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A Study in the Existence of Long Memory in Petroleum Production of Oil-Rich Areas

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ABSTRACT

Differencing parameter has a special importance in the modeling of time series with a long memory. There are lots of techniques concerning the existence of long memory and estimation of differencing parameter. These techniques include graphical, parametric, and semi-parametric methods.
This paper deals with some of the common methods in the study of the existence of long memory. Then smoothed periodogram method, detrended fluctuation analysis method, and whittle estimator were chosen as the best ones to study the existence of long memory in USA, Persian Gulf rim countries, OPEC, OAPEC, and the world's petroleum production.
The results of this study proved that the long memory exists in USA petroleum production only.
JEL Classification: C1; C13; C14
KEY WORDS: Long Memory; Smoothed Periodogram; Detrended Fluctuation Analysis; Whittle Estimator.

INTRODUCTION

A long memory can be characterized by its autocorrelation function that decays at a hyperbolic rate. Such a decay rate is much slower than that of the time series, which has short memory. Take fractional white noise as an example such a process can be expressed as $(1-B)^d x_t = \varepsilon_t$ where ε_t is white noise. If d=0, x_t has short memory, and its ACF decreases to 0 quickly. If d=1, x_t is a random walk, and its variance is infinite. If 0 < d < 1, x_t has long memory which if 0 < d < 0.5, x_t is a long memory process with finite variance, and if $0.5 \leq d < 1$, x_t is a long memory process with infinite variance. The best-known model for modeling long memory is ARFIMA model. The ARFIMA model process was first introduced by Granjer and Joyeux (1980).

Many methods are available for detecting the existence of long memory and estimating the fractional differencing parameter. These techniques include graphical methods, parametric methods, and semi-parametric methods: Rescaled range analysis (R/S method), Log periodogram method ($\widehat{d}_{\text{GPH}}$), Smoothed periodogram approach ($\hat{d}_{\text{sp}}$), Detrended fluctuation analysis (DFA), Whittle estimator ($\hat{d}_W$), Maximum likelihood estimation (ML estimate).

An important step in building long memory models is fractional differencing, in empirical studies, most economists use first-order differencing as an alternative. Convenient as it is, such replacement will undoubtedly cause over-differencing, which will lead to the loss of information of the time series.

MATERIALS AND METHODS

ARFIMA model and fractional differencing

ARFIMA model

Let $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ be a white noise process with zero mean and variance $\sigma_\varepsilon^2 > 0$, and B the backward-shift operator, i.e., $B^k(X_t) = X_{t-k}$. If $\{X_t\}_{t\in\mathbb{Z}}$ is a linear process satisfying

$$\Phi(B)(1-B)^d X_t = \Theta(B)\varepsilon_t, \quad t\in\mathbb{Z}. \tag{1}$$

Where d ∈ (-0.5, 0.5), $\Phi(.)$, $\Theta(.)$ are polynomials of degree p and q, respectively, given by $\Phi(B) = 1-\phi_1 B - \cdots - \phi_p B^p$,$\Theta(B) = 1-\theta_1 B - \cdots - \theta_q B^q$ Where $\phi_i, 1 \leq i \leq p,\ \theta_j, 1 \leq j \leq q,$ are real constants, than $\{X_t\}_{t\in\mathbb{Z}}$ is called general fractional differentiation ARFIMA (p, d, q) process, where d is the degree or fractional differentiation parameter. If d ∈ (-0.5;0.5), then $\{X_t\}_{t\in\mathbb{Z}}$ is a stationary, and an invertible process. The most important characteristic of an ARFIMA (p, d, q) process is the property of long

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dependence, when d ∈ (0.0;0.5), short dependence, when d=0, and intermediate dependence, when d ∈ (-0.5;0.0).

Remark 1. If $\{X_t\}_{t\in\mathbb{Z}}$ is defined by expression (1), then its spectral density function is given by

$$f_X(\omega) = f_U(\omega)[2\sin\left(\frac{\omega}{2}\right)]^{-2d} \qquad 0 < \omega \le \pi \tag{2}$$

Where $f_U(.)$ denotes the spectral density function of an ARMA (p, q) process, U_t , given by $(1-B)^d x_t = U_t \quad , \quad t \in \mathbb{Z}$.

