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Investigation of Viscoelastic Fluid with Uniform and Non-Uniform Magnetic Source

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ABSTRACT

This paper is analyzed for magnetohydrodynamic flow of Maxwell fluid on exponentially moving plate. Both plate and fluid accelerates in variable velocity $V(0,t)=B_0H(t)Exp(\alpha t)$. The solutions of governing equations have been perused analytically under the influence of mathematical transformations (Fourier and Laplace). The expressions of velocity and shear stress are generalized for limiting cases under the existence and non-existence of magnetic field. The particularized cases have been traced out for Newtonian fluids. The graphs are depicted for rheological parameters on the fluid flows in which four types of models are described namely (i) Maxwell fluid in the existence of magnetic field (ii) Maxwell fluid in the non-existence of magnetic field (iii) Newtonian fluid in the existence of magnetic field and (iv) Newtonian fluid in the non-existence of magnetic field.
KEYWORDS: MHD Maxwell fluid, mathematical transformations, analytic results, graphical analysis.

1. INTRODUCTION

Rheology of non-Newtonian fluid is completely differed from Newtonian fluids because rheology of nonlinear fluids is of special interest due to practical and technological applications in engineering and industry. Various models of fluids have been consulted in order to check the mysteries of nature of non-Newtonian fluids. Among these models, the integral type, the rate type and the differential type have diverted the interest of researchers of fluid mechanics. For these models, the differential type model is subclass model termed as viscoelastic model (Maxwell model). Maxwell model has capacity of description of differences of normal stress [1-2]. Having in mind the significance of non-Newtonian fluid, we selected problem on non-Newtonian fluid under the existence and non-existence of magnetic field under the influence of accelerated plate of Maxwell fluid. Nazar et al. worked on the second grade fluid on oscillating plate for exact solutions without considering magnetic field using mathematical transformations [3]. Nandkeolyar et al. explored chemically reactive fluid and heat radiating for heat and mass transfer flow under the influence of ramped wall temperature with flat porous plate [4]. In last few decades, non-Newtonian fluids are being obeyed for their extensive applications in pharmaceutical, chemical, and cosmetic industries for instance, in the manufacture of numerous oil and gas, chemicals, Syrup, paint, cleanser, juice, deodorizer and many others. There is no denying fact that non-Newtonian fluids lead superior challenge to mathematicians and engineers. The characteristics of non-Newtonian fluids even cannot be described by Navier-Stokes' equations [5]. The study of flow of electrically conducting fluid is termed as magnetohydrodynamics (MHD) which is very much important in industrial processes this is due to the fact that influence of a magnetic field on fluid flows. The effects of magnetic field are highly useful in purification of crude oil, magnetic materials processing, glass manufacturing, paper production, MHD electrical power generation, geophysics and many others [6]. Under constant mass diffusion the analytical solution for the flow of vertical plate is investigated by [7]. The vertical plate with Newtonian heating for the magnetohydrodynamic flow with oscillation is obtained by [8]. Here we include various other investigations for magnetohydrodynamic flow problems which are most recently studied by [9-23]. By motivations of above investigations, our aim is to analyze magnetohydrodynamic flow of Maxwell fluid on exponentially moving plate. Both plate and fluid accelerates in variable velocity $V(0,t)=B_0H(t)Exp(\alpha t)$. The solutions of governing equations have been perused analytically under the influence of mathematical transformations (Fourier and Laplace). The expressions of velocity and shear stress are generalized for limiting cases under the existence and non-existence of magnetic field. The particularized cases have been traced out for Newtonian fluids. The graphs are depicted for rheological parameters on the fluid flows in which four types of models are described namely (i) Maxwell fluid in the existence of magnetic field (ii) Maxwell fluid in the non-existence of magnetic field (iii) Newtonian fluid in the existence of magnetic field and (iv) Newtonian fluid in the non-existence of magnetic field.

2. FLOW EQUATIONS

The basic constitutive equations for electrically conducting flow of an incompressible fluid in the absence of body forces are

$$\nabla.\vec{\boldsymbol{V}}=0, \qquad \rho(\vec{\boldsymbol{V}}.\nabla)\vec{\boldsymbol{V}}+\sigma M_0^2\vec{\boldsymbol{V}}=-\rho\vec{\boldsymbol{V}}_t, \tag{1}$$

Where, ρ, $\vec{\boldsymbol{V}}$, σ, $\boldsymbol{M_0}$, t, $\boldsymbol{\nabla}$ *is*density, velocity field, electrical conductivity of the fluid, applied magnetic field, time, dell operator respectively. It is pointed out that the fluid is electrically conducting in the existence of unvarying magnetic field. The Cauchy stress is given by

