



## Model Making for Heat Transfer in a Fluidized Bed Dryer

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### ABSTRACT

This research intends to study heating transfer in a fluidized bed dryer by the use of 3-phases model making including D group of Geldart classification. There are some modifications in Vitor (2004) and Rizzi (2009) researches. Some of these modifications are bed porosity in interstitial gas and involving changes of diameter and velocity of bubbles at bed height with modified Horio&Nonaka relations. Three-phase model making equations are linear differential ones which have been solved by finite difference method. There is a good compatibility in comparing the results out of model making with Rizzi (2009) experimental results in this research.

**KEY WORDS:** Fluidized bed dryer, Heat transfer coefficient, Three-phase model making- drying process.

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### INTRODUCTION

Fluidized bed dryers are so much important from among all types of dryers. Some of the major advantages of these dryers are: High level of heat transfer, mass transfer due to very good contact between solid and gas dryers and also good combination of materials in dryer chamber. There are a lot of models for studying of heat transfer and mass transfer in this type of dryers. Tommy & Johnston (1952) introduced Two-phase simple model for fluidized bed dryer for the first time. According to this model “all gas in excess of that necessary to just fluidize the bed passes through in the form of bubbles” while the emulsion is always at minimum fluidization condition. According to the Two-phases model we have  $\varepsilon_e$  and gas superficial velocity in emulsion with all gas currency  $u_0$  in  $\varepsilon_{mf}$  and  $u_{mf}$ . Geldart and Abrahamson (1980) found out that theories of that model were not completely in compliance with experimental data and changing of  $\varepsilon_e$  and  $u_e$  with  $u_0$ . Widehagen et al., (2002) used the three-phase model for fluidized bed dryer including prosperity alumina. They assumed a solid phase with perfect mixing and Bubble phase and interstitial phase under plug flow. They presented equilibrium equations between mass and energy under this three-phase model making.

They excluded any energy wastes out of bed walls in this research. Vitoret. Al (2004) studied Biomass drying process belonging to B group of Geldart classification with the same previous model making process. Of course they considered any energy wasting of walls with regard to experimental data and by the use of three phase model making equations and heat transfer between the bed and wall for calculation of heating transfer coefficient and mass transfer coefficient according to the Reynolds number. Then Rizziet,al (2009) studied heat transfer in a fluidized bed dryer by applying a three-phase model including (Grass seeds) belonging to D group of Geldart classification. Also they considered heating losses from bed wall but they solved only relevant equations of heat transfer due to the low level of moisture percentage of solid particles and by assuming any mass transfer between different phases.

#### Numeric model making:

Rizzi et al (2009) equations used in this research and considered that:

- 1- Solid phase & bubble phase are separately in transfer situation of mass and energy in general condition. Following image shows the mentioned relation but due to the low level of moisture we ignored any mass transfer therefore we only considered energy equilibrium equations.
- 2- According to the made researches by Hiraki et.al (1969), all bubbles may include a small part of the particles maximum %1. This is because there is no more heat transfer and mass transfer between bubble phase and solid phase.
- 3- Solid phase is perfectly mixed and bubble phase under plug flow and interstitial gas phase can flow in the arbitrary flow regime (perfectly mixed, plug flow or any intermediate situation).

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- 4-inRizzi research Porosity situation in interstitial gas phase inside a network is assumed in minimum situation (Two- phase theory) but its quantity is assumed to be equal to porosity in extended field.
  - 5- The velocity of bubbles related to Two-phase currency is assumed related to diameter of bubble and bubblevelocity with other in Rizzi research but the relevant relations of bubble diameter and velocity of which have been modified with other relations.
  - 6- Heating losses from the walls of bed only in relation to interstitial gas.
- Following figure shows any relation among different phases:

