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Unfolding a Polygon to a Closed Surface

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ABSTRACT

In this article we discuss how can we unfold a triangle to obtain a closed surface taking into consideration that if two closed surfaces M_1 *and* M_2 come from unfolding a polygon P_n and have the same quintuplet $(k, n, \alpha, \beta, \gamma)$, then they are topologically equivalent.

Keyword: Closed surface, folding , Unfolding polygon

INTRODUCTION

Definitions:

1- Let K and L be Simplicial complexes. A Simplicial map $f : K^{(0)} \to L^{(0)}$ from K to L is a Simplicial folding from K into L if for every I and all $\sigma \in K^{(i)}, f(\sigma) \in L^{(i)}$ [1].

2-Let K be a finite Simplicial n-complex, let $f : K \to K$ be a Simplicial folding With $f(K) \neq K$ .Denote by $I_k = (\lambda_{ij}^k)$, the K^{th} incidence matrix of incidence members of simplexes σ^k and σ^{k-1} of K which is defined by :

$$\lambda_{ij} = \begin{cases} \pm 1 & \text{if } \sigma^{k-1} \text{ is a face of } \sigma^k \\ 0 & \text{if } \sigma^{k-1} \text{ is not a face of } \sigma^k . \end{cases}$$

Where if σ^{k-1} is a face of σ^k and $\sigma^{k-1} = v_1 v_2 .... v_k$,then $\sigma^k = \pm v_1 v_2 .... v_k$ for some additional matrix v .And the incidence number

$$\lambda_{ij} = \begin{cases} +1 & \text{if } \sigma^{k-1} = v_1 v_2 .... v_k \\ -1 & \text{if } \sigma^k = v_1 v_2 .... v_k . \end{cases}$$

The matrix I_k s of order $r \times s$ where r and s are number of k and $(k-1)$ -simplexes in K.Also the incidence matrices satisfy $I_k I_{(k-1)} = 0, k = 1,2,...,(n-1)$.The set of all Simplicial foldings $f : K \to L$ is denoted by $\Delta(K, L)$ and $\Delta(K)$ denoted by The set of all Simplicial foldings of K into itself .

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Regular folding to a polygon

Let $f : M \to P_n$ be a regular folding with α vertices , β edges and γ faces. Then the following relations between α, β, γ, n and k .

$(1)\, n\gamma = 2\beta;$

$(2)\, n\gamma \geq 4\alpha, hence\; eM = \alpha - \beta + \gamma \leq \alpha\left(\frac{4}{n-1}\right);$

(3) If f is a regular folding with valancey k , then: $k\alpha = n\gamma = 2\beta.$

(4) If $M = M_g$ is an orintable surface with genus g , then we have:

$g = 1 + \left(\frac{(k-2)(n-2)-4}{4n}\right)\alpha, [3].$

Proposition:

Let K and L be finite Simplicial $n-$complexes, and let $f \in \Delta(K, L)$,then $f(K) < L.$

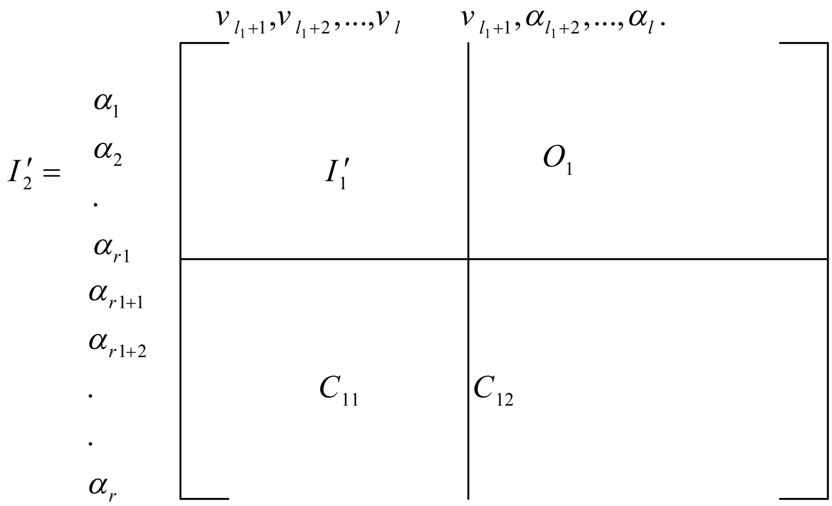
In particular if $f \in \Delta(K)$ with $f(K) = K' \neq K$ then $K' < K, [2].$

This proposition suggests that the incidence matrices $I'_k . k = 1, 2, ..., n$ of $f(K) = K' \neq K$ are sub matrices of incidence matrices $I_k, k = 1, 2, ..., n$ of $K\,(or\, L)$ in general possibly after rearranging its rows and columns. To better visualize consider the 2^{nd} and 1^{st} incidence matrices I_2, I_1 of K and I'_2, I'_1 of $f(K) = K' \neq K$, where $f \in \Delta(K)$ .Its claimed that the matrix can be partitioned into 4 blocks such that I'_2 appear in the upper corner block and a zero matrix O_2 in the upper right one. If the complementary matrix C_{21} of O_2 is arranged in such away such that the $2-$simplexes $\sigma_1, \sigma_2 ..., \sigma_{r1}$ are the images of $2-$simplexes $\sigma_{r1+1}, \sigma_{r1+2}, ..., \sigma_r$ respectively ,see Fig(1).Then the matrix C_{22} , then the complement of I'_2 in I_2 will be sub matrix of I'_2 possibly after deleting the columns of I'_2 which are not images of any of the edges $\alpha_{s_1+1}, \alpha_{s_1+2}, ..., \alpha_s$.