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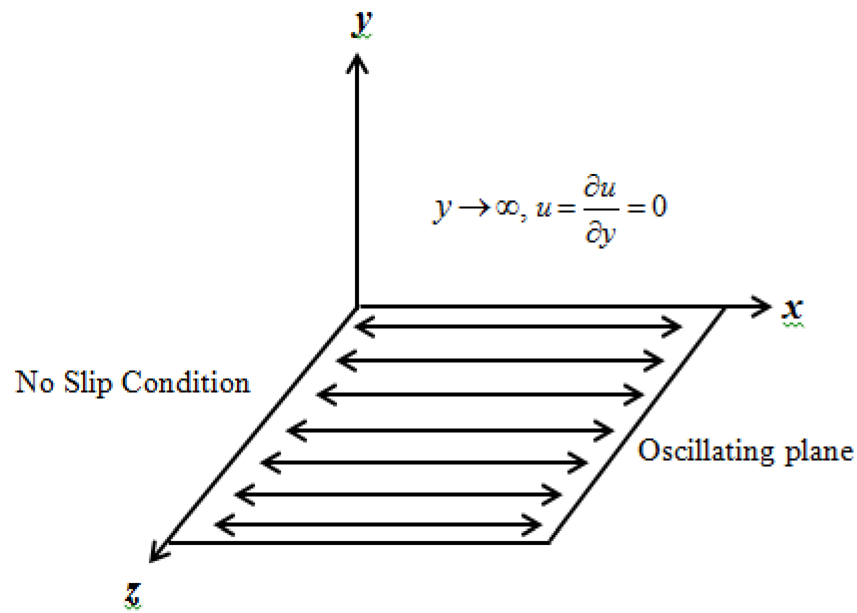
$$\boldsymbol{T} = \boldsymbol{S} - \ p\,\boldsymbol{I}, \quad \left(\dot{\boldsymbol{S}} - \boldsymbol{S}\boldsymbol{L}^{T} - \boldsymbol{L}\boldsymbol{S}\right)\lambda = \mu \boldsymbol{A} - \boldsymbol{S}, \tag{2}$$

where $\boldsymbol{T}, -p\mathbf{I}, \boldsymbol{S}, \boldsymbol{L}, \boldsymbol{A}, \mu, \Upsilon, T$ denotes Cauchy stress, indeterminate spherical stress, extra-stress tensor, velocity gradient, first Rivlin Ericksen tensor, dynamic viscosity of the fluid, relaxation time, transpose operation. The model (2) encompasses as distinctive case the Newtonian fluid model when $\Upsilon \to 0$. we take up extra-stress tensor **S** and velocity field **V** as

$$\boldsymbol{V} = \boldsymbol{V}(y,t) = u(y,t)\boldsymbol{i}\,, \qquad \boldsymbol{S} = \boldsymbol{S}(y,t), \tag{3}$$



Geometrical Configuration of the Problem.

For such flows, limitation of incompressibility is automatically fulfilled. If the fluid is at rest up to the moment $t = 0$, then

$$\boldsymbol{S} = (y,0) = 0, \qquad \boldsymbol{V} = (y,0) = 0\,, \tag{4}$$

imply $S_{zz} = S_{yz} = S_{yy} = S_{xz} = 0$, and

$$V_y(y,t)\mu - \tau(y,t) - \Upsilon\tau_t(y,t) = 0\,, \tag{5}$$

While non existence of body forces, the sense of balance of linear momentum decreases to

$$\tau_t(y,t) - \sigma M_0^2 V(y,t) - p_x - \rho V_t(y,t) = 0, -p_y - \sigma M_0^2 v(y,t) = 0, -p_z - \sigma M_0^2 V(y,t) = 0. \tag{6}$$

Under simplifications(5) and $(6)_1$, we find the governing equation under the form

$$V_y(y,t) + \Upsilon V_{yt}(y,t) + \frac{1}{\rho}(p_x + \Upsilon p_{xt}) - vV_{tt}(y,t) + \frac{\sigma M_0^2}{\rho}\left(V(y,t) + \Upsilon V_t(y,t)\right) = 0, y, t > 0, \tag{7}$$

The governing equations to MHD Maxwell fluid are

$$V_y(y,t) + \Upsilon V_{yt}(y,t) - vV_{tt}(y,t) + MV(y,t) + \Upsilon V_t(y,t) = 0, \tag{8}$$

$$\tau(y,t) + \Upsilon\tau_t(y,t) - \mu V_y(y,t) = 0, \tag{9}$$

where $M = \frac{\sigma M_0}{\rho}$.