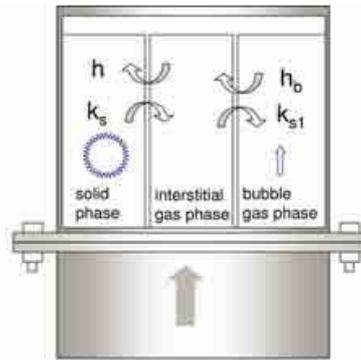


Figure1 : Any relation among different phases in 3phase model making

Where:  $h$  and  $k_s$  are heat & mass transfer coefficients between inter-bed solid phase& gas phase and  $h_b$  &  $k_{s1}$  as heat & mass transfer coefficient between probe phase and inter-be gas phase

Regarding all above-mentioned theories it is possible to find inter-phases energy balance equations obtained from Rizi&Vitor researches in table 1

Table 1: Energy balance equations from Vitor(2004) model

	$f_{E1} = ha(T_{gi} - T_r); f_{E2} = h_b a_1 (T_{gb} - T_r)$ $f_{E1T} = \frac{1}{N} \sum_{k=1}^N f_{E1}^{(k)}; f_{E2T} = \frac{1}{N} \sum_{k=1}^N f_{E2}^{(k)}$ <p>with: <math>a = \frac{6(1 - c_{mf})}{d_p \phi}</math></p> $h_b a_1 = \frac{(h_{bc} a_b)(h_{cg} a_c)}{h_{bc} a_b + h_{cg} a_c}$ $h_{bc} a_b = \delta \left\{ 4.5 \left( \frac{G_{mf} c_{pg}}{d_b} \right) + 5.85 \left[ \frac{(k_G c_{pg} \rho_g)^{1/2} g^{1/4}}{d_b^{1.25}} \right] \right\}$ $h_{cg} a_c = \delta \left\{ 6.78 (\rho_g c_{pg} k_G)^{1/2} \left[ \frac{\epsilon_{mf} u_x}{\rho_g d_b^3} \right]^{1/2} \right\}$									
<p><b>subscripts:</b>                  b bubble;                  c cloud;                  g gas;                  bc bubble-cloud;                  cg cloud-gas interstitial.</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 33%;">Solid Phase</th> <th style="width: 33%;">Interstitial gas phase</th> <th style="width: 33%;">Bubble phase</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;"> <math>(1 - \epsilon) \rho_s \frac{dH_s}{dt} = f_{F1}</math> (1a)                             </td> <td style="text-align: center;"> <math>(1 - \delta) \epsilon \rho_g \frac{d\bar{H}_i}{dt} + G_{gi} \beta_T \frac{\bar{H}_i - H_0}{I_r} = f_{H2T} - f_{F1} - E_w</math> (2a)                             </td> <td style="text-align: center;"> <math>\delta \rho_g \frac{\partial H_b}{\partial t} + G_{gb} \frac{\partial H_b}{\partial z} = -f_{E2}</math> (3a)                             </td> </tr> <tr> <td style="text-align: center;"> <math>H_s = (c_{ps} + Y_s c_{pw}) (T_s - T_r)</math> (1b)                             </td> <td style="text-align: center;"> <math>\bar{H}_i = Y_g \lambda + (c_{pgi} + Y_g c_{pvi}) \times (\bar{T}_{gi} - T_r)</math> (2b)                             </td> <td style="text-align: center;"> <math>H_b = Y_g \lambda + (c_{pgb} + Y_g c_{pvb}) \times (T_{gb} - T_r)</math> (3b)                             </td> </tr> </tbody> </table>	Solid Phase	Interstitial gas phase	Bubble phase	$(1 - \epsilon) \rho_s \frac{dH_s}{dt} = f_{F1}$ (1a)	$(1 - \delta) \epsilon \rho_g \frac{d\bar{H}_i}{dt} + G_{gi} \beta_T \frac{\bar{H}_i - H_0}{I_r} = f_{H2T} - f_{F1} - E_w$ (2a)	$\delta \rho_g \frac{\partial H_b}{\partial t} + G_{gb} \frac{\partial H_b}{\partial z} = -f_{E2}$ (3a)	$H_s = (c_{ps} + Y_s c_{pw}) (T_s - T_r)$ (1b)	$\bar{H}_i = Y_g \lambda + (c_{pgi} + Y_g c_{pvi}) \times (\bar{T}_{gi} - T_r)$ (2b)	$H_b = Y_g \lambda + (c_{pgb} + Y_g c_{pvb}) \times (T_{gb} - T_r)$ (3b)
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<b>Initial and boundary conditions and considerations:</b>										
$T_s(0) = T_{s0}$	$T_{gi}(0, z) = T_{gb}(0, z) = T_{s0}$ $T_{gi}(t, 0) = T_{gb}(t, 0) = T_{s0}$									
$Y_s = \text{constant} = Y_s^*$ (moisture content in equilibrium to $Y_n^*$ )	$Y_g = \text{constant} = Y_g^*$ ; $1 < \beta_T \leq 1.5 \Rightarrow$ arbitrary flow; $\beta_T = \left( \frac{L}{\bar{H}_i - H_0} \right) \frac{\partial H_i}{\partial z} \Rightarrow$ plug-flow; $\beta_T = 1 \Rightarrow$ perfect mixing.									