Importance of fractional differencing

Most practical time series are non-stationary, with their means and covariance fluctuating in time. Therefore, how to transform a non-stationary time series into a stationary one became an important problem in the field of time series analysis. For a long period of time, it has become a standard practice for time series analysts to consider differencing their time series to achieve stationary time series. However, econometricians were somewhat reluctant to accept this, believing that they may lose something of importance.

Over-differencing could also impact the behavior of variance. Before achieving stationary time series, the variance of series kept decreasing, and once over-differencing occurred, the variance would increase again. Accordingly, fractional difference was necessary if we wanted both stationary time series and avoidance of over-difference.

Consider an MA(1) time series $x_t = (1-\theta B)\varepsilon_t$. After over-differencing, we have $\Delta x_t = (1-B)(1-\theta B)\varepsilon_t$. Then, we can obtain
$\Delta x_t = (1-\theta_1 B - \theta_2 B^2)\varepsilon_t$. This is a more complex model, which has two parameters. Compare their variance, $V(x_t) = (1+\theta^2)\sigma^2$, and
$V(\Delta x_t) = 2(1+\theta+\theta^2)\sigma^2$. Then $V(\Delta x_t) - V(x_t) = (1+\theta)^2\sigma^2 > 0$.

The result illustrates that the variance of over-differenced time series is longer than that of the original MA (1) series. The variance will keep decreasing when the time series is getting closer to stationary series; however, once the over-differencing occurred, the variance will increase immediately.

Meaning of fractional differencing

For time series x_t, fractional differencing of order d can be illustrated by the formula $w_t = (1-B)^d x_t$
First, consider a binomial expansion of $(1-B)^d$

$$(1-B)^d = 1 - dB - \frac{d(d-1)B^2}{2!} - \cdots. \tag{3}$$

For every real number d > -1, (3) can be expressed by a hyper-geometric function

$$(1-B)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} B^k$$

The fractional differencing method proposed in this paper illustrates the essence of long memory. It shows the connection between fractional differencing parameter d and long memory. Take ARFIMA (0, d, 0) as an example such a process can be expressed as $(1-B)^d x_t = \varepsilon_t$, often called fractional white noise. When d=0, x_t is merely a white noise, and its ACF decreases to 0 quickly. When d=1, x_t is a random walk, whose value of ACF is 1, and it can be regarded as a white noise after the first order differencing. When d is non-integer, the ith element of the fractional differenced time series, is actually the weighted sum of $x_i, x_{i-1}, \ldots, x_0$, elements of the original time series. w_i is not only determined by x_i and x_{i-1} , but also influenced by all historical data ahead of x_i , this is just the characteristic of long memory. The fractional differencing method illustrates relationship between fractional differencing and long memory.

Methods for detecting the existence of long memory

Rescaled range analysis

The best known test for long memory or long range dependence is probably the rescaled range, or range over standard deviation, or simply R/S statistic, which was originally proposed by Hurst (1951).

For a given set of observations $(x_t, t \ge 0)$, with the mean over period $n\bar{X}_n = \frac{1}{n}\sum_{t=1}^{n} x_t$, and sample variance $S_n^2 = \frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x}_n)^2$, the R/S statistic is defined by

R/S(n) $= \frac{1}{S(n)} = [max_{1\le k\le n}\sum_{t=1}^{k}(x_t - \bar{x}_n) - min_{1\le k\le n}\sum_{t=1}^{k}(x_t - \bar{x}_n)]$.

For each different logn there is an R/S (n), H could then be estimated by solving for slope through an ordinary least square regression, using the formula

$\log R/S(n) = \log(c) + H \times \log(n)$.

If 0.5 < H <1, we could draw the conclusion that there is long memory in the series. H= 0.5 + d, which H is known as the Hurst coefficient to measure the long memory in x_t. The longer H is the longer memory the stationary process has. H and d can be used interchangeably as the measure of long memory.

R/S analysis is a graphical method for estimating the fractional differencing parameter d in ARFIMA (p, d, q) model.