3. MATHEMATICAL FORMULATION OF FLUID FLOW

Consider flow problem of electrically conducting Maxwell fluid owning the space on an infinitely stretched plane that is positioned in xz plane and vertical to y-axis. At the beginning, the fluid is at rest and at the moment $t = 0^{+}$ the plane is begin to move in plane. The fluid above the plane is progressively moved because of shear. The governing equations are given by equations (8) and (9) having initial and boundary conditions are

$$y > 0, t \geq 0, V(0,t) = B_0 H(t) Exp(\alpha t), \frac{\partial V(y,0)}{\partial t} = 0, V(y,0) = 0, \tau(y,0) = 0, \tag{10}$$

where $H(t)$ is the Heaviside function. Moreover, the natural conditions

$$t > 0, \quad As\ \ y \to \infty\ \ then\ \ V(y,t), \frac{\partial V(y,t)}{\partial t} \to 0, \tag{11}$$

have to be also satisfied.

4. DETERMINATION OF PROBLEM

4.1 CALCULATION OF VELOCITY FIELD

Using Fourier sine transform on (8)and (10), we get

$$V_{st}(\eta,t)+\Upsilon V_{stt}(\eta,t)+\eta^2\nu V_s(\eta,t)-B_0\sqrt{\frac{2}{\pi}}\eta\nu\, H(t)Exp(\alpha t)+M\left(1+\Upsilon\frac{\partial}{\partial t}\right)V_s(\eta,t)=0, \tag{12}$$

has to fulfill the following expressions

$$\eta>0, V_s(\eta,0)=V_{st}(\eta,0)=0, \tag{13}$$

By applying the Laplace transform to (12) and having in mind the initial conditions (13), we find that

$$\bar{V_s}(\eta,\Delta)=\frac{\sqrt{2}B_0\,\nu\,\eta}{\sqrt{\pi}(\Delta+\alpha)[\Upsilon\Delta^2+(1+\Upsilon M)\Delta+M+\nu\eta^2]}. \tag{14}$$

equivalently, equation (14) can be expressed as

$$\bar{V_s}(\eta,\Delta)=\frac{B_0\,\nu\,\eta}{(M+\nu\eta^2)}\sqrt{\frac{2}{\pi}}\left[\frac{1}{(\Delta+\alpha)}-\frac{\Delta(1+\Upsilon\Delta+\Upsilon M)}{(\Delta+\alpha)[\Upsilon\Delta^2+(1+\Upsilon M)\Delta+M+\nu\eta^2]}\right]. \tag{15}$$

Using inverse Fourier sine transform to (15), we get

$$\bar{V}(y,\Delta)=\frac{2B_0\nu}{\pi}\int_0^\infty\frac{\eta\sin(y\eta)}{(M+\nu\eta^2)}\left[\frac{1}{(\Delta+\alpha)}-\frac{\Delta(1+\Upsilon\Delta+\Upsilon M)}{(\Delta+\alpha)\left[\Upsilon\Delta^2+(1+\Upsilon M)\Delta+M+\nu\eta^2\right]}\right]d\eta. \tag{16}$$

Finally, we apply inverse Laplace transform to (16) with convolution product, and considering the fact

$$\int_0^\infty\frac{\eta\sin(\varphi\eta)}{\theta^2+\eta^2}d\eta=\frac{\pi}{2}e^{-\theta\varphi},\ \ \theta>0, \tag{17}$$

we find velocity field in multiple integral,

$$\begin{aligned}V(y,t)&=B_0H(t)Exp\left(\alpha t-y\sqrt{\frac{M}{\nu}}\right)-\frac{2\,B_0\,H(t)\,\nu}{\pi\,\Upsilon(\Delta_1-\Delta_2)}\int_0^\infty\int_{\mathbf{0}}^{\boldsymbol{t}}\frac{\eta\sin(y\eta)}{(M+\nu\eta^2)}Exp(t-q)\\&\times\{(1+\Upsilon\Delta_1)e^{\Delta_1 q}-(1+\Upsilon\Delta_2)e^{\Delta_2 q}\}\,d\eta\,dq\;+\frac{2\,B_0\,H(t)\,\nu\,M}{\pi(\Delta_1-\Delta_2)}\int_0^\infty\int_0^t\frac{\eta\sin(y\eta)}{(M+\nu\eta^2)}\\&\times Exp(t-q)(e^{\Delta_1 q}-e^{\Delta_2 q})\,d\eta\,dq.\end{aligned} \tag{18}$$

where

$$\Delta_1,\Delta_2=\frac{-(1+\Upsilon M)\pm\sqrt{(1+\Upsilon M)^2-4\Upsilon(M+\nu\eta^2)}}{2\Upsilon}, \tag{10}$$

are the roots of the algebraic equation $\Upsilon\Delta^2+(1+\Upsilon B)\Delta+M+\nu\eta^2=0.$