**Numeric model making & manner of solving the equations**

Firstly we should replace any relations among enthalpy of the phases and relevant relations of energy exchange rate in major equations. Then for solving this system of equations, we may benefit from finite difference method with explicit viewpoint of governing equations on separation:

Solid phase:

$$T_s^{n+1} = \frac{ha}{(1 - \epsilon)\rho_s(c_{ps} + Y_s c_{pw})} \Delta t (\overline{T_{gi}^n} - T_s^n) + T_s^n$$

Interstitial gas phase:

$$\begin{aligned} \overline{T_{gi}^{n+1}} = \overline{T_{gi}^n} - & \left( \frac{G_{gi} \times \Delta t}{(1 - \delta)\epsilon\rho_g \times \Delta Z} \right) (T_{gi}^n(z) - T_{gi}^n(z - 1)) \\ & + \left( \frac{\Delta t}{(1 - \delta)\epsilon\rho_g(c_{pgi} + Y_g c_{pvi})} \right) (h_b a_1 (T_{gb}^n - \overline{T_{gi}^n}) - ha(\overline{T_{gi}^n} - T_s^n) - E_w) \end{aligned}$$

Bubble phase:

$$T_{gb}^{n+1}(z) = T_{gb}^n(z) - \left( \frac{G_{gb} \times \Delta t}{\delta \times \rho_g \times \Delta Z} \right) (T_{gb}^n(z) - T_{gb}^n(z - 1)) - \left( \frac{\Delta t}{\delta\rho_g(c_{pgb} + Y_g c_{pvb})} \right) (T_{gb}^n(z) - T_{gi}^n(z))$$

It is possible to find out any heating loss rate from bed wall through the following relation:

$$E_w = \alpha_w \frac{A_L}{V_{bed}} (\overline{T_{bed}} - T_{amb}) = \alpha_{wa} \frac{A_L}{V_{bed}} (\overline{T_w} - T_{amb})$$

The coefficient  $\alpha_w$  in Rizzi et. Al(2009) research has been explained by the use of experimental data and by the parameter estimation method as follows:

$$\alpha_{wa} = 2.75 \times 10^1$$

Since the temperature of solid phase is a function of time without any change in height of length (theory No. 3), therefore the primary temperature of primary solid phase is calculated by primary condition.

In order to calculate primary distribution out of bubble phase & Interstitial gas phase temperatures inside the network it is possible to use plug flow properties and firstly we should omit any time limitation in differential equations and consider localization equations. Then with regard to territory conditions in downstream territory separate any equations of all three phases in accordance with Z parameter which is the same bed height and by deduction method. Then we may obtain a primary distribution of gas phase temperatures with bubble phase at height length. By bearing the primary distribution of temperatures at each phase it is possible to calculate new step amounts with regard to previous time amounts.

