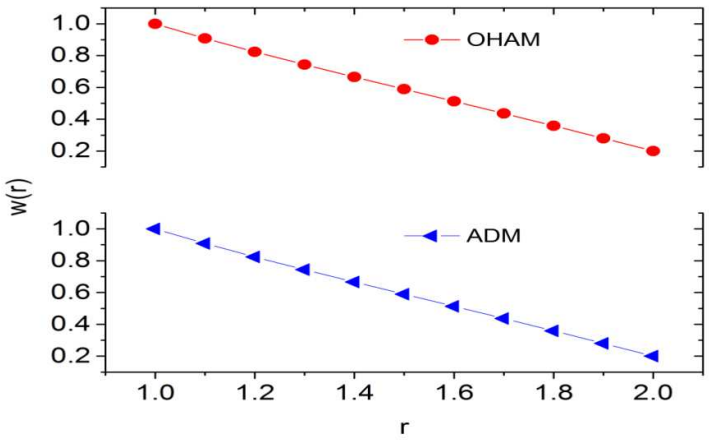

Figure 3: Velocity comparison of ADM and OHAM
$\beta = 0.3, \delta = 2, C_1 = -0.0367873, C_2 = 0.7067062, M = 0.1, U = 0.2.$

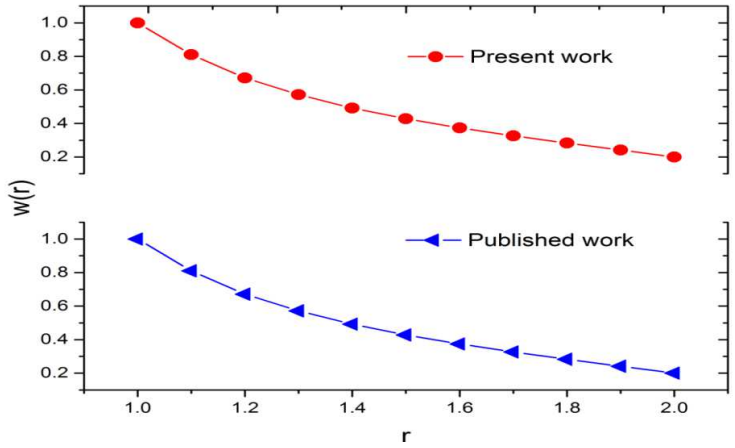

Figure 4: Velocity comparison of the present work and published work [23]for $\beta = 0.3, \delta = 2, C_1 = -0.0367873, C_2 = 0.7067062, M = 0.01, U = 0.2.$

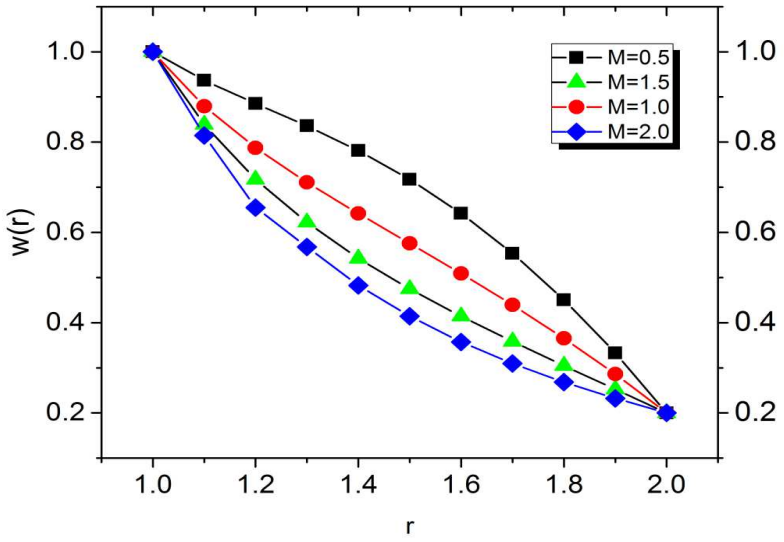

Figure5: Dimensionless velocity profile for different values of magnetic parameter M **when** $\beta = 0.2, \delta = 2, U = 0.2.$

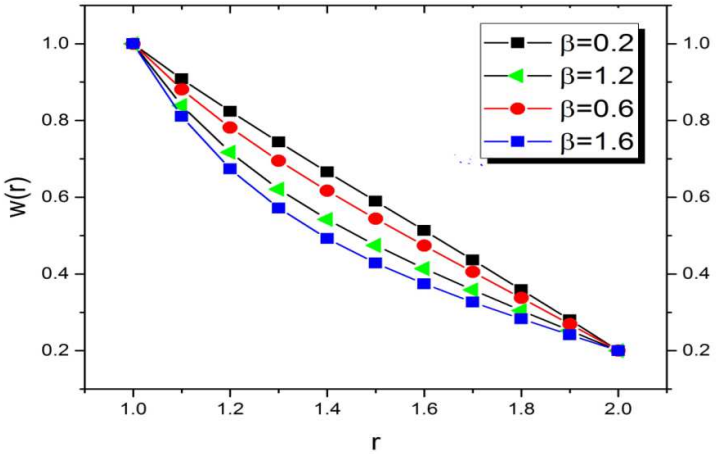

Figure6:Dimensionless velocity profile for different values of non-Newtonian parameter β **when** $\delta = 2, M = 0.1, U = 0.2..$

