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Simply Support-Clamped Boundary Conditions for Isotropic Cylindrical Shell

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ABSTRACT

Isotropic cylindrical shells composed of stainless steel are important. Material properties are graded in the thickness direction of the shell according to volume fraction power law distribution. The study is carried out using third order shear deformation shell theory. The governing equations of motion of isotropic cylindrical shells are derived based on shear deformation theory. Results are presented on the frequency characteristics, influence of constituent volume fractions and the effects of simply support-clamped boundary conditions.

KEYWORDS: Cylindrical shell, Simply Support, Clamped.

INTRODUCTION

Researches on free vibrations of cylindrical shells have been carried out extensively [1-5]. Recently, the present authors presented studies on the influence of boundary conditions on the frequencies of a multi–layered cylindrical shell [6]. In all the above works, different thin shell theories based on Love–hypothesis were used.

Vibration of cylindrical shells with ring support is considered by Loy and Lam [7]. The concept of functionally graded materials (FGMs) was first introduced in 1984 by a group of materials scientists in Japan [8-9] as a means of preparing thermal barrier materials. Since then, FGMs have attracted much interest as heat-shielding materials.

FGMs are made by combining different materials using power metallurgy methods [10]. They possess variations in constituent volume fractions that lead to continuous change in the composition, microstructure, porosity, etc., resulting in gradients in the mechanical and thermal properties [11-12]. Vibration study of FGM shell structures is important. However, study of the vibration of FGM shells with ring supports is limited.

The FGMs considered are composed of stainless steel and nickel where the volume fractions follow a power-law distribution. The study is carried out based on third order shear deformation shell theory. Studies are carried out for cylindrical shells with simply support-clamped (SS-C) boundary conditions. Results presented include the frequency characteristics of cylindrical shells, and the influence of boundary conditions. The present analysis is validated by comparing results with others in the literature.

1- THIRD ORDER THEORY OF REDDY

The third- order theory of Reddy used in the present study is based on the following displacement field

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) + \alpha_3^2.\psi_1(\alpha_1,\alpha_2) + \alpha_3^3.\beta_1(\alpha_1,\alpha_2) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) + \alpha_3^2.\psi_2(\alpha_1,\alpha_2) + \alpha_3^3.\beta_2(\alpha_1,\alpha_2) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \quad (1)$$

These equations can be reduced by satisfying the stress-free conditions on the top and bottom faces of the laminates, which are equivalent to $\in_{13}=\in_{23}=0$ at $Z=\pm\frac{h}{2}$ Thus,

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$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) - C_1.\alpha_3^3(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1}) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) - C_1.\alpha_3^3(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2}) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{2}$$

$$\begin{Bmatrix} \in_{11} \\ \in_{22} \\ \in_{12} \end{Bmatrix} = \begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} + \alpha_3 \begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} k'_{11} \\ k'_{22} \\ k'_{12} \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \in_{13} \\ \in_{23} \end{Bmatrix} = \begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} + \alpha_3^2 \begin{Bmatrix} \gamma_{13}^2 \\ \gamma_{23}^2 \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} \gamma_{13}^3 \\ \gamma_{23}^3 \end{Bmatrix} \tag{4}$$

where

$$\begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{A_1}\frac{\partial u_1}{\partial\alpha_1} + \frac{u_2}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2} + \frac{u_3}{R_1}) \\ (\frac{1}{A_2}\frac{\partial u_2}{\partial\alpha_2} + \frac{u_1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1} + \frac{u_3}{R_2}) \\ \frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}(\frac{u_2}{A_2}) + \frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}(\frac{u_1}{A_1}) \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{A_1}\frac{\partial\phi_1}{\partial\alpha_1} + \frac{\phi_2}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}) \\ (\frac{1}{A_2}\frac{\partial\phi_2}{\partial\alpha_2} + \frac{\phi_1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}) \\ (\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}(\frac{\phi_2}{A_2}) + \frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}(\frac{\phi_1}{A_1})) \end{Bmatrix} \tag{6}$$