As it is obvious in separated equations, there are some coefficients which should be calculated with suitable relations. Hereinafter we may point out to some of the applied relations:

**Heat transfer coefficients:**

We used the table1 for calculation of  $h_b$ . Rizziet, al considered these parameters as an equation of Reynolds for calculation of h parameter. Then it is possible to calculate these equations by the use of lab data and estimation method for unknown coefficients parameter and finally the following correlation is proposed:

$$h = \frac{k_{gi}}{d_p} (4.11 \times 10^{-2} Re_p 1.55 \times 10^{-1})$$

**Specifications of experimental model of Rizzi et al.**

Different experiments of Rizzi et al were used for confirming the correctness of numeric model making and for D Geldart particles. Following figure shows a plan of this system which is a fluidized dryer.

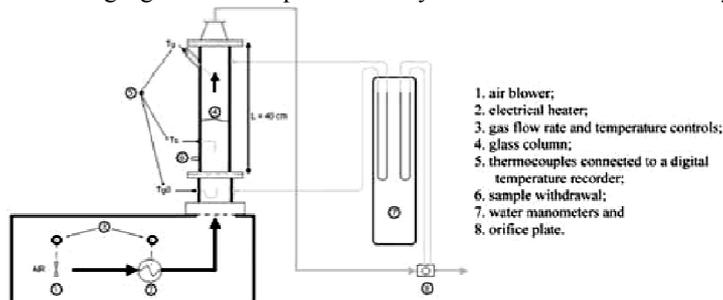


Figure2: experimental setup from Rizzi et al(2009) research

Table 2 shows different specifications of the bed and used seeds.

column diameter	0.07 m
bed height	0.4 m
particle sphericity	0.92
Particle diameter	2.23 mm
Solid density	1018 kg/m <sup>3</sup>
specific heat	428 j/kg k

Table2: bed and seeds properties in Rizzi et al(2009) research

This is necessary to mention that matured moisture at average temperature of entrance gas in these tests is equal with following quantities:

$$Y_s^* = 0.06 \quad Y_g^* = 0.012$$

Data collection was performed by fixed temperature of gas at entrance part to the bed, temperature of solid particles and temperature of exit gas at 38cm height and fixed temperature of wall through the time. It is necessary to attention that the used solid particles in these tests are dry. The table3 shows the required conditions of test:

Table3:Experiment condition in Rizzi et al(2009) research

Test #	$G_{M,0}$ (kg m <sup>-2</sup> s <sup>-1</sup> )	$T_{g0}$ (°C)	$T_{g0}$ (°C)	$T_{amb}$ (°C)	$T_w$ (°C)	L (m)
1	0.948	51.4	26.2	27.3	33.9	0.182
2	1.301	30.9	18.1	19.6	22.3	0.227
3	1.075	40.9	26.4	29.2	32.8	0.204
4	1.088	40.9	21.6	24.4	29.4	0.212
5	1.259	51.5	22.6	25.1	33.5	0.235
6	1.073	40.7	25.4	27.3	30.8	0.200
7	1.001	30.8	17.1	17.1	20.7	0.189
8	1.198	51.1	27.2	29.2	37.2	0.220
9	1.064	41.3	28.1	29.9	32.2	0.205

### Calculation method of bubble fluidized bed properties

Bubble diameter:

Bubble diameter is one of the modified parameters of this study for which the Horio&Nonako method was used. A repeating method is used for solving this nonlinear equation.

$$\left( \frac{\sqrt{d_b} - \sqrt{d_{be}}}{\sqrt{d_{b0}} - \sqrt{d_{be}}} \right)^{1-\gamma_M/\eta} \left( \frac{\sqrt{d_b} + \sqrt{\delta}}{\sqrt{d_{b0}} + \sqrt{\delta}} \right)^{1+\gamma_M/\eta} = \text{Exp}\left(-0.3 \frac{z - z_0}{D_c}\right)$$

$$\frac{\delta}{d_{bm}} = \frac{(\gamma_M + \eta)^2}{4}$$

$$\gamma_M = \frac{\alpha u_{mf}^{-p} (D_c)^{-0.5}}{0.9 k_b (g)}$$

$$\frac{\alpha}{k_b} = \begin{cases} 2.3 \times 10^{-2} \text{ m/s}^2 & \text{for } p = 1 \\ 6.5 \times 10^{-3} \text{ m/s}^2 & \text{for } p = 1.2 \end{cases}$$

