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Folding of ruled manifolds and its deformations

K. Khalifa

Department of mathematics-EL-Arish Faculty of Education Seuz Canal University -EL-Arish –Egypt

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ABSTRACT

In this paper we introduce a new types of folding on a manifolds we call it ruled folding ,and if we have topologically folded a ruled manifold the image need not be ruled manifold, but the image of isocurvature folding of a ruled manifold is ruled. The relation between ruled folding and the induced by exponential map are discussed, also the relation between ruled folding and the induced by the inverse exponential map are obtained.

Key Words: Folding, ruled, manifold.

INTRODUCTION

The folding of manifolds is defined by S.A.Robertson, 1977$\left[8\right]$, and the folding of a manifold into another manifold or into itself studied by E.EL-Kholy $\left[5\right]$and M.EL-Ghoul$\left[2,3\right]$, but the folding of minimal manifolds and its deformations studied by M.EL-Ghoul and K.Khalifa $\left[4\right]$.

Definitions:

1- The Gaussian curvature at a point p of a surface S is K where $K = k_1 k_2$, $k_1 \, and \, k_2$ are the principle curvatures.

2- A subset A of a topological space X is called a retract of X if there exist a continuous map $r : X \rightarrow A$ such that:

(i) X is open,

(ii) $r(a) = a, a \in A$.

3- A subset A of a topological space X is called a deformation retract of X if there exist a retraction $r : X \rightarrow A$ and a homotopy $f : X \times I \rightarrow A$ such that:

$f(x,0) = x, f(a,t) = a, x \in X, a \in A, t \in I$ $\left[6\right]$.

4- A regular surface S is called a ruled surface if the Gaussian curvature is zero $\forall p \in S$, $\left[1\right]$.

5- Let M, N are two manifolds with dimensions m, n respectively, a map $f : M \rightarrow N$

***Corresponding Author:** Dr. K .Khalifa, Department of mathematics-EL-Aris Faculty of Education Seuz Canal University -EL-Arish –Egypt .

is said to be an isometric folding of M into N if every piecewise geodesic path $\gamma : I \to M$ the induced path $f \circ \gamma : I \to N$ is piecewise geodesic and of the same length $[8]$.

6 - Let M, be an m-dimensional manifold, a map $f : M \to N$ is said to be isocurvature folding if f preserves the curvature at corresponding points $[7]$.

The main results:

We will introduce the following definitions:

Definition (1):

Let M, be an m-dimensional manifold, M is said to be ruled m- manifold if the Gaussian curvature vanishes everywhere; $i.e. K = 0 \forall p \in M$.

Definition (2):

A subset A of ruled m- manifold M is called a ruled retraction of if there exist a continuous map $r : M \to A$ such that:

(i) M is open,

(ii) $r(M) = A$,

(iii) $r(a) = a, a \in A$.

(iv) $r(M)$ ruled manifold.

Definition (3):

A subset A of ruled m- manifold M is called a ruled deformation retraction of M if there exist a continuous map $r : M \to A$ and a homotopy $f : M \times I \to A$ such that:

$(i) f(x, 0) = x, \forall x \in M$

$(ii) f(x, 1) = r(x), x \in X$,

$(iii) f(a, t) = a, a \in A, t \in I$.

Definition (4):

Let M, N are two smooth manifolds with dimensions m, n respectively, a map $f_r : M \to N$ is said to be a ruled folding if and if f_r preserves the Gaussian

Curvature vanishes at corresponding points.

Theorem (1):

The topological folding of ruled manifold M into itself is not necessarily a ruled manifold, but the isocurvature folding of ruled manifold M into itself is necessarily a ruled manifold.

Proof: : Let M be ruled manifold, $f : M \to M$ topological folding.

Since the topological folding does not preserves the curvature, then $\forall p \in M$ the Gaussian curvature $K(\sigma, x) = 0$, but $K'(\sigma', y)$, where $f(x) = y$ is not necessarily zero at each point.

