Soft Ultra-Filters and Soft G-Filters of MTL-Algebras

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ABSTRACT

The aim of this paper is to present the ideas of soft ultra filters & soft G filters in MTL Algebras, some examples are given and some results are proved using these concepts.


1. INTRODUCTION

The logic MTL, Monoidal t-norm based logic was presented by F. Estva and L. Godo and discuss several properties of MTL-algebra in [6] The Boolean logic (BL) was introduced by P. Hajek and discuss the properties in [1]. The fuzzy reasoning and implication operators was discussed by Ying in ([2],[3]). A proper logical system for fuzzy propositional calculus and a innovative arithmetical configurations (R_0-algebras) is proposed by Wang (see [4] and [5]). In ([7],[8]) Jun and Zhang deliberate the fuzzy configuration of filters and further studied the classifications of fuzzy filters in MTL-algebras. Jun and Zhang also explored the fuzzi-fication of Boolean & MV-filters. In paper [10] Moldtsov gives the idea of soft set theory. Maji-et-al further work on soft set and defines some operations on soft set (see [11]). These operations were corrected by Ali et al [12]. X.H. Zhang et al gives the concept of fuzzy ultra filters and fuzzy G-filters in [9] as a continuation of this research paper, we additional study Soft ultra filters, soft prime filters and soft G-filters in a monoidal t-norm logic (MTL) algebras. We gives some examples and prove some results.

2. PREMİLİNİRİES

We reminiscence some concepts and their significant properties, by a lattice we mean a moderatelyorderly set in which every two components has a supremum& infimum (this may be alsoidentified aassmallest upper join &highest lower meet, respectively)

By a residuated framework by mean a lattice \( E = (E, \leq, \land, \lor, \rightarrow, 0, 1) \) having the smallest component 0 and the biggestcomponent 1, &capable with the two binary processes \( \otimes \) (called product) and \( \rightarrow \) (called residuum) like,

(i) \( \otimes \) is an isotone, commutative & associative.

(ii) \( x \otimes 1 = x \) for all \( x \in E \).

(iii) The Galois correspondence grips, which is

\[ x \otimes y \leq z \Rightarrow x \leq y \rightarrow z \]

for all \( z, y, x \in E \).

2.1 Definition [6]

A residuated framework \( E = (E, \leq, \land, \lor, \otimes, \rightarrow, 0, 1) \) is entitled an MTL-algebra if it fulfills the pre-linearity equation

\[ (x \rightarrow y) \lor (y \rightarrow x) = 1 \]

for all \( x, y \in E \).

2.2 Proposition [6, 9]

The subsequent possessions grip in any resituated lattice \( E = (E, \leq, \land, \lor, \otimes, \rightarrow, 0, 1) \)
We describe

In MTL-algebra, the subsequent are accurate

\[ \forall y \in E \quad x \not\leq (y \rightarrow x) \rightarrow x. \]

2.3 Definition [6]

Let \( E \) is MTL-algebra. A not empty subset \( F \) of \( E \) is named a filter of \( E \) if it fulfills

\[ F \]

2.4 Proposition [6]

A not empty subset \( F \) of the MTL-algebra \( E \) is the filter of \( E \) if &only if it fulfills the following

\[ F \]

2.5 Definition [5]

Let \( F \) be a not empty subset of an MTL-algebra \( E \). Then \( F \) is called a prime filter of \( E \) if, \( F \) is a proper filter and \( x \not\leq (y \rightarrow z) \rightarrow y \not\leq z \rightarrow x. \)

2.6 Definition [5]

Let \( F \) be a not empty subset of an MTL-algebra \( E \). Then \( F \) is called an ultrafilter of \( E \) if \( F \) is a proper filter and \( x \not\leq (y \rightarrow z) \rightarrow y \not\leq z \rightarrow x. \)

2.7 Definition [5]

Let \( F \) be a not empty subset of an MTL-algebra \( E \). So \( F \) is called a G-filter if

\[ F \]

2.8 Theorem

A softset \( (F, E) \) above \( U \) is a softfilter in an MTL-algebra \( E \) if it fulfills the subsequent circumstances

\[ F \]
2.9 Definition [1, 7, 8]
Suppose that $U$ is a preliminary universal set and $E$ is all likely considerations set below consideration with reverence to $U$. The power set of $U$ (i.e., the set of all sub set of $U$) is symbolized by $P(U)$ & $A$ is subset of $E$. Typically, limitations are characteristics, qualities or possessions of an substances in $U$.
A couple $(F & A)$ is named a soft set above $U$, here $F$ is a plotting given by

$$F : A \rightarrow P(U)$$

Additionally, a softset above $U$ is a parameterized intimate of sub-sets of the universal set $U$. For $e \in A$, $F(e)$ might be measured as a set of $e$ – estimated elements of a soft-set $(F, A)$.