Log periodogram method

Geweke and Porter-Hudak (1983) proposed a semi-parametric approach to the testing for long memory. This well-known and widely used approach is based on simple linear regression of the log periodogram on a deterministic regressor. Taking logarithms on both sides of (2) and evaluating the spectral density at the Fourier frequencies $\omega_{\mathrm{j}} = \frac{2\pi j}{\mathrm{n}}$, we have

$$\log f(\omega_{\mathrm{j}}) = \log \mathrm{f_U}(0) - d\log[2\sin\frac{\omega_{\mathrm{j}}}{2}]^2 + \log\left[\frac{f_U(\omega}{f_U(0)}\right] \tag{4}$$

On the other hand, the logarithm of the periodogram $I(\omega_{\mathrm{j}})$ may be written as

$$\log I(\omega_{\mathrm{j}}) = \log[\frac{I(\omega_{\mathrm{j}})}{f(\omega_{\mathrm{j}})}] + \log f(\omega_{\mathrm{j}}) \tag{5}$$

Now, combining (4) and (5) we have

$$\log I(\omega_{\mathrm{j}}) = \log \mathrm{f_U}(0) - d\log[2\sin\frac{\omega_{\mathrm{j}}}{2}]^2 + \log\left\{\frac{I(\omega_j)[2\sin\frac{\omega_j}{2}]^{2d}}{f_U(0)}\right\}$$

By defining $y_{\mathrm{j}} = \log I(\omega)_{\mathrm{j}}$, $\alpha = \log \mathrm{f_U}(0)$, $\beta = -d$, $x_{\mathrm{j}} = \log[2\sin\frac{\omega_{\mathrm{j}}}{2}]^2$, and $\varepsilon_{\mathrm{j}} = \log\{\frac{I(\omega_{\mathrm{j}})[2\sin\frac{\omega_{\mathrm{j}}}{2}]^{2d}}{f_U(0)}\}$, we obtain the regression equation $y_{\mathrm{j}} = \alpha + \beta x_{\mathrm{j}} + \varepsilon_{\mathrm{j}}$.

The least squares estimate of the long memory parameter d is given by

$$\hat{d}_{\mathrm{GPH}} = -\frac{\sum_{j=1}^{g(n)}(x_j - \bar{x})(y_j - \bar{y})}{\sum_{\mathrm{j}=1}^{\mathrm{g(n)}}(\mathrm{x_j} - \bar{\mathrm{x}})^2}$$

Where $0 < \alpha < 1$, $g(n) = n^{\alpha}$, $\bar{x} = \left(\frac{1}{g(n)}\right)\sum_{j=1}^{\mathrm{g(n)}} \mathrm{x_j}$, $\bar{\mathrm{y}} = \left(\frac{1}{g(n)}\right)\sum_{j=1}^{\mathrm{g(n)}} \mathrm{y_j}$.

For simple fractional processes denoted ARFIMA (0,d,0), this method is proper.

Smoothed periodogram approach

The semi-parametric smoothed periodogram method, proposed by Reisen (1994; see also Reisen, Abraham, & Toscano, 2000).

The regression estimator $\hat{d}_{\mathrm{sp}}$ is obtained by replacing the periodogram function in expression (5) by the smoothed periodogram function with the parzen lag window. The parameter m in the lag window generator, usually referred to as the truncation point, is a function of the sample size chosen as $m = n^{\beta}$, for $0 < \beta < 1$. The value of $g(n)$ is chosen as in the $\hat{d}_{\mathrm{GPH}}$ method. In contrast to the GPH method which constrains d not to be greater than 0.5, the smoothed periodogram method is not restricted in this way. Therefore, it can be applied directly to non-stationary data.

Detrended fluctuation analysis methodology

In order to determine the presence of long memory in time series, the detrended fluctuation analysis (DFA) proposed by peng et al. (1994) is applied. Shieh (2006) argues that DFA permits the detection of embedded intrinsic self-similarity when compared with other methods such as the Fourier analysis. The DFA is employed to calculate the Hurst exponent through a logical set of operations. The first step involves the following estimation:

$$x(k) = \sum_{t=1}^{k}(x_t - \bar{x}) \tag{6}$$

Where k is the number of observations in the time series, $\bar{x}$ is mean. In step number two, the accumulated time series is divided into boxes of same length of n. in each box (of length n), the trend is estimated by applying ordinary least squares (OLS) method. The OLS line in each box is identified as $x_n(i)$. to remove the trend in each box, $x_n(i)$. is subtracted from *x* (i). Then, the following quantity is calculated.