4.2 CALCULATION OF SHEAR STRESS

Applying Laplace transform to equation (9), we have

$$\bar{\tau}(y,\Delta)=\frac{\mu\bar{V_y}(y,\Delta)}{(\Upsilon\Delta+1)}, \tag{20}$$

differentiating partially equation (17) with respect to y, we get

$$\bar{V_y}(y,\Delta)=\frac{2B_0\nu}{\pi}\int_0^\infty\frac{\eta^2\cos(y\eta)}{(M+\nu\eta^2)}\left[\frac{1}{(\Delta+\alpha)}-\frac{\Delta(1+\Upsilon\Delta+\Upsilon M)}{(\Delta+\alpha)\left[\Upsilon\Delta^2+(1+\Upsilon M)\Delta+M+\nu\eta^2\right]}\right]d\eta, \tag{21}$$

Replacing equation (21) in (20), we have

$$\bar{\tau}(y,\Delta)=\frac{2B_0\nu\mu}{\pi(\Upsilon\Delta+1)}\int_0^\infty\frac{\eta^2\cos(y\eta)}{(M+\nu\eta^2)}\left\{\frac{1}{(\Delta+\alpha)}-\frac{\Delta(1+\Upsilon\Delta+\Upsilon M)}{(\Delta+\alpha)\left[\Upsilon\Delta^2+(1+\Upsilon M)\Delta+M+\nu\eta^2\right]}\right\}d\eta, \tag{22}$$

Simplifying equation (22), we obtain

$$\begin{aligned}\bar{\tau}(y,\Delta)&=\frac{B_0\,\mu\sqrt{\frac{M}{\nu}}}{(\Delta+\alpha)(1+\Upsilon\Delta)}Exp\left(-y\sqrt{\frac{M}{\nu}}\right)-\frac{2B_0\mu\nu}{\pi}\int_0^\infty\frac{\eta^2\cos(y\eta)}{(M+\nu\eta^2)}\left(\frac{1}{(\Delta+\alpha)(\Delta-\Delta_1)(\Delta-\Delta_2)}\right)d\eta\\&+\frac{2\,M\,B_0\Upsilon\,\mu}{\pi}\int_0^\infty\frac{\nu\,\eta^2\cos(y\eta)}{(M+\nu\eta^2)}\left(\frac{1}{(1+\Upsilon\Delta)(\Delta+\alpha)(\Delta-\Delta_1)(\Delta-\Delta_2)}\right)d\eta,\end{aligned} \tag{23}$$

By decomposing equation (23) as

$$\bar{\tau}(y,\Delta)=\frac{B_0\,\mu\sqrt{\frac{M}{\nu}}Exp\left(-y\sqrt{\frac{M}{\nu}}\right)}{(\Delta+\alpha)(1+\Upsilon\Delta)}-\frac{2B_0\mu\nu}{\pi}\int_0^\infty\frac{\eta^2\cos(y\eta)}{(M+\nu\eta^2)}\frac{1}{(\Delta+\alpha)}\left(\frac{1}{(\Delta-\Delta_1)}-\frac{1}{(\Delta-\Delta_2)}\right)d\eta$$

$$+\frac{2\,M\,B_0\Upsilon\,\mu}{\pi}\int_0^\infty \frac{\nu\,\eta^2\cos(y\eta)}{(M+\nu\eta^2)}\left\{\frac{1}{(\Delta-\Delta_1)(1+\Delta_1\Upsilon)}-\frac{1}{(\Delta-\Delta_2)(1+\Delta_2\Upsilon)}\right.$$
$$\left.+\frac{\Upsilon^2(\Delta_1-\Delta_2)}{(1+\Delta_1\Upsilon)(1+\Delta_2\Upsilon)(1+\Delta\Upsilon)}\right\}d\eta. \tag{24}$$