$$\eta = (\gamma_M^2 + \frac{4d_{bm}}{D_c})^{0.5}$$

$$\frac{d_{be}}{D_c} = \frac{\left(-\gamma_M + \left(\gamma_M^2 + \frac{4d_{bm}}{D_c}\right)^{0.5}\right)^2}{4}$$

$$d_{bm} = 0.65 \left[\frac{\pi}{4} D_c^2 (u_0 - u_{mf})\right]^{0.4}$$

$$d_{b0} = 3.685(u_0 - u_{mf})^2/g$$

### The speed of probe increase:

The velocity of bubble increase in this project was a modified parameter out of the relation by Werther (1981).

$$u_b = \psi(u_0 - u_{mf}) + \alpha u_{br}$$

$$u_{br} = 0.711(g d_b)^{1/2} \frac{d_b}{D_c} < 0.125$$

$$u_{br} = 0.711(g d_b)^{1/2} \frac{1}{2} \exp\left(-1.49 \frac{d_b}{D_c}\right) \quad 0.125 < \frac{d_b}{D_c} < 0.6$$

$\alpha$  is a coefficient of any violation of bed probes out of individual increasing probes.

Table4:valus $\alpha$  in Wertherequ.

D	B	A	Geldart Group
0.87	$2 D_c^{1/2}$	$3.2 D_c^{1/2}$	$\alpha$
0.1 – 1	0.1 – 1	0.05 – 1	$D_c(m)$

Followings are other relations:

$$\delta = 1 - \frac{l_{mf}}{L}$$

$$G_{gb} = \psi(G_0 - G_{mf}) = \rho_g \psi(u_0 - u_{mf})$$

$$G_g = G_0 - G_{gb}$$

$$\psi = 0.26 \quad \text{for Geldart D}$$

$$G_{gb} = \psi(G_0 - G_{mf})$$

$$L(1 - \varepsilon) = L_{mf}(1 - \varepsilon_{mf})$$

$$G_{mf} = 0.682 \frac{kg}{m^2s} ; \text{ Rizzi et al(2009) research}$$

$$\varepsilon_{mf} = 0.392 ; \text{ Rizzi et al(2009) research}$$

### DISCUSSION AND CONCLUSION

As it was mentioned before Rizzi experimental data was used in order to have correct consideration of the results. There is a comparison for tests 8 & 9 with following results of numeric model making and by applying relevant modifications beside experimental data as mentioned in following figure

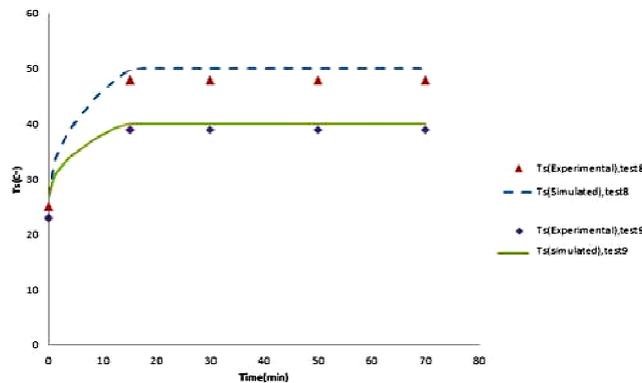


Figure4.comparison of numerical modeling with laboratory data of Rizzi for temperature of solid particles

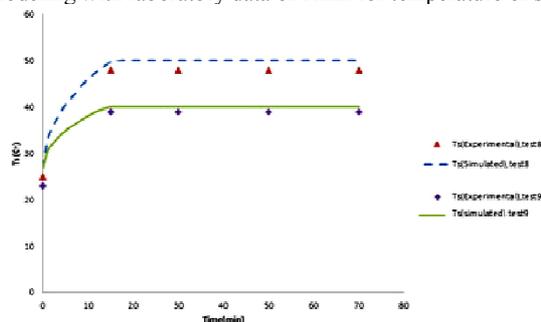


Figure5.comparison of numerical modeling with laboratory data of Rizzi for temperature of outputting gas

As it is obvious there is a suitable compliance between the obtained results out of numerical modeling with laboratory data of Rizzi and his colleagues. This is necessary to mention that all performed modifications on model making process resulted in better compliance of the results out of numeric & experimental model making process.