Let M be ruled manifold, $f : M \to M$ isocurvature folding.

Since f maps area with Gaussian curvature $K(\sigma, x)$ into area with Gaussian curvature $K'(\sigma', y)$ and since M is ruled, $K(\sigma, p) = K'(\sigma', y) = 0, f(x) = y$.

Then the isocurvature folding $f : M \to M$ is ruled folding.

Theorem (2):

Let M be a surface embedded in R^3. Then the limit of topological folding of M into itself is a ruled surface; $i.e. \lim_{n\to a} f_n(M) = M', M'$ is a ruled surface.

Proof: The proof of this theorem comes by topologi 339 g successfully M.

$f_1 : M \to M$,

$f_2 : f_1(M) \to f_1(M)$,

$f_3 : f_2(f_1(M)) \to f_2(f_1(M))$,....,

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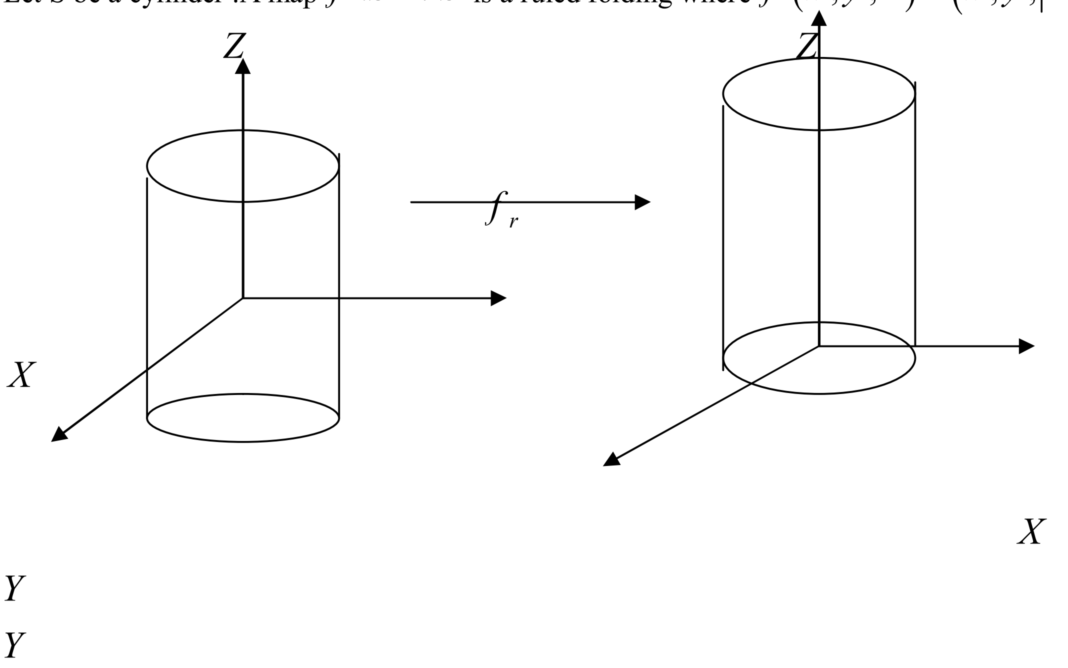
Then $\exists$ a number "a " such that :

$f_a : f_{a-1}(f_{a-2} \ldots f_1(M)) \to f_{a-1}(f_{a-2} \ldots f_1(M))$,

Such that $f_a(M) = M'$ is a plane which is ruled in R^3.

Example (1):

Let S be a cylinder .A map $f : S \to S$ is a ruled folding where $f(x, y, z) = (x, y, |z|)$ fig(1).



fig(1).

Theorem (3):

Let M, N are two smooth manifolds with dimensions m, n respectively, $m \le n$.

Any topological folding $f : M \to N$ induces a ruled folding $\overline{f_r} : T_x M \to T_y N$,

$f(x) = y$.