$$\begin{Bmatrix} k'_{11} \\ k'_{22} \\ k'_{12} \end{Bmatrix} = C_1 \begin{Bmatrix} (\frac{1}{A_1}(-\frac{\partial A_1}{R_1\partial\alpha_1} + \frac{\partial\phi_1}{\partial\alpha_1} + \frac{\partial^2 u_3}{A_1\partial\alpha_1^2} - \frac{\partial A_1}{A_1^2\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_1}) + \frac{\partial A_1}{\partial\alpha_2}\frac{1}{A_1A_2}(\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2})) \\ (\frac{1}{A_2}(-\frac{\partial A_2}{R_2\partial\alpha_2} + \frac{\partial\phi_2}{\partial\alpha_2} + \frac{\partial^2 u_3}{A_2\partial\alpha_2^2} - \frac{\partial A_2}{A_2^2\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_2}) + \frac{\partial A_2}{\partial\alpha_1}\frac{1}{A_1A_2}(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1})) \\ (\frac{A_2}{A_1}(\frac{\partial}{R_2\partial\alpha_1}(\frac{u_2}{A_2}) + \frac{\partial}{\partial\alpha_1}(\frac{\phi_2}{A_2}) + \frac{1}{A_2}\frac{\partial^2 u_3}{\partial\alpha_1\partial\alpha_2} - \frac{1}{A_2^2}\frac{\partial A_2}{\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_2}) + \\ + \frac{A_1}{A_2}(\frac{\partial}{R_1\partial\alpha_2}(\frac{u_1}{A_1}) + \frac{\partial}{\partial\alpha_2}(\frac{\phi_1}{A_1}) + \frac{1}{A_1}\frac{\partial^2 u_3}{\partial\alpha_1\partial\alpha_2} - \frac{1}{A_1^2}\frac{\partial A_1}{\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_1})) \end{Bmatrix} \tag{7}$$

$$\begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} = \begin{Bmatrix} (\phi_1 - \frac{u_1}{R_1} + \frac{1}{A_1}\frac{\partial u_3}{\partial\alpha_1}) \\ (\phi_2 - \frac{u_2}{R_2} + \frac{1}{A_2}\frac{\partial u_3}{\partial\alpha_2}) \end{Bmatrix} \tag{8}$$

$$\begin{Bmatrix} \gamma_{13}^2 \\ \gamma_{23}^2 \end{Bmatrix} = 3C_1 \begin{Bmatrix} (-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1}) \\ (-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2}) \end{Bmatrix} \tag{9}$$

$$\begin{Bmatrix} \gamma_{13}^3 \\ \gamma_{23}^3 \end{Bmatrix} = C_1 \begin{Bmatrix} \frac{(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1})}{R_1} \\ \frac{(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2})}{R_2} \end{Bmatrix} \tag{10}$$

2- FORMULATION

For a thin cylindrical shell, the stress -strain relationship are defined as

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{44} & 0 & 0\\ 0 & 0 & 0 & Q_{55} & 0\\ 0 & 0 & 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\in_{11}\\ \in_{22}\\ \in_{23}\\ \in_{13}\\ \in_{12}\end{Bmatrix} \tag{11}$$

For a isotropic cylindrical shell the reduced stiffness Q_{ij} (i , j=1, 2 and 6) are defined as

$$Q_{11}=Q_{22}=\frac{E}{1-\nu^2}\ ,Q_{12}=\frac{\nu\ E}{1-\nu^2} \tag{12}$$

$$Q_{44}=Q_{55}=Q_{66}=\frac{E}{2(1+\nu)} \tag{13}$$

where E is the Young's modulus and ν is Poisson's ratio. Defining

$$\left(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},G_{ij},H_{ij}\right)=\int_{-h/2}^{h/2}Q_{ij}\left(1,\alpha_3,\alpha_3^2,\alpha_3^3,\alpha_3^4,\alpha_3^5,\alpha_3^6\right)d\alpha_3 \tag{14}$$

where Q_{ij} are functions of z for functionally gradient materials. Here A_{ij} denote the extensional stiffness, D_{ij} the bending stiffness, B_{ij} the bending-extensional coupling stiffness and $E_{ij},F_{ij},G_{ij},H_{ij}$ are the extensional, bending, coupling, and higher-order stiffness. For a thin cylindrical shell the force and moment results are defined as