$$F(n) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[x(i) - x_n(i)]^2} \tag{7}$$

The process described in the second step is repeated for every scale, n. The slope of the line relating log F(n) to log n determines the scaling (Hurst H) exponent which is also referred to as the self-similarity parameter. Assume that kth order autocovariance is defined as:

$$\gamma(k) = covariance[x_t, x_{t+k}] \; . \tag{8}$$

Then the kth order autocorrelation can be determined as:

$$\rho_k = \frac{\gamma(k)}{\sqrt{var(x_t)}\sqrt{var(x_{t+k})}} = \frac{\gamma(k)}{\gamma(0)}. \tag{9}$$

Peters (1996) defines the autocorrelation function, as:

$$\rho = 2^{(2H-1)} - 1. \quad (10)$$

If time series are random and uncorrelated, then the present has no effect on the future. The Hurst exponent, H, helps to infer the presence or otherwise of long memory in a time series. For example, if H=0.5, then in Eq. (9), the autocorrelation, equals zero. The time series is then described as a Brownian motion which is also called a white noise process with independent increments that are identically normally distributed. It is also known as a random walk series. This type of process can be modeled using an ARMA (p, q) process. If $0 \le H < 0.5$, then the underlying time series is said to be anti-persistent (Oh and Kim, 2006). The series is said to "mean-reverting". The closer H is to zero, ρ moves closer to -0.5 indicating a negative autocorrelation. On the other hand, if $0.5 < H \le 1.0$, then the time series is said to be persistent. Persistent time series are also referred to as fractional Brownian series or biased random walk. Peters (1996), notes that in fractional Brownian series, correlation exits between events across time scales. Therefore, the probability of two events following each other is not fifty-fifty. The Hurst exponent is employed to describe such a probability. For example, with H=0.70, there is a higher probability that if the last change in a time series is positive the next change will also be a positive. The higher probability is the source of "bias" in the random walk process.

Whittle estimator

The parametric approach proposed by Fox, and Taqqu (1986). This estimator involves the function

$$Q(\eta) = \int_{-\pi}^{\pi} \frac{I(\omega)}{f_X(\omega;\eta)} d\omega.$$

Where $f_X(.;\eta)$ is the spectral density function of the $\{X_t\}_{t\in\mathbb{N}}$, and η denotes the vector of unknown parameters. The $\hat{d}_W$ estimator is the value of η which minimizes the function $Q(.)$ (see [21]). When we are dealing with the situation where p=0=q, η is given only by the parameter d. For computational purpose, it is easier to minimize the function

$$\mathcal{L}_n(n) = \frac{1}{2n} \sum_{j=1}^{n-1} \left\{ ln f_X(\omega_j;\eta) + \frac{I(\omega_j)}{f_X(\omega_j;\eta)} \right\}.$$

Instead of $(.)$, where ω_j are the Fourier frequencies, for j=1,…,n-1. For general ARFIMA (0, d, 0) Gaussian processes Fox, and Taqqu have shown in [7] that the maximum likelihood estimator of d is strongly consistent, and asymptotically normally distributed.

Maximum likelihood estimates

Assuming that the processes $\{x_t\}$ is Gaussian, the log likelihood function may be expressed as

$$\mathcal{L}_n(\theta) = -\frac{1}{2} log det T(f_{\boldsymbol{\theta}}) - \frac{1}{2} X' T(f_{\boldsymbol{\theta}})^{-1} X.$$

Where θ is the parameter vector, $f_{\boldsymbol{\theta}}$ is the spectral density given in (2),
$X = (X_1, \dots, X_n)'$ and T is the variance-covariance matrix of $\{x_t\}$, see for example Beran (1994, section 5.3). The ML estimates are obtained by maximizing $\mathcal{L}_n(\theta)$. The range of d is confined to [0 , 0.5] that is, only persistent stationary series are considered.

RESULTS AND DISCUTIONS

Comparison of estimation methods

Sometimes both long and short-range components are present in the data. Though R/S analysis is robust to time series that exhibits only long memory, it cannot distinguish between short memory and long memory when they exist in time series simultaneously.