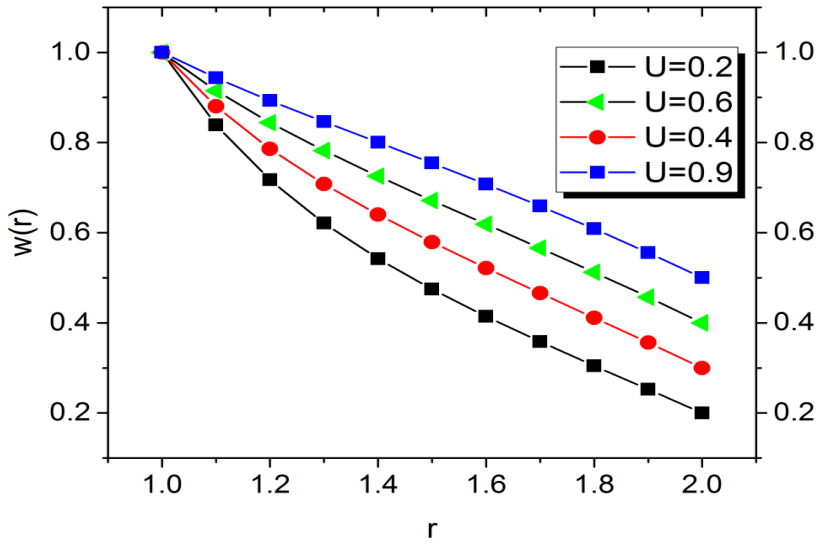

Figure7: Dimensionless velocity profiles for different values of U when $\beta = 0.2, \delta = 2, M = 0.2$.

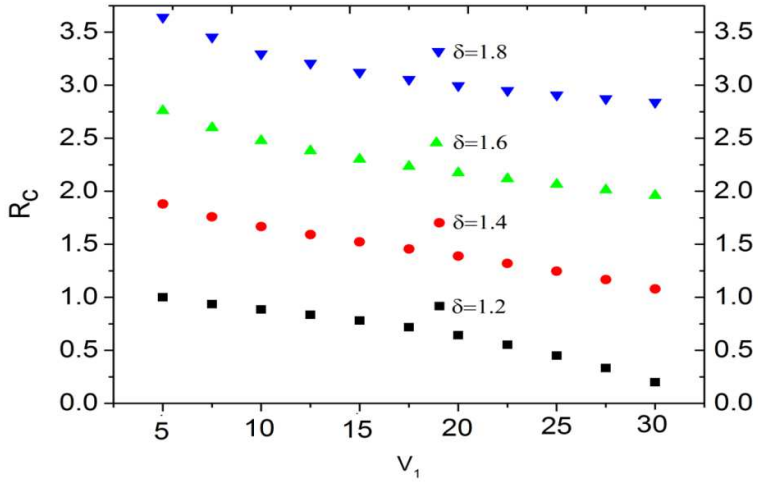

Figure 8: Thickness of coated wire for different alues of radii ratio δ when fixed $\beta = 0.3, U = 0.2, M = 0.1.$

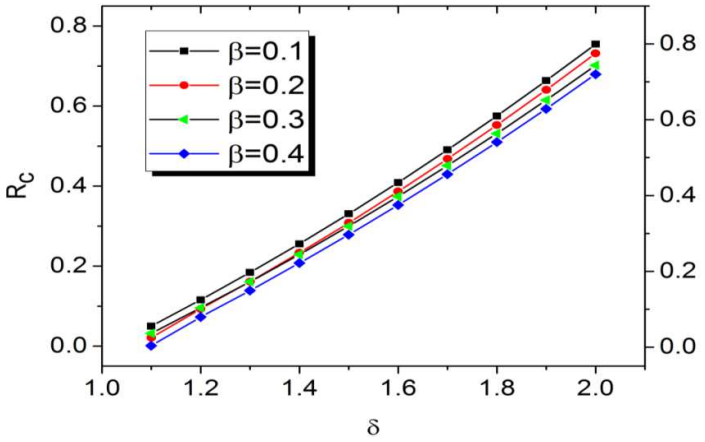

Figure 9:Thickness of coated wire for different values of β verses δ when fixed $U = 0.2, M = 0.1.$

6. Conclusion

The wire coating is necessary in order to provide protection from signal attenuation and mechanical damage. In the present analysis, the problem of wire coating by withdrawing from a bath of magneto-hydrodynamic (MHD) third grade fluid is investigated. The solution of the governing nonlinear problem is established using OHAM. Results are also verified by ADM. The effect of emerging parameters on the velocity and thickness of coated wire is discussed in detail. Additionally, the present work is also compared with published result.