3. SOFT FILTERS

No, we describe soft filter of a MTL-algebra. Some characterizations of a soft filter are investigated. Throughout this paper $E$ is an MTL-algebra and $U$ is not blank set.

3.1 Definition
The softset $(F, E)$ above $U$ is entitled a soft filter of an MTL-algebra $E$ if $F$ satisfies

1. $F(x \otimes y) \supseteq F(x) \cap F(y)$ for all $x, y \in E$

2. $F$ is order-preserving, that is, $x \leq y \Rightarrow F(x) \subseteq F(y)$, for all $x, y \in E$.

3.2 Example
Let $E = \{0, a, b, c, d, 1\}$, where $0 < b < a < 1$, $0 < d < a < 1$, and $0 < d < c < 1$.
Define $\otimes$ and $\rightarrow$ as follows

\[
\begin{array}{cccccccc}
\otimes & 0 & a & b & c & d & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a & 0 & b & b & d & 0 & a \\
b & 0 & b & b & 0 & 0 & b \\
c & 0 & d & 0 & c & d & c \\
d & 0 & 0 & 0 & d & 0 & d \\
1 & 0 & a & b & c & d & 1
\end{array}
\]

\[
\begin{array}{cccccccc}
\rightarrow & 0 & a & b & c & d & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 \\
a & d & 1 & a & c & c & 1 \\
b & c & 1 & 1 & c & c & 1 \\
c & b & a & b & 1 & a & 1 \\
d & a & 1 & a & 1 & 1 & 1 \\
1 & 0 & a & b & c & d & 1
\end{array}
\]

Then $E$ is an MTL-algebra. Let's describe the soft sets $(F_1, E)$, $(F_2, E)$ and $(F_3, E)$ over $E$ by

$$F_1 : E \rightarrow P(U)$$

$$F_1(x) = \begin{cases} \{a\} & \text{if } x \in \{0, c, d\}, \\ \{U\} & \text{if } x \in \{a, b, 1\}, \end{cases}$$

$$F_2 : E \rightarrow P(U)$$

$$F_2(x) = \begin{cases} \{a\} & \text{if } x \in \{0, a, b, c, d\}, \\ \{U\} & \text{if } x = 1 \end{cases}$$

and

$$F_3 : E \rightarrow P(U)$$

$$F_3(x) = \begin{cases} \{a\} & \text{if } x = 0, \\ \{U\} & \text{if } x \in \{a, b, c, d, 1\}, \end{cases}$$

where $A = \{y, z\} \subset U = \{x, y, z\}$. Then $F_1$, $F_2$ and $F_3$ are soft filters of $E$.

3.3 Theorem
The softset $(F, E)$ in $E$ is soft-filter in $E$ only if it fulfills
3.4 Boolean soft filter
A soft filter $F$ of $E$ is supposed to be a Boolean if the subsequent equality fulfills

$$F(x \lor x^*) = F(1) \quad (x \in E) \quad \text{Where} \quad x^* = x \to 0$$

3.5 Proposition
Each Boolean soft-filter $F$ of $E$ fulfills the subsequent situation

$$F(x \to z) \supseteq F(x \to (z^* \to y)) \cap F(y \to z).$$

3.6 Theorem
Let $F$ be a soft filter of $E$ then subsequent declarations are equal

1. $F$ is Boolean.

2. $F(x \to z) \supseteq F(x \to (z^* \to y)) \cap F(y \to z)$.

3. $(\forall x, y, z \in E) F(x \to y) \equiv F((x \to y) \to x)$.