$$\begin{Bmatrix}N_{11}\\ N_{22}\\ N_{12}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}d\alpha_3\ ,\quad \begin{Bmatrix}M_{11}\\ M_{22}\\ M_{12}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}\alpha_3^3 d\alpha_3 \tag{15}$$

$$\begin{Bmatrix}P_{11}\\ P_{22}\\ P_{12}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}\alpha_3^3 d\alpha_3\ ,\begin{Bmatrix}P_{13}\\ \\ P_{23}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{13}\\ \\ \sigma_{23}\end{Bmatrix}\alpha_3^3 d\alpha_3 \tag{16}$$

$$\begin{Bmatrix}Q_{13}\\ \\ Q_{23}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{13}\\ \\ \sigma_{23}\end{Bmatrix}d\alpha_3\quad ,\quad \begin{Bmatrix}R_{13}\\ \\ R_{23}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{13}\\ \\ \sigma_{23}\end{Bmatrix}{\alpha_3}^2 d\alpha_3 \tag{17}$$

3- THE EQUATIONS OF MOTION FOR VIBRATION OF A GENERIC SHELL

The equations of motion for vibration of a generic shell can be derived by using Hamilton's principle which is described by

$$\delta\int_{t_1}^{t_2}(\Pi-K)dt=0\quad ,\quad \Pi=U-V \tag{18}$$

4- Equations of motion for vibration of cylindrical shell

The curvilinear coordinates and fundamental from parameters for a cylindrical shell are:

$$R_2=a,\frac{1}{R}=0,A_2=a,A_1=0,\alpha_3=\alpha_3,\alpha_2=\theta,\alpha_1=x \tag{19}$$

the equations of motions for vibration of cylindrical shell with the third-order theory of Reddy are converted to

$$a\frac{\partial N_{11}}{\partial x}+\frac{\partial N_{12}}{\partial \theta}=I_0\ddot{u}_1+(I_1-C_1I_3)\ddot{\phi}_1-C_1I_3\frac{\partial \ddot{u}_3}{\partial x} \tag{20}$$

$$\frac{\partial N_{22}}{\partial \theta}+C_1\frac{\partial P_{12}}{\partial x}+Q_{23}-3C_1R_{23}+C_1P_{23}=(I_0+2\frac{C_1}{a}I_3+\frac{C_1^2}{a^2}I_6)\ddot{u}_2+$$
$$(I_1-C_1I_3+\frac{C_1}{a}I_4-\frac{C_1^2}{a}I_6)\ddot{\phi}_2-(\frac{C_1}{a}I_3-\frac{C_1^2}{a^2}I_6)\frac{\partial \ddot{u}_3}{\partial \theta} \tag{21}$$

$$-C_1a\frac{\partial^2 P_{11}}{\partial x^2}+N_{22}-\frac{C_1}{a}\frac{\partial^2 P_{22}}{\partial \theta^2}-2C_1\frac{\partial^2 P_{12}}{\partial x\partial \theta}-a\frac{\partial Q_{13}}{\partial x}+3C_1a\frac{\partial R_{13}}{\partial x}-\frac{\partial Q_{23}}{\partial \theta}+$$
$$+3C_1\frac{\partial R_{23}}{\partial \theta}-\frac{C_1}{a}\frac{\partial P_{23}}{\partial \theta}=-C_1I_3\frac{\partial u_1}{\partial x}-\frac{C_1}{a}I_3\frac{\partial u_2}{\partial \theta}+(-C_1I_4+C_1^2I_6)\frac{\partial \ddot{\phi}_1}{\partial x}+$$
$$(\frac{C_1}{a}I_4+\frac{C_1^2}{a}I_6)\frac{\partial \ddot{\phi}_2}{\partial \theta}-\frac{C_1^2}{a^2}I_6\frac{\partial \ddot{u}_2}{\partial \theta}+C_1^2I_6\frac{\partial^2 \ddot{u}_3}{\partial x^2}+\frac{C_1^2}{a}I_6\frac{\partial^2 \ddot{u}_3}{\partial \theta^2}-\ddot{u}_3I_0 \tag{22}$$