$$I'_2 = \begin{array}{c} \\ \sigma_1 \\ \sigma_2 \\ . \\ \sigma_{r1} \\ \sigma_{r1+1} \\ \sigma_{r1+2} \\ . \\ . \\ \sigma_r \end{array} \begin{array}{c|c} \alpha_1, \alpha_2, ..., \alpha_{s_1} & \alpha_{s_1+1}, \alpha_{s_1+2}, ..., \alpha_s . \\ \hline I'_2 & O_2 \\ \hline C_{21} & C_{22} \end{array}$$

Fig(1)

In the same way I_1 can be partitioned such that I_1' appears in the upper left corner block and a zero matrix O_1 in the upper right one see Fig(2).Again if the complementary matrix C_{11} of O_1 is arranged in such that the edges $\alpha_1, \alpha_2, ..., \alpha_{s_1}$ are the images of $\alpha_{s_1+1}, \alpha_{s_1+2}, ..., \alpha_s$ respectively . Then the matrix C_{12} will be sub matrix of I_1' possibly after deleting the rows and columns of I_1' which are not images of any of the edges $\alpha_{s_1+1}, \alpha_{s_1+2}, ..., \alpha_s$ and vertices $v_{l_1+1}, v_{l_1+2}, ..., v_l$ respectively.

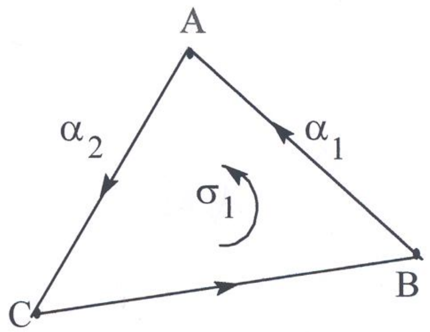


Fig(2)

The main results:

1- Unfolding a triangle to a sphere

Suppose a polygon P_3 triangle, and we want to unfold it symmetrically along one of edges say $\alpha_1 = AB$.
The $1-$incidence matrix of the given triangle is given by
Which of order 1×3 see Fig(3)



Fig(3)

Now we construct the $1-$incidence matrix after unfolding P_3 along α_1 we delay the elements of extending columns α_1, and construct a matrix C_{22} to has the same (opposite) elements as the rest elements of the matrix C_{11}.Thus C_{22} may have the following form:

$$C_{22} = [1 \quad 1]$$

Then we construct a zero a matrix C_{12} of order 1×2 and we call the columns of order α_4, α_5

$$C_{12} = \begin{bmatrix} \alpha_4 & \alpha_5 \\ 0 & 0 \end{bmatrix}$$

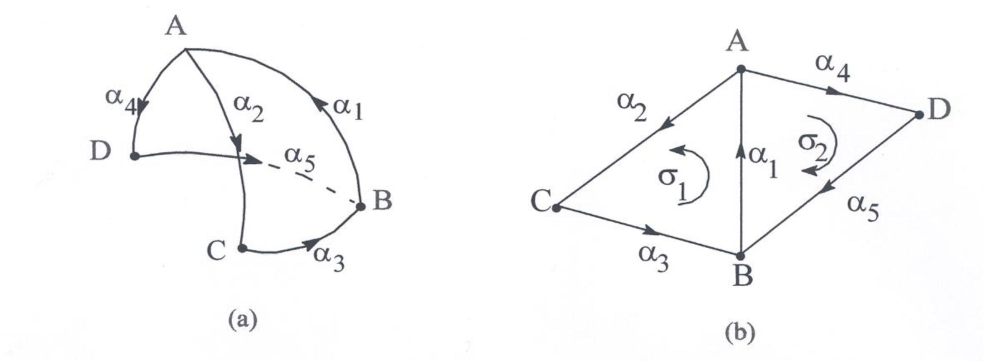
Lastly we construct C_{21} matrix of order 1×3 by putting all elements zeros except the element of extending (unfolding) column α_1 , is the same as in C_{11} but with opposite sign i.e.;

$$C_{21} = \sigma_2 \begin{bmatrix} -1 & 0 & 0 \end{bmatrix}$$

If we collect these four matrices in one matrix we obtain the 1- incidence matrix of unfolding P_3 along α_1 in the following form.

$$I_2 = \begin{matrix} \\ \sigma_1 \\ \\ \sigma_2 \end{matrix} \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \vdots & \alpha_4 & \alpha_5 \\ 1 & 1 & 1 & \vdots & 0 & 0 \\ - & - & - & - & - & - \\ -1 & 0 & 0 & \vdots & 1 & 1 \end{bmatrix}$$

The unfolding surface with I_2 as 1- incidence matrix can be realized as in Fig (4) (a) or (b) which is not stratifications of closed surface. So to have a closed surface we unfold again along the two edges α_3 and α_5 (or α_2 and α_4).



Fig(4)

Once again we construct 1- incidence matrix of the extending Simplicial complex by same procedure as above, hence

$$C'_{11} = \begin{matrix} \\ \sigma_1 \\