3.7 MV-soft filter
A softset $(F, E)$ in $E$ is said to be MV-soft filter if it is a soft filter of $E$ that fulfills the subsequent situation

$$((\forall x, y \in E) F(x \to y) \subseteq F(((y \to x) \to x) \to y)).$$

3.8 Soft Ultra Filter
A soft filter $F$ of a MTL-algebra $E$ is said to be soft ultra filter of $E$ if $F$ fulfills the following condition

$$F(x) = F(1) \quad \text{or} \quad F(x^*) = F(1) \quad (\forall x \in E).$$

3.9 Example
Let $E = \{0, a, b, c, d, 1\}$ the residuum $\to$ & product $\otimes$ are well clear as

<table>
<thead>
<tr>
<th>$\otimes$</th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
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$\to$ & $\otimes$ are well clear as

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So $E(\land, \lor, \otimes, \to, 0, 1)$ is a MTL-algebra.

Let softset $(F, E)$ in $E$ is define as

$$F(x) = \begin{cases} E & \text{if } x \in \{1, a, d\}, \\ \{0\} & \text{otherwise}. \end{cases}$$

$F$ is soft filter of $E$ and satisfies

$$F(x) = F(1) \quad \text{or} \quad F(x^*) = F(1) \quad (\forall x \in E).$$

So $F$ is a soft ultra filter of $E$.

3.10 Soft prime filter
A soft filter $F$ of a MTL-algebra $E$ is said to be a soft prime filter of $E$ when $F$ fulfills the following condition

$$((\forall x, y \in E) F(x \lor y) \subseteq F(x) \cup F(y)).$$

3.11 Theorem
Let’s $(F, E)$ be a soft set of a MTL Algebra $E$. Then $F$ is a soft ultra filter of $E$ if and only if it satisfies:

1. $F$ is the soft Boolean filter of $E$,
2. $F$ is the soft prime filter of $E$. 

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Proof Let us assume that the soft set $F$ in $E$ is a soft Boolean & soft prime filter of $E$.

We know that

$\forall x \in E$ We have

$$F(1) = F(x \vee x^*)$$  
Definition 3.4

$$\subseteq F(x) \cup F(x^*)$$  
Definition 3.8

$$\Rightarrow F(1) \subseteq F(x) \cup F(x^*)$$

We know from Theorem 3.3

$F(x) \subseteq F(1)$ & $F(x^*) \subseteq F(1)$

We have

$F(x^*) = F(1)$ Or $F(x^*) = F(1)$

So $F$ is a soft ultra filter of $E$.

Conversely Let $F$ be a soft ultra filter of $E$. Since we know that $\forall x \in E$

$x \leq x \vee x^*$, $x^* \leq x \vee x^*$

by Definition 3.1,

$F(x) \subseteq F(x \vee x^*)$ and $F(x^*) \subseteq F(x \vee x^*)$.

By Definition of soft ultra filter we have $(\forall x \in E)$

$$F(x) = F(1) \text{ or } F(x^*) \subseteq F(1)$$

Thus,

$$F(1) \subseteq F(x \vee x^*)$$ \hspace{1cm} (A)

By (A) and Theorem 3.3, we get

$$F(x \vee x^*) = F(1)$$

Thus $F$ is a soft Boolean-filter of $E$.

Now by Proposition 2.2 $(u_z)$

$$x \vee y = ((x \rightarrow y) \rightarrow y) \land ((y \rightarrow x) \rightarrow x)$$

$$\leq (x \rightarrow y) \rightarrow y \text{ \hspace{1cm} \because } a \land b \leq a$$

By Definition d3

$$F(x \vee y) \subseteq F((x \rightarrow y) \rightarrow y)$$ \hspace{1cm} (B)

Since $0 \leq y$ and By Suggestion 2.2 $(u_6)$

by Definition 3.1,

$$F((x \rightarrow y) \rightarrow y) \subseteq F(x^* \rightarrow y)$$

From (B) we have

$$F(x \vee y) \subseteq F(x^* \rightarrow y)$$ \hspace{1cm} (1)

For several $x, y \in E$ if

$$F(x) = F(1)$$ \hspace{1cm} (C)

