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Nickel on Outer Surface Cylindrical Shell

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ABSTRACT

Cylindrical shell has nickel on its outer surface. The study is carried out based on third order shear deformation shell theory. The objective is to study the natural frequencies, the influence of constituent volume fractions and the effects of configurations of the constituent materials on the frequencies. The properties are graded in the thickness direction according to the volume fraction power-law distribution. The governing equations are obtained using energy functional with the Rayleigh-Ritz method.
KEYWORDS: Nickel, Metal, Cylindrical.

INTRODUCTION

The study of the free vibrations of cylindrical shells has been carried out extensively. Among those who have studied the vibrations of cylindrical shells include Arnold and Warburton [1], Ludwig and Krieg [2], Chung [3], Soedel [4], Forsberg [5], Bhimaraddi [6], Soldatos and Hajigeoriou [7], Bert and Kumar [8].

They possess variations in constituent volume fractions that lead to continuous change in the composition, microstructure, porosity, etc. and this results in gradients in the mechanical and thermal properties [9] and [10]. The objective is to study the natural frequencies, the influence of constituent volume fractions, the effects of configurations of the constituent materials on the frequencies. The analysis of the functionally graded cylindrical shell is carried out using third order shear deformation shell theory and solved using Rayleigh-Ritz method with energy functional, obtained using an energy approach.The displacement fields employ consist of some beam eigenfunctions of vibrations that guarantee satisfaction of edge boundary conditions.

1- THIRD ORDER THEORY

where A_1 and A_2 are the fundamental form parameters or Lame parameters, U_1, U_2 and U_3 are the displacement at any point ($\alpha_1, \alpha_2, \alpha_3$), R_1 and R_2 are the radius of curvature related to α_1, α_2 and α_3 respectively. The third- order theory of Reddy used in the present study is based on the following displacement field:

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) + \alpha_3^2.\psi_1(\alpha_1,\alpha_2) + \alpha_3^3.\beta_1(\alpha_1,\alpha_2) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) + \alpha_3^2.\psi_2(\alpha_1,\alpha_2) + \alpha_3^3.\beta_2(\alpha_1,\alpha_2) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{1}$$

These equations can be reduced by satisfying the stress-free conditions on the top and bottom faces of the laminates, which are equivalent to $\in_{13} = \in_{23} = 0$ at $Z = \pm\frac{h}{2}$ Thus for third order theory

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) - C_1.\alpha_3^3(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1}) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) - C_1.\alpha_3^3(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2}) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{2}$$

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$$\begin{Bmatrix} \in_{11} \\ \in_{22} \\ \in_{12} \end{Bmatrix} = \begin{Bmatrix} \in_{11}^{0} \\ \in_{22}^{0} \\ \in_{12}^{0} \end{Bmatrix} + \alpha_3 \begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} k'_{11} \\ k'_{22} \\ k'_{12} \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \in_{13} \\ \in_{23} \end{Bmatrix} = \begin{Bmatrix} \gamma_{13}^{0} \\ \gamma_{23}^{0} \end{Bmatrix} + \alpha_3^2 \begin{Bmatrix} \gamma_{13}^{2} \\ \gamma_{23}^{2} \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} \gamma_{13}^{3} \\ \gamma_{23}^{3} \end{Bmatrix} \tag{4}$$

2- FORMULATION

For a thin cylindrical shell, plane stress condition can be assumed. The constitutive relation for a thin cylindrical shell is consequently given by the tow-dimensional Hook's law as

$$\{\sigma\} = [Q]\{\varepsilon\} \tag{5}$$

where, $\{\sigma\}$ is the stress vector, $\{\varepsilon\}$ is the strain vector and $[Q]$ is the reduced stiffness matrix. The stress vector for plane stress condition is

$$\{\sigma\}^T = \{\sigma_{11} \quad \sigma_{22} \quad \sigma_{12} \; \sigma_{13} \; \sigma_{23}\} \tag{6}$$

where σ_{11} is the stress in x direction, σ_{22} the stress in the θ direction and σ_{12} is the shear stress on the $x\theta$ plane and σ_{13} is the shear stress on the $x\,z$ plane and σ_{23} is the shear stress on the $\theta\,z$ plane. The strain vector is defined as

$$\{\varepsilon\}^T = \{\varepsilon_{11} \; \varepsilon_{22} \; \varepsilon_{12} \quad \varepsilon_{13} \quad \varepsilon_{23}\} \tag{7}$$

where ε_{11} is the strain in x direction, ε_{22} the strain in the θ direction and ε_{12} is the shear strain on the $x\theta$ plane and ε_{13} is the shear strain on the $x\,z$ plane and ε_{23} is the shear strain on the $\theta\,z$ plane. The reduced stiffness $[Q]$ matrix is given as