Proof: Let $f : M \to N$ a topological folding of M into N, then for all $y = f(x)$,

$K(\sigma, x) = K'(\sigma', y)$ Where K, K' is the Gaussian curvature of M and N respectively?

Since $T_x M = R^m$ and $T_y N = R^n$ with Gaussian curvature vanishes everywhere.

Then the induced a ruled folding $\overline{f_r} : T_x M \to T_y N$ is a ruled folding.

Theorem (4):

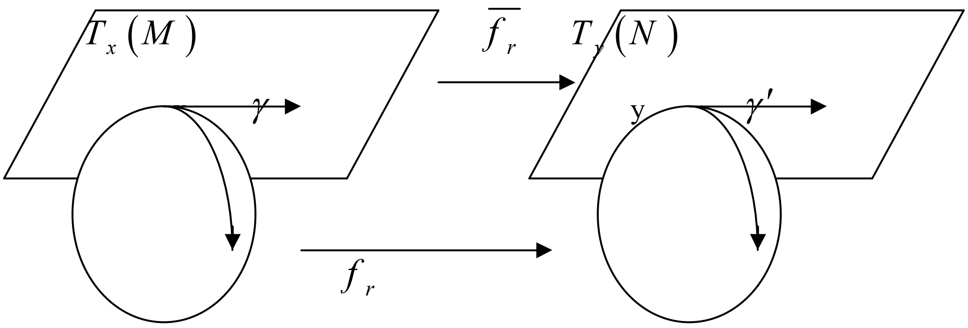
Let M, N are two ruled manifolds without conjugate points. Any ruled folding $f_r : M \to N$ induces a ruled folding on the tangent spaces by the inverse exponential map. But a ruled folding $f_r : T_x M \to T_y N$ need not induces a ruled

folding $\overline{f} : M \to N$ by the exponential map.

Proof:

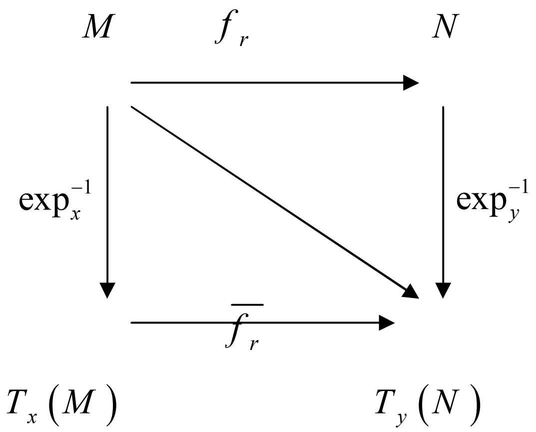
First: Let M, N are two ruled h manifolds without conjugate points, $\exp_x^{-1} : M \to T_x M$ be given by $\exp_x^{-1}(\alpha) = \gamma, \alpha$ is a curve in M,

$\alpha(0) = \gamma(0) = x, L(\alpha) = L(\gamma)$. Then $K(\sigma, x) = 0 \; \forall x \in T_x, (M)$, since $T_x(M)$ is a ruled . See fig (2)



M fig (2) N

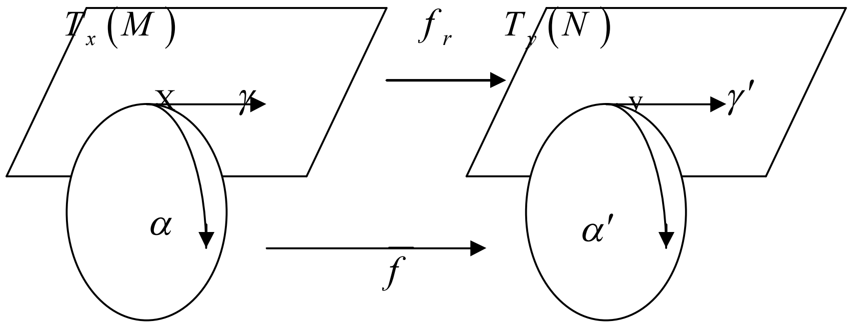
Since $T_x M, T_y N$ are ruled, then $\overline{f_r} : T_x M \to T_y N$ is ruled and the following diagram is commutative for lengths of the curves and Gaussian curvature ; *i.e.*