Finally, employing inverse Laplace transform on equation (24), we get shear stress in multiple integral

$$\tau(y,t)=-\frac{B_0\,\mu\,H(t)}{\Upsilon}\sqrt{\frac{M}{\nu}}Exp\left(-y\sqrt{\frac{M}{\nu}}\right)\int_0^t Exp\left(t-q+\frac{q}{\Upsilon}\right)dq-\frac{2\,B_0\,H(t)\,\nu\,\mu}{\pi\,\Upsilon(\Delta_1-\Delta_2)}\int_0^\infty\int_0^t\frac{\eta^2\cos(y\eta)}{(M+\nu\eta^2)}$$
$$\times Exp(t-q)(e^{\Delta_1 q}-e^{\Delta_1 q})d\eta\,dq\;+\frac{2MB_0H(t)\nu\mu}{\pi(\Delta_1-\Delta_2)}\int_0^\infty\int_0^t\frac{\eta^2\cos(y\eta)}{(M+\nu\eta^2)}Exp(t-q)$$
$$\times\left\{\frac{e^{\Delta_1 q}}{(1+\Delta_1\Upsilon)}-\frac{e^{\Delta_2 q}}{(1+\Delta_2\Upsilon)}+\frac{\Upsilon(\Delta_1-\Delta_2)e^{-q/\Upsilon}}{(1+\Delta_1\Upsilon)(1+\Delta_2\Upsilon)}\right\}d\eta\,dq. \tag{25}$$

5. PARTICULAR CASES

In order to check the influence of magnetic field, the Maxwell fluid is analyzed in presence and absence of magnetic field. In this connection, one can have limiting cases as enumerated below:

5.1. MAXWELL FLUID IN THE NON-EXISTENCE OF MAGNETIC FIELD $M=0$

Taking $B=0$ into equations (18) and (25)

$$V_M(y,t)=B_0H(t)Exp(\alpha t)-\frac{2\,B_0\,H(t)}{\pi\,\Upsilon(\Delta_1-\Delta_2)}\int_0^\infty\int_0^t\frac{\sin(y\eta)}{\eta}Exp(t-q)$$
$$\times\ \{(1+\Upsilon\Delta_1)e^{\Delta_1 q}-(1+\Upsilon\Delta_2)e^{\Delta_2 q}\}\,d\eta\,dq\,, \tag{26}$$
$$\tau_M(y,t)=-\frac{2\,B_0H(t)\,\mu}{\pi\,\Upsilon(\Delta_1-\Delta_2)}\int_0^\infty\int_0^t\cos(y\eta)\,Exp(t-q)(e^{\Delta_1 q}-e^{\Delta_2 q})\,d\eta\,dq, \tag{27}$$

are obtained.

5.2. NEWTONIAN FLUID IN THE EXISTENCE OF MAGNETIC FIELD $\Upsilon\neq 0$ AND $M=0$

Taking$\Upsilon=0$ into equations (19) and (26) with following truths

$$\lim_{\Upsilon\to 0}\Delta_1=-(M+\nu\eta^2)\,,\quad \lim_{\Upsilon\to 0}\Delta_2=-\infty\,,\quad and\quad \lim_{\Upsilon\to 0}\Upsilon(\Delta_1-\Delta_2)=1\,,$$

the solutions for MHD Newtonian fluid for the velocity fields and the shear stresses

$$V_N(y,t)=B_0H(t)Exp\left(\alpha t-y\sqrt{\frac{M}{\nu}}\right)-\frac{2B_0H(t)\nu}{\pi}\int_0^\infty\int_0^t\frac{\eta\sin(y\eta)}{(M+\nu\eta^2)}Exp(t-q-M-\nu\eta^2)\,d\eta dq \tag{28}$$
$$\tau_N(y,t)=-\mu\,B_0H(t)\sqrt{\frac{M}{\nu}}\;Exp\left(-\sqrt{\frac{B}{\nu}}\;y\right)-\frac{2\,B_0\,H(t)\mu}{\pi}\int_0^\infty\int_0^t\frac{\nu\,\eta^2\cos(y\eta)}{(M+\nu\eta^2)}$$
$$\times\,Exp(t-q-M-\nu\eta^2)\,d\eta\,dq\,, \tag{29}$$

are achieved.