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \tag{8}$$

For an isotropic cylindrical shell the reduced stiffness Q_{ij} (i, j=1, 2 and 6) are defined as

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2} \tag{9}$$

$$Q_{12} = \frac{\nu\, E}{1-\nu^2} \tag{10}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1+\nu)} \tag{11}$$

where E is the Young's modulus and ν is Poisson's ratio. For a thin cylindrical shell the force and moment results are defined as

$$\begin{Bmatrix} N_{11} \\ N_{22} \\ N_{12} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} d\alpha_3 \;, \quad \begin{Bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \alpha_3^3 d\alpha_3 \tag{12}$$

$$\begin{Bmatrix} P_{11} \\ P_{22} \\ P_{12} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \alpha_3^3 d\alpha_3 \ , \begin{Bmatrix} P_{13} \\ \\ P_{23} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{13} \\ \\ \sigma_{23} \end{Bmatrix} \alpha_3^3 d\alpha_3 \tag{13}$$

$$\begin{Bmatrix} Q_{13} \\ Q_{23} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} d\alpha_3 \quad , \quad \begin{Bmatrix} R_{13} \\ R_{23} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \alpha_3^{\,2} d\alpha_3 \tag{14}$$

The constitutive equation is obtained as

$$\{N\} = [S]\{\varepsilon\} \tag{15}$$

where $\{N\}$ and $\{\varepsilon\}$ are, respectively, defined as

$$\{N\}^T = \{N_{11} N_{22} N_{12} M_{11} M_{22} M_{12} P_{11} P_{22} P_{12} P_{13} P_{23} Q_{13} Q_{23} R_{13} R_{23}\} \tag{16}$$

$$\{\varepsilon\}^T = \{\varepsilon_{11}^0 \ \varepsilon_{22}^0 \ \varepsilon_{12}^0 \ k_{11} k_{22} k_{12} k'_{11} k'_{22} k'_{12} \gamma_{23}^0 \ \gamma_{13}^0 \ \gamma_{23}^2 \ \gamma_{13}^2 \ \gamma_{23}^3 \ \gamma_{13}^3\} \tag{17}$$

and $[S]$ is defined as

$$[S] = \begin{pmatrix} [A] & [B] & [E] \\ [B] & [D] & [F] \\ [E] & [F] & [H] \\ [E'] & [G] & [H'] \\ [A'] & [D'] & [E'] \\ [D'] & [F'] & [G] \end{pmatrix} \tag{18}$$

where A, B, E, D, F, H and G are the extensional, coupling and bending stiffness matrices and Q_{ij} are functions of z for functionally gradient materials. Here A_{ij} denote the extensional stiffness, D_{ij} the bending stiffness, B_{ij} the bending-extensional coupling stiffness and $E_{ij}, F_{ij}, G_{ij}, H_{ij}$ are the extensional, bending, coupling, and higher-order stiffness. Defining

$$\{A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij}, H_{ij}\} = \int_{-h/2}^{h/2} Q_{ij} \{1, \alpha_3, \alpha_3^2, \alpha_3^3, \alpha_3^4, \alpha_3^5, \alpha_3^6\} d\alpha_3 \tag{19}$$

The strain energy and kinetic energy of a cylindrical shell can be defined as

$$U = \frac{1}{2} \iiint \{\varepsilon\}^T \{\sigma\} \quad dV \tag{20}$$

$$T = \frac{1}{2} \iiint \rho \left[\left(\frac{\partial u_1}{\partial t}\right)^2 + \left(\frac{\partial u_2}{\partial t}\right)^2 + \left(\frac{\partial u_3}{\partial t}\right)^2 + \left(\frac{\partial \phi_1}{\partial t}\right)^2 + \left(\frac{\partial \phi_2}{\partial t}\right)^2 \right] dV \tag{21}$$

where, ρ is the mass density, $\{\varepsilon\}$ is the strain vector and $\{\sigma\}$ is the stress vector. By substituting from Eq. (5), the strain and kinetic energies can be written as

$$U = \frac{1}{2} \int_0^L \int_0^{2\pi} \{\varepsilon\}^T [S] \{\varepsilon\} \, R d\theta \, dx \tag{22}$$

$$T = \frac{1}{2} \int_0^L \int_0^{2\pi} \rho_T \left[\left(\frac{\partial u_1}{\partial t}\right)^2 + \left(\frac{\partial u_2}{\partial t}\right)^2 + \left(\frac{\partial u_3}{\partial t}\right)^2 + \left(\frac{\partial \phi_1}{\partial t}\right)^2 + \left(\frac{\partial \phi_2}{\partial t}\right)^2 \right] R d\theta dx \tag{23}$$