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Appendix

$$\Lambda_1 = -\frac{1}{4}M^2 C_1$$

$$\Lambda_2 = \beta C_1 - 3U\beta C_1 + 3U^2\beta C_1 - U^3\beta C_1$$

$$\Lambda_3 = \frac{1}{4\delta^2 \ln \delta^4}(-4\beta C_1 + 12U\beta C_1 - 12U^2\beta C_1 + 4U^3\beta C_1 + 4\beta\delta^2 C_1 - 12U\beta\delta^2 C_1 + 12U^2\beta\delta^2 C_1$$
$$-4U^3\beta\delta^2 C_1 - M^2\delta^2 \ln \delta^3 C_1 + M^2\delta^4 \ln \delta^3 C_1)$$

$$\Lambda_4 = \frac{1}{4}(-M^2 C_1 - M^2 C_1^2 - M^2 C_2)$$

$$\Lambda_5 = \frac{1}{4\delta^4 \mathrm{Log}[\delta]^7}((4\beta C_1 - 12U\beta C_1 + 12U^2\beta C_1 - 4U^3\beta C_1 + 4\beta C_1^2 + 3M^2\beta C_1^2 - 12U\beta C_1^2 - 6M^2U$$
$$\beta C_1^2 + 12U^2\beta C_1^2 + 3M^2U^2\beta C_1^2 - 4U^3\beta C_1^2 - 3M^2\beta\delta^2 C_1^2 + 6M^2U\beta\delta^2 C_1^2 - 3M^2U^2\beta\delta^2 C_1^2 + 4\beta C_2 -$$
$$12U\beta C_2 + 12U^2\beta C_2 - 4U^3\beta C_2)\delta^4 \ln \delta^4 + (12\beta^2 C_1^2 - 60U\beta^2 C_1^2 + 120U^2\beta^2 C_1^2 - 120U^3\beta^2 C_1^2 + 60U^4$$
$$\beta^2 C_1^2 - 12U^5\beta^2 C_1^2)\delta^2 \ln \delta - (12\beta^2 C_1^2 + 60U\beta^2 C_1^2 - 120U^2\beta^2 C_1^2 + 120U^3\beta^2 C_1^2 - 60U^4\beta^2 C_1^2$$

$$\Lambda_6 = \frac{\beta^2 C_1^2}{4 \ln \delta^3}(12 - 60U + 120U^2 - 120U^3 + 60U^4 - 12U^5)$$

$$\Lambda_7 = \frac{1}{4\delta^4 \ln \delta^7}((M^2 + M^2\delta^2)\delta^4 \ln \delta^6 C_1 + (-12 + 60U - 120U^2 + 120U^3 - 60U^4 + 12U^5 + 24r^4\delta^2 -$$
$$120U\delta^2 + 240U^2\delta^2 - 240U^3\delta^2 + 120U^4\delta^2 - 24U^5\delta^2 - 12\delta^4 + 60U\delta^4 - 120U^2\delta^4 + 120U^3\delta^4 - 60U^4\delta^4$$
$$+12U^5\delta^4)\beta^2 C_1^2 + (-12 + 60U - 120U^2 + 120U^3 - 60U^4 + 12U^5 + 12r^4\delta^4 - 60U\delta^4 + 120U^2\delta^4 - 120U^3\delta^4$$
$$+60U^4\delta^4 - 12U^5\delta^4)\beta^2 \ln \delta C_1^2 + (-4 - 3M^2 + 12U + 6M^2U - 12U^2 - 3M^2U^2 + 4U^3 + 4\delta^2 + 6M^2\delta^2 -$$
$$12U\delta^2 - 12M^2U\delta^2 + 12U^2\delta^2 + 6M^2U^2\delta^2 - 4U^3\delta^2 - 3M^2\delta^4 + 6M^2U\delta^4 - 3M^2U^2\delta^4 - M^2\delta^2 + M^2\delta^4)$$
$$\beta\delta^2 \ln \delta^3 C_1^2 + (-4 + 12U - 12U^2 + 4U^3 + 4\delta^2 - 12U\delta^2 + 12U^2\delta^2 - 4U^3\delta^2 - M^2\frac{1}{\beta}\delta^2 +$$
$$M^2\frac{1}{\beta}\delta^4 \ln \delta^3)\beta\delta^2 \ln \delta^3 C_2$$