Then By Theorem 3.3,

$$F(x \vee y) \subseteq F(1)$$

$$= F(x) \text{ \hspace{1cm} By (C)}$$

$$\subseteq F(x) \cup F(y) \text{ \hspace{1cm} \because } a \leq a \vee b$$

$$\Rightarrow F(x \vee y) \subseteq F(x) \cup F(y).$$
By Definition 3.8, if
\[ F(x) \neq F(1)(\forall x \in E). \]
Then by Definition 3.8,
\[ F(x^+) = F(1) \ldots (D), \]
thus by Theorem 3.3,
\[ F(y) \supseteq F(x^+) \cap F(x^+ \rightarrow y) \]
\[ = F(1) \cap F(x^+ \rightarrow y) \quad \text{By } (D) \]
\[ = F(x^+ \rightarrow y) \quad \because F(x) \subseteq F(1) \]
\[ \Rightarrow F(y) \supseteq F(x^+ \rightarrow y) \ldots .(2) \]
Combining (1) and (2) we have
\[ F(x \lor y) \supseteq F(x^+ \rightarrow y) \]
\[ \subseteq F(y) \]
\[ \subseteq F(x) \cup F(y) \]
\[ \Rightarrow F(x \lor y) \subseteq F(x) \cup F(y). \]
Thus \( F \) is a soft prime filter of \( E \).

This complete the proof.

3.12 Soft G-Filter
A soft filter \( F \) of MTL algebra \( E \) is said to be the soft G-filter of \( E \) if it fulfills the subsequent situation
\[ (\forall x, y \in E), F(x \otimes x \rightarrow y) \subseteq F(x \rightarrow y) \]

3.13 Example
Let \( E = \{0, a, b, c, d, 1\} \) in which \( \rightarrow, \otimes \) is clear as

\[
\begin{array}{cccccc}
\otimes & 0 & a & b & c & d & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a & 0 & a & c & c & 0 & a \\
b & 0 & c & b & c & d & b \\
c & 0 & c & c & c & 0 & c \\
d & 0 & 0 & d & 0 & 0 & d \\
1 & 0 & a & b & c & d & 1 \\
\end{array}
\quad
\begin{array}{cccccc}
\rightarrow & 0 & a & b & c & d & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 \\
a & d & 1 & b & b & d & 1 \\
b & 0 & a & 1 & a & d & 1 \\
c & d & 1 & 1 & 1 & d & 1 \\
d & a & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & a & b & c & d & 1 \\
\end{array}
\]

So \( E(\wedge, \lor, \otimes, \rightarrow, 0, 1) \) is a MTL algebra. Let the soft-set \( (F, E) \) in \( E \) is define as
\[ F(x) = \begin{cases} E & \text{if } x \in \{1, a\}, \\ \{1\} & \text{otherwise}. \end{cases} \]
\( F \) is soft-filter of \( E \) and fulfills
\[ (\forall x, y \in E), F(x \otimes x \rightarrow y) \subseteq F(x \rightarrow y) \]
So \( F \) is a soft G-filter of \( E \).

3.14 Theorem
A soft set \( (F, E) \) in \( E \) is Boolean soft-filter of \( E \) if & only if it fulfills the following
a. \( F \) is soft G filter of \( E \).
\[ F \text{ is soft MVfilter of } E. \]
\[ F \text{ is soft MVfilter of } E. \]
\[ F \text{ is soft MVfilter of } E. \]
Proof. Let us assume that the \( F \) is soft Boolean filter of \( E \).
Meanwhile
We get
\[ y \leq ((y \rightarrow x) \rightarrow x) \rightarrow y \]

By means of
\[ ((y \rightarrow x) \rightarrow x) \rightarrow y \leq y \rightarrow x \quad \because y \leq x \Rightarrow x \rightarrow z \leq y \rightarrow z. \]

(1) \[ x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z) \]

(2) \[ x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z) \]

(3) \[ y \leq x \Rightarrow x \rightarrow z \leq y \rightarrow z \]

(4) \[ ((y \rightarrow x) \rightarrow x) \rightarrow y \leq y \rightarrow x. \]

We get
\[ x \rightarrow y \leq ((y \rightarrow x) \rightarrow x) \rightarrow ((y \rightarrow x) \rightarrow y) \]
\[ = (y \rightarrow x) \rightarrow (((y \rightarrow x) \rightarrow x) \rightarrow y) \]
\[ \leq (((((y \rightarrow x) \rightarrow x) \rightarrow y) \rightarrow x) \rightarrow (((y \rightarrow x) \rightarrow x) \rightarrow y)) \]
\[ x \rightarrow y \leq (((((y \rightarrow x) \rightarrow x) \rightarrow y) \rightarrow x) \rightarrow (((y \rightarrow x) \rightarrow x) \rightarrow y)) \]