The parameter ρ_T is the density per unit length defined as

$$\rho_T = \int_{-h/2}^{h/2} \rho \ dz \tag{24}$$

The displacement fields for a cylindrical shell can be written as:

$$\begin{aligned} u_1 &= \overline{A}\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t) \\ u_2 &= \overline{B}\,\phi(x)\sin(n\theta)\cos(\omega t) \\ u_3 &= \overline{C}\phi(x)\cos(n\theta)\cos(\omega t) \\ \phi_1 &= \overline{D}\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t) \\ \phi_2 &= \overline{E}\phi(x)\sin(n\theta)\cos(\omega t) \end{aligned} \tag{25}$$

where, $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{D}$ and $\overline{E}$ are the constants denoting the amplitudes of the vibrations in the x,θ and z directions, $\phi(x)$ is the axial function that satisfies the geometric boundary conditions, n denotes the number of circumferential waves in the mode shape and ω is the natural angular frequency of the vibration. To determine the natural frequencies, the Rayleigh-Ritz method is used. The energy functional Π defined by the Lagrangian function as

$$\Pi = T_{\max} - U_{\max} \tag{26}$$

Substituting Eq. (25) into Eqs. (22) and (23) and minimizing the energy functional Π with respect to the unknown coefficients as follows,

$$\frac{\partial\Pi}{\partial\overline{A}} = \frac{\partial\Pi}{\partial\overline{B}} = \frac{\partial\Pi}{\partial\overline{C}} = \frac{\partial\Pi}{\partial\overline{D}} = \frac{\partial\Pi}{\partial\overline{E}} = 0 \tag{27}$$

The eigenvalue equations are solved by imposing the non-trivial solutions condition and equating the determinant of the characteristic matrix $[C_{ij}]$ to zero. Expanding this determinant, a polynomial in even powers of ω is obtained

$$\beta_\circ\omega^{10} + \beta_1\omega^8 + \beta_2\omega^6 + \beta_3\omega^4 + \beta_4\omega^2 + \beta_5 = \circ \tag{28}$$

where $\beta_i\,(i = 0,1,2,3,4,5)$ are some constants. Eq. (28) is solved five positive and five negative roots are obtained. The five positive roots obtained are the natural angular frequencies of the cylindrical shell in the x , θ and z directions. The smallest of the five roots is the natural angular frequency studied in the present study.

3- RESULTS AND DISCUSSION

In this paper studies are presented on influence of constituent volume fractions is studied by varying the volume fractions of the nickel. Table 1 shows the variations of the volume fractions V_N of Nickel, respectively, in the thickness direction z for a cylindrical shell. The volume fraction for decreased from 1 at $z = -0.5h$ to 0 at $z = 0.5h$ and the volume fraction of Nickel V_{fN} increased from 0 at $z = -0.5h$ to 1 at $z = 0.5h$.

TABLE I
VARIATION OF THE STAINLESS STEEL FOR CYLINDRICAL SHELL

z	V_{fN}					
	N=0.5	***N=0.7***	***N=1***	***N=2***	***N=5***	***N=15***
-0.5h	0	0	0	0	0	0
-0.4h	0.3162	0.1995	0.1	0.01	0.00001	$10^{-15}\times 1$
-0.3h	0.4472	0.3241	0.2	0.04	0.00032	$10^{-11}\times 3.27$
-0.2h	0.5477	0.4305	0.3	0.09	0.00243	$10^{-8}\times 1.43$
-0.1h	0.6324	0.5265	0.4	0.16	0.01024	0.00000107
0	0.707	0.6155	0.5	0.25	0.03125	0.00003051
0.1h	0.7745	0.6993	0.6	0.36	0.07776	0.0004701
0.2h	0.8366	0.7790	0.7	0.49	0.1680	0.004747
0.3h	0.8944	0.8553	0.8	0.64	0.3276	0.03518
0.4h	0.9486	0.9289	0.9	0.81	0.5904	0.20589
0.5h	1	1	1	1	1	1

4- Conclusions

A study on the cylindrical shell composed of Nickel has been presented. The study was carried out for cylindrical shells where the configurations of the constituent materials. The analysis of the functionally graded cylindrical shell is carried out using third order shear deformation shell theory and solved using Rayleigh-Ritz method with energy functional, obtained using an energy approach. Studied were made on study the natural frequencies, the influence of constituent volume fractions, the effects of configurations of the constituent materials on the frequencies.

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