5.3. NEWTONIAN FLUID IN THE EXISTENCE OF MAGNETIC FIELD $\Upsilon=0$ AND $M=0$

The solutions are well known in literature for Newtonian fluid investigated by Kashif [24]

6. NUMERICAL RESULTS AND CONCLUDING REMARKS

In this portion, some relevant physical aspects of analytical solutions under the existence and non-existence of magnetohydrodynamic (MHD) are discussed. The profiles of velocity and shear stress are illustrated numerically for the induced fluid motion. Several rheological parameters of our interest are discussed with respect to magnetic field. Graphs are depicted for four types of models in which comparison are shown for fluid motion under influence of magnetic field. At last following major outcomes are listed below:

- The modeled governing partial differential equations are evaluated for exact solutions from which various similar solutions can be their limiting cases, the general solutions equations (18) and (25) are expressed in terms of multiple integral, convolution product and elementary functions.
- Fig.1 represents the fluid motions is increasing on the whole domain of plane within the variation of time parameter and remaining all other rheological parameters are fixed.
- Relaxation phenomenon of fluid is displayed in Fig.2, it is clearly shown that by variations the small values of relaxation parameter, the fluid flow is scattering in shear stress while velocity profile is sequestrating and narrowing around plate.

- In Fig. 3, the velocity profile decreases with increase in viscosity, this is due to fact of shear thickening and shear thinning.
- It is pointed out from Fig.4 that as magnetic parameter increases the fluid flow is slowdown on the complete domain over accelerated plate.
- The impact of boundary condition parameter α is identical in qualitative sense.
- The comparison is worth mentioning in good agreement for four models shown in Fig.6 and 7, in which the existence and non-existence of magnetohydrodynamic (MHD) has significant impact on motion of fluid flow has reversal behavior, i-e velocity is increasing while shear stress is decreasing at two different times on the boundary layer.

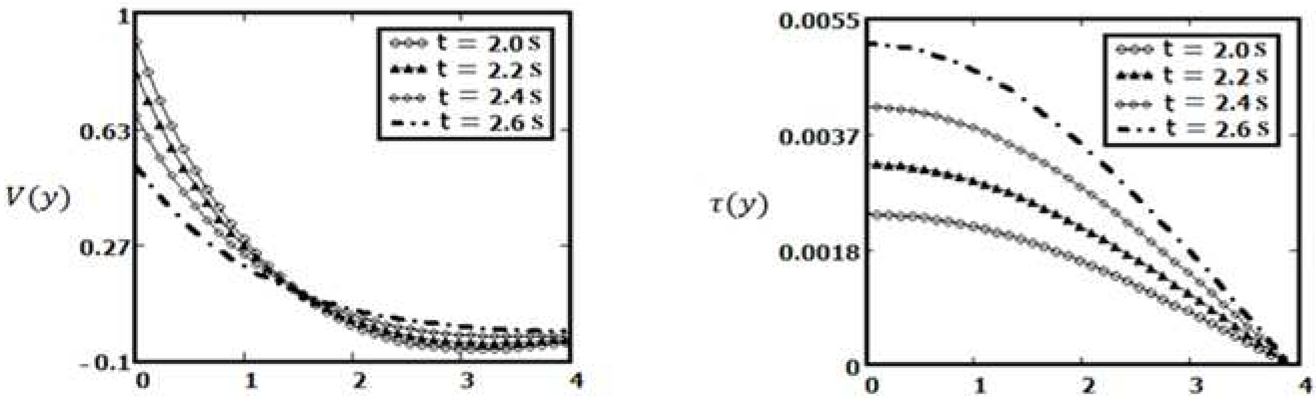


Fig. 01. Plot of velocity field and shear stress for Maxwell fluid in presence of magnetic source for different values time parameter.

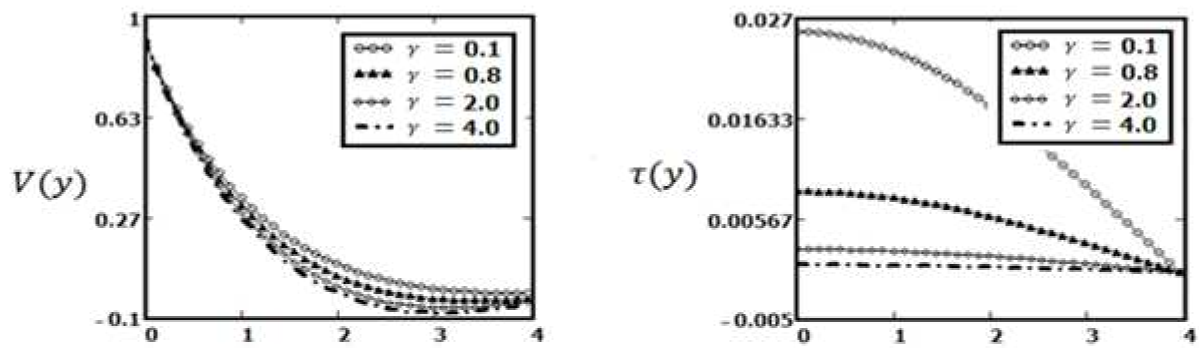


Fig. 02. Plot of velocity field and shear stress for Maxwell fluid in presence of magnetic source for different values relaxation phemenon.