$$-a\frac{\partial M_{11}}{\partial x}+C_1a\frac{\partial P_{11}}{\partial x}-\frac{\partial M_{12}}{\partial \theta}+C_1\frac{\partial P_{12}}{\partial \theta}-3C_1R_{13}a+aQ_{13}=-I_1\ddot{u}_1+C_1I_3\ddot{u}_1+$$
$$(-I_2+2C_1I_4-C_1^2I_6)\ddot{\phi}_1+(C_1I_4-C_1^2I_6)\frac{\partial \ddot{u}_3}{\partial x} \tag{23}$$

$$-\frac{\partial M_{22}}{\partial \theta}-C_1\frac{\partial P_{22}}{\partial \theta}-a\frac{\partial M_{12}}{\partial x}+C_1a\frac{\partial P_{12}}{\partial x}-3C_1R_{23}a+aQ_{23}+C_1R_{23}=(-I_1$$
$$C_1I_3-\frac{C_1}{a}I_4)\ddot{u}_2+(-I_2+2C_1I_4)\ddot{\phi}_2-\frac{C_1}{a}I_4\frac{\partial \ddot{u}_3}{\partial \theta} \tag{24}$$

The displacement fields for a cylindrical shell and the displacement fields which satisfy these boundary conditions can be written as

$$\begin{aligned}
u_1&=\overline{A}\frac{\partial \phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\
u_2&=\overline{B}\,\phi(x)\,\sin(n\theta)\cos(\omega t)\\
u_3&=\overline{C}\,\phi(x)\cos(n\theta)\,\cos(\omega t)\\
\phi_1&=\overline{D}\frac{\partial \phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\
\phi_2&=\overline{E}\,\phi(x)\,\sin(n\theta)\cos(\omega t)
\end{aligned} \tag{25}$$

where, $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{D}$ and $\overline{E}$ are the constants denoting the amplitudes of the vibrations in the x,θ and z directions, ϕ_1 and ϕ_2 are the displacement fields for higher order deformation theories for a cylindrical shell, $\phi(x)$ is the axial function that satisfies the geometric boundary conditions. The axial function $\phi(x)$ is chosen as the beam function as

$$\phi(x)=\gamma_1\cosh(\frac{\lambda_m x}{L})+\gamma_2\cos(\frac{\lambda_m x}{L})-\zeta_m(\gamma_3\sinh(\frac{\lambda_m x}{L})+\gamma_4\sin(\frac{\lambda_m x}{L})) \tag{26}$$

Substituting Eq. (25) into Eqs. (20) - (24) for third order theory we can be expressed

$$\det(C_{ij}-M_{ij}\,\omega^2)=0 \tag{27}$$

Expanding this determinant, a polynomial in even powers of ω is obtained

$$\beta_\circ\omega^{10}+\beta_1\omega^8+\beta_2\omega^6+\beta_3\omega^4+\beta_4\omega^2+\beta_5=\circ \tag{28}$$

where $\beta_i\,(i=0,1,2,3,4,5)$ are some constants. Eq. (28) is solved five positive and five negative roots are obtained. The five positive roots obtained are the natural angular frequencies of the cylindrical shell

based third-order theory. The smallest of the five roots is the natural angular frequency studied in the present study.

5- RESULTS AND DISCUSSION

In this paper, studies are presented for a cylindrical shell with simply support-clamped boundary conditions is considered. Table 1 shows the variation of the natural frequency with the circumferential wave number n for a functional graded cylindrical shell. The frequencies for the simply support-clamped boundary conditions increased with the circumferential wave number.

Table 2
The natural frequencies for an isotropic cylindrical shell under (SS-C) boundary conditions
(m = 1, h / R=0.01, L / R=20).

m	n	Ω (HZ)
1	1	**0.376439**
	2	**0.472346**
	3	**0.496120**
	4	**0.506473**
	5	**0.513098**
	6	**0.520332**
	7	**0.530908**
	8	**0.544434**
	9	**0.562121**
	10	**0.587658**

6- CONCLUSIONS

The study is carried out using third order shear deformation shell theory. Studies are carried out for cylindrical shells with simply support-clamped (SS-C) boundary conditions. The study showed that in this boundary conditions the frequency first decreases and then increases as the circumferential wave number n increases.

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