By Theorem 3.6 and Definition 3.1,
\[ F(((y \rightarrow x) \rightarrow x) \rightarrow y) \supseteq F(((y \rightarrow x) \rightarrow x) \rightarrow y) \rightarrow x \rightarrow (((y \rightarrow x) \rightarrow x) \rightarrow y)) \]
\[ 
\]
\[ F(x \rightarrow y) \]
\[ \Rightarrow F(x \rightarrow y) \subseteq F(((y \rightarrow x) \rightarrow x) \rightarrow y). \]

Hence verify that \( F \) is a MV-softfilter of \( E \). Let \( x, y \in E \) and by Proposition 3.5 we have,
\[ F(x \rightarrow y) \supseteq F(x \rightarrow (x \rightarrow y)) \cap F(x \rightarrow x) \]
\[ = F(x \rightarrow (x \rightarrow y)) \cap F(1) \quad \because (x \rightarrow x = 1) \]
\[ = F(x \rightarrow (x \rightarrow y)) \quad \because (F \subseteq F(1)) \]
\[ = F(x \otimes x \rightarrow y) \quad \text{By Proposition (}\ u_3 \text{)} \]
\[ \Rightarrow F(x \rightarrow y) \supseteq F(x \otimes x \rightarrow y). \]

Thus \( F \) is a soft G-filter of \( E \).

Conversely

By supposition that \( F \) is a soft G-filter and soft MV-filter of \( E \).

Soby Proposition 2.2 \( (u_3) \)
\[ x \leq (x \rightarrow y) \rightarrow y \]

and by proposition 2.2 \( (u_6) \)
\[ \Rightarrow (x \rightarrow y) \rightarrow x \leq (x \rightarrow y) \rightarrow (x \rightarrow y) \rightarrow y. \]

By suggestion 2.2 \( (u_5) \)
\[ \Rightarrow (x \rightarrow y) \rightarrow x \leq ((x \rightarrow y) \otimes (x \rightarrow y)) \rightarrow y, \]

and by Definition 3.1 we have
\[ F((x \rightarrow y) \rightarrow x) \subseteq F((x \rightarrow y) \otimes (x \rightarrow y)) \rightarrow y) \]
\[ \subseteq F((x \rightarrow y) \rightarrow y) \quad \text{By Definition 3.12} \]
\[ F((x \rightarrow y) \rightarrow x) \subseteq F((x \rightarrow y) \rightarrow y) \ldots \ldots (1) \]

Now by proposition 2.2 \( (u_5) \)
\[ (x \rightarrow y) \rightarrow y \leq (x \rightarrow (x \rightarrow y)) \rightarrow (x \rightarrow y). \]

By proposition 2.2 \( (u_6) \)
By Definition 3.1,
\[
((x \rightarrow (x \rightarrow y)) \rightarrow (x \rightarrow y)) \rightarrow x \leq ((x \rightarrow y) \rightarrow y) \rightarrow x
\]
By Explanation 3.7 and Explanation 3.1 we consume
\[
F(((x \rightarrow (x \rightarrow y)) \rightarrow (x \rightarrow y)) \rightarrow x) \subseteq F(((x \rightarrow y) \rightarrow y) \rightarrow x) \quad \text{.....(A)}
\]
By Theorem 3.3,
\[
F((x \rightarrow y) \rightarrow x) \subseteq F(((x \rightarrow (x \rightarrow y)) \rightarrow (x \rightarrow y)) \rightarrow x)
\]
\[
\subseteq F(((x \rightarrow y) \rightarrow y) \rightarrow x) \quad \text{by (A)}
\]
\[
\Rightarrow F((x \rightarrow y) \rightarrow x) \subseteq F(((x \rightarrow y) \rightarrow y) \rightarrow y) \rightarrow x) \quad \text{.....(2)}
\]
From (1) and (2) we get
\[
F((x \rightarrow y) \rightarrow x) \subseteq F((x \rightarrow y) \rightarrow y) \cap F(((x \rightarrow y) \rightarrow y) \rightarrow x).
\]
By Theorem 3.3,
\[
F((x \rightarrow y) \rightarrow x) \subseteq F(x).
\]
So by Theorem 3.6 \( F \) is soft Boolean filter of \( E \).
This complete the proof.

REFERENCES