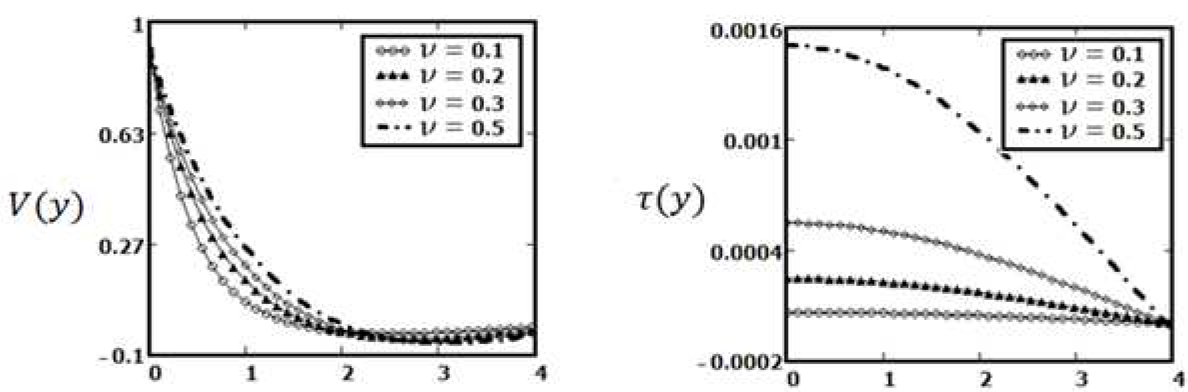


Fig. 03. Plot of velocity field and shear stress for Maxwell fluid in presence of magnetic source for different values viscosity.

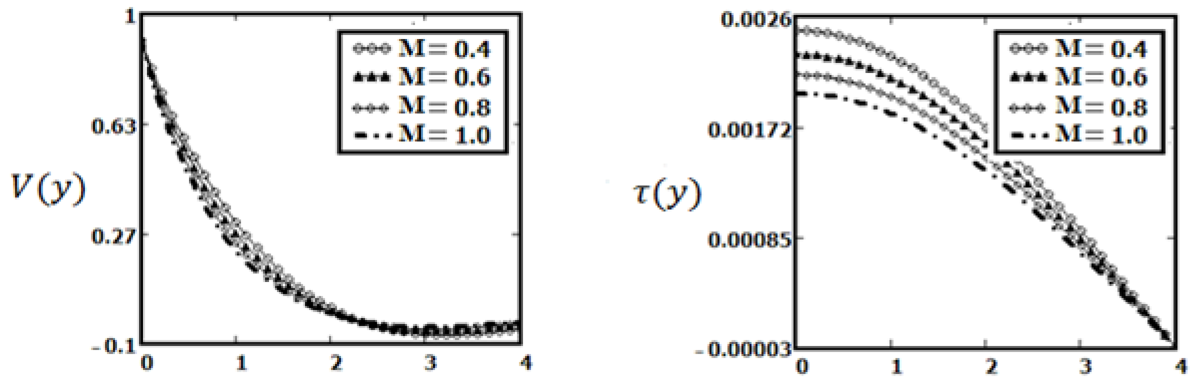


Fig. 04. Plot of velocity field and shear stress for Maxwell fluid in presence of magnetic source for different values magnetic parameter.

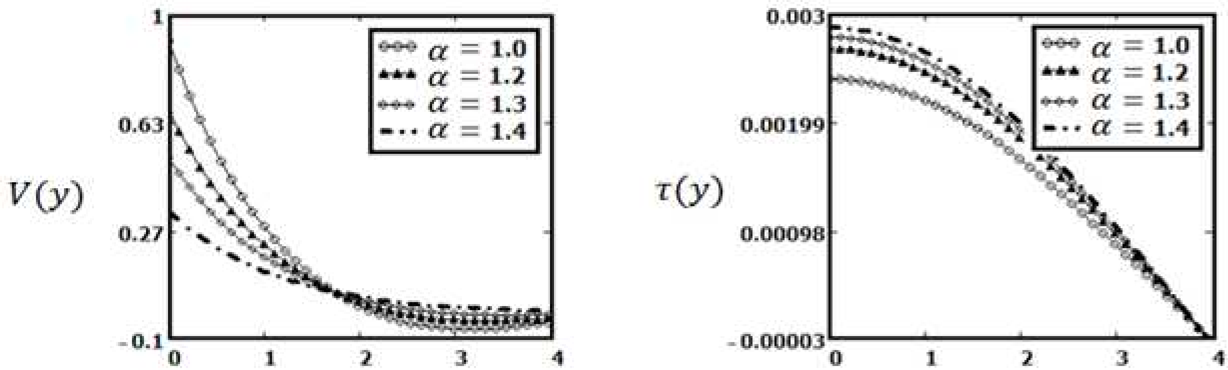


Fig. 05. Plot of velocity field and shear stress for Maxwell fluid in presence of magnetic source for different values exponential parameter.