### Assumption of gas flow model at Interstitial gas phase:

There are different assumptions for model making of Interstitial gas phase which may have perfectly mixed, plug flow or any intermediate situation. Following figure shows the relevant results of gas phase temperature for test No. 8 made by Rizzi *et al.* and considering the type of Interstitial gas phase currency in three forms:

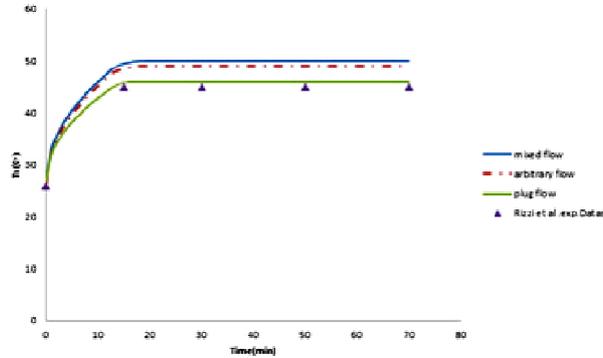


Figure 6. temperature of solid particles by different assumptions of Interstitial gas phase and its comparison to experience8 Rizzi  
As it is obvious, plug flow assumption is more compliance with laboratory data.

### NOTATION:

specific exchange superficial area between gas and particles,	$m^{-1}$	$a$
lateral area of the bed	$m^2$	$A_L$
specific heat	$J/kg.K$	$c$
diameter	$cm$	$d$
column diameter	$m$	$D_c$
rate of energy loss through column wall per unit of bed volume	$W/m^3$	$E_w$
heat transfer rate Between particles and interstitial gas	$W/m^3$	$f_{E1}$
heat transfer rate Between bubble phase and interstitial gas	$W/m^3$	$f_{E2}$
air mass flow rate per cross sectional area of the column	$kg/m^2.s$	$G$
heat transfer coefficient between particles and interstitial gas	$W/m^2.K$	$h$
heat transfer coefficient between bubble phase and interstitial gas	$W/m^2.K$	$h_b$
specific enthalpy	$J/kg$	$H$
expanded bed height	$m$	$L$
gas thermal conductivity	$W/m.K$	$k_g$
temperature	$^{\circ}C$	$T$
time	$s$	$t$
superficial gas velocity	$m/s$	$U$
rise velocity of an isolated bubble	$m/s$	$u_b$
volume	$m^3$	$V$
water content in dry basis	-	$Y$
equilibrium moisture content	-	$Y^*$
axial coordinate	$m$	$Z$
equilibrium bubble diameter	$m$	$d_{be}$
The size of the stochastic bubble	$cm$	$d_{bm}$
initial value of bubble diameter	$cm$	$d_{b0}$

**Greek Symbols**

heat transfer coefficient between column walls and air ambient	$j/m^2.K.s$	$\alpha_w$
coefficient related with the interstitial gas phase	-	$\beta_r$
coefficient related to Horio and NonakaEq	-	$\gamma_m$
volumetric bubble concentration	-	$\delta$
bed porosity	-	$\epsilon$
latent heat of vaporization	$j/kg$	$\lambda$
viscosity	$kg/s.m$	$\mu$
density	$kg/m^3$	$\rho$
particle sphericity	-	$\varphi$
ratio of the visible bubble flow to the excess gasvelocity	-	$\psi$

**Special Subscripts**

- 0 = initial value
- Amb= ambient
- Bed= bed
- B=bubble
- Exp= experimental
- g= gas
- gL= exit gas
- i= interstitial gas
- m= gas-solid mixture
- mb= minimum bubbling condition
- mf= minimum fluidization condition
- p= particle
- s= solid
- v =water vapor
- w= wall

**Dimensionless Numbers:**

$$Re = \frac{G_{gi} d_p}{\mu_{gi}}$$

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