$\overline{f_r} \circ \exp_x^{-1} = \exp_y^{-1} \circ f_r$ for lengths of the curves and Gaussian curvature.

Second: Let $f_r : T_x M \to T_y N$ be a ruled folding defined by $f_r(\gamma) = \gamma'$ where $\gamma \in T_x M$ and $\gamma' \in T_y N$.

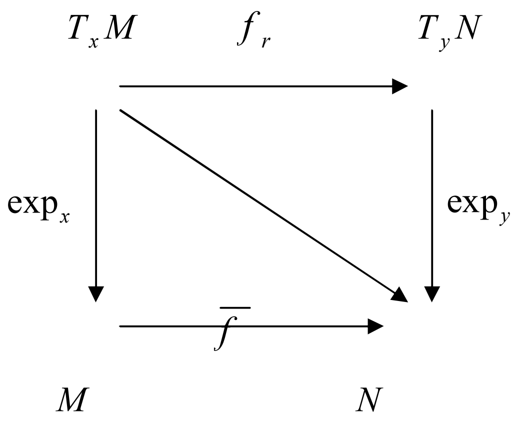
Since $\exp_x : T_x M \to M$ maps γ into α where $\alpha(0) = \gamma(0) = x$, $L(\gamma') = L(\alpha')$, see fig(3).



M fig(3) N

and $K(\gamma) \neq K(\alpha)$,then $K \neq K'$ at corresponding points ,also $L(\gamma') = L(\alpha')$, but $K(\gamma') \neq K(\alpha')$ then $K' \neq K''$.

Thus $\overline{f} : M \to N$ which maps α into α' is not ruled and the following diagram is commutative for lengths of the curves but not for Gaussian curvatures ;*i.e*.



$\overline{f} \circ \exp_x = \exp_y \circ f_r$ for lengths of the curves but not for Gaussian curvatures.

If M is ruled m- manifolds the deformation of M not necessarily a ruled manifold. The deformation of M is ruled manifold if there is a homeomorphism f maps area with Gaussian curvature $K(\sigma, x) = 0$ into area with Gaussian curvature $K'(\sigma', y) = 0$,$f(x) = y$.

Theorem (5):

The deformation of ruled surface may be ruled or not.

Proof: Let S be a ruled surface and $d_K : S \rightarrow S'$ conditional deformation with

$d_K(S) = S'$, where $K = \dfrac{eg - f}{EG - F}, K' = \dfrac{e'g' - f'}{E'G' - F'}$ are the Gaussian curvature of S and S' respectively where $X(u,v)$ be a parameterization of ruled surface, $e = \langle X_u, X_u \rangle, f = \langle X_v, X_v \rangle, g = \langle X_u, X_v \rangle, e, f, g, e', f', g'$ are the coefficient of the first fundamental form at ruled x and x' respectively.

$$E = -\langle N_u, X_u \rangle, f = -\langle N_v, X_U \rangle, G = \langle N_V, X_v \rangle, E, F, G, E', F', G'$$

are the coefficient of the second fundamental form at ruled x and x' respectively.

Suppose $d_K(S) = S'$ such that d_K is the deformation which makes :

$$\frac{eg - f}{EG - F} = \frac{e'g' - f'}{E'G' - F'} = 0.$$

In this case only the deformation of ruled surface is a ruled surface.

Any other deformation $d' : S \rightarrow S''$ such that $K \rightarrow K', K = 0, K' \neq 0$ is not ruled surface.

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