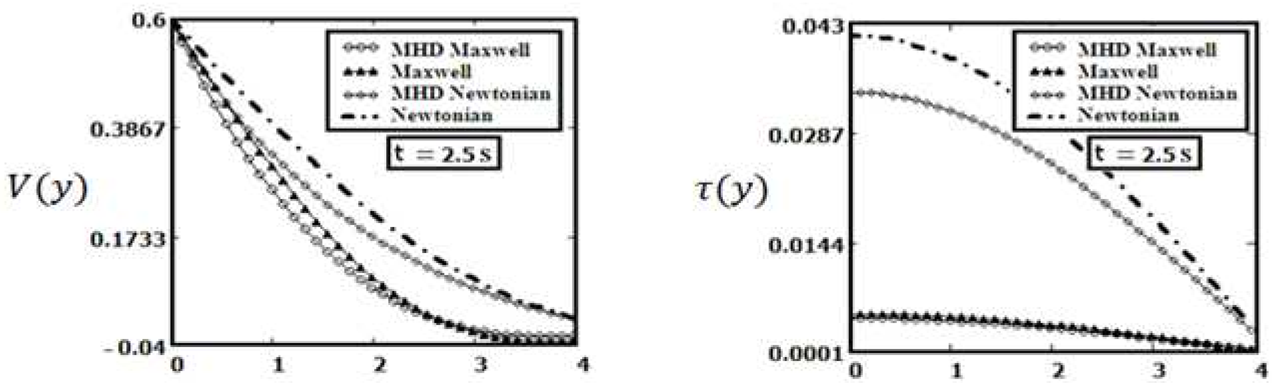


Fig. 06. Comparative anlyais of velocity field and shear stress for four models i-e (i) MHD Maxwell fluid,(ii) MHD Maxwell fluid, (iii) MHD Newtonian fluid and (i) Newtonian fluid for smaller time.

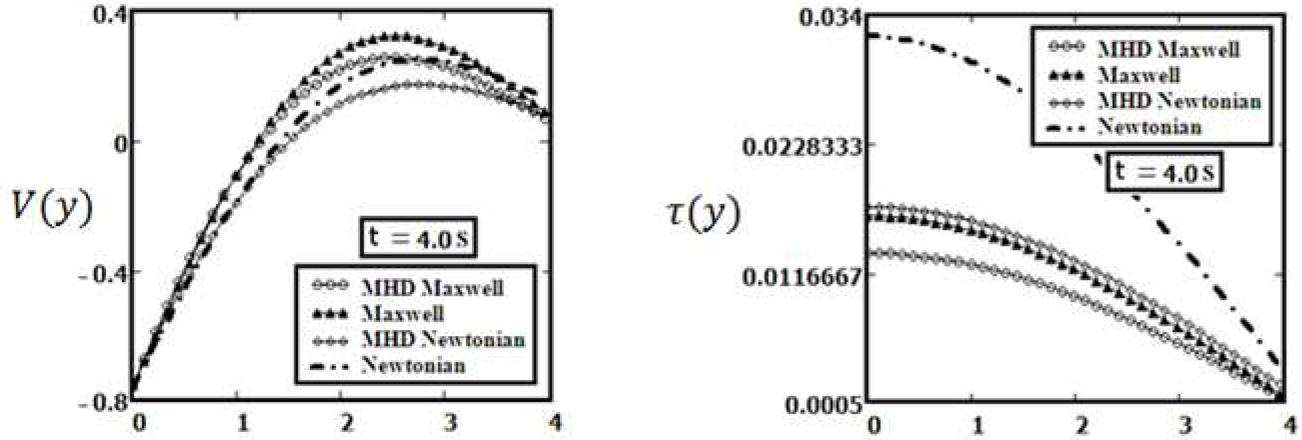


Fig. 07. Comparative anlyais of velocity field and shear stress for four models i-e (i) MHD Maxwell fluid,(ii) MHD Maxwell fluid, (iii) MHD Newtonian fluid and (i) Newtonian fluid for larger time.

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