A primary disadvantage of the GPH technique is that the specified relationship between the spectral density $f(\omega_j)$ and harmonic frequency ω_j is only approximate for the ARFIMA (p, d, q) specification when the sample length T is small. Consequently, the technique yields upwardly biased estimates of d when the function x_t also contains AR and MA components (Sowell, 1992). However, for simple fractional processes denoted ARFIMA (0, d, 0), this relationship is exact. Another possible problem with the GPH estimator is that this estimator is only appropriate for stationary long memory processes with d satisfying -0.5 < d < 0.5. The maximum likelihood method differs from the GPH procedure in that it allows all the ARFIMA parameters to be estimated simultaneously. ML estimator can be applied to stationary data. Therefore, if there is no prior information about the magnitude of the long memory parameter before estimation, we need a more flexible estimation technique and inference for both the stationary and non-stationary cases.

Smoothed periodogram and DFA methods can be applied directly to non-stationary data. The advantage of the DFA over the other methods is that it avoids the spurious detection of apparent long- range correlation that is an artifact of non-stationary.

The whittle estimator is appropriate for series with finite as well as infinite variance therefore, it can be applied to non-stationary data.

Empirical study

The data employed in this study are the monthly USA, Persian Gulf rim countries[1], OPEC[2], OAPEC[3], and the world's petroleum production from January 2001 to December 2009. The data are obtained from the Energy Information Administration of the U.S. Department of Energy. Test for stationary carried out with augmented Dickey-Fuller methods. The ADF test checks whether a series has a unit root. The default null hypothesis is that the series does have a unit root.

Table 1: ADF test

	USA	OPEC	OAPEC	Persian Gulf	World
ADF test statistic	-1.77	-1.68	-1.99	-2.29	**-1.42**
p-value	0.67	0.71	0.58	0.46	**0.81**

The examination of the table 1 shows that there is non-stationary in the observed data. The null hypothesis is accepted at 5% level of significance.

Therefore, we use smoothed periodogram approach, DFA method, and whittle estimator because these methods are appropriate for both the stationary and non-stationary cases.
All calculations were carried out using a R program which the results are given in table 2.

Table 2: Long memory tests

Areas	Long memory tests		
	$\hat{d}_{sp}$	h_{DFA}	$\hat{d}_w$
USA	0.45	1	**0.62**
OPEC	1	0.48	**1**
OAPEC	1	0.47	**1**
Persian Gulf	1	0.36	**1**
World	1	0.48	**1**

The results of the long memory tests show that the long memory exists in USA petroleum production only. Therefore, in order to achieve stationary, USA petroleum production data need to be fractionally differenced. Then we can establish ARFIMA model for it.

For OPEC, OAPEC, Persian Gulf, and world's petroleum production data, the Hurst exponent are 0.48, 0.47, 0.36, and 0.48 respectively, which indicate that all of these four time series do not have long memory.

CONCLUSIONS

In this paper, we studied some of the common methods for detecting the existence of long memory and estimating the fractional differencing parameter d. The oldest and best-known method for detecting long memory is the R/S analysis but it generally is inaccurate and sensitive to short range serial correlations. We expressed ML estimator which this estimator is only appropriate for stationary long memory processes. On the other hand, semi-parametric estimation methods have attracted much research recently. The most common of these semi-parametric estimations is the GPH estimator is that this estimator is only appropriate for stationary long memory processes.

Therefore, we characterized a more flexible estimation technique (DFA methods, smoothed periodogram approach, and whittle estimator) because these methods are appropriate for both the stationary and non-stationary cases.

Then we used these methods for detecting the existence of long memory in USA, Persian Gulf rim countries, OPEC, OAPEC, and the world's petroleum production. The results of this study proved that the long memory exists in USA petroleum production only.

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1- The Persian Gulf countries are Bahrain, Iran, Iraq, Kuwait, Qatar, Saudi Arabia, and United Arab Emirates.
2- OPEC: Organization of the Petroleum Exporting Countries: Algeria, Angola, Ecuador, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, the United Arab Emirates, and Venezuela.
3- OAPEC: Organization of Arab Petroleum Exporting Countries: Algeria, Bahrain, Egypt, Iraq, Kuwait, Libya, Qatar, Saudi Arabia, Syria, Tunisia, and the United Arab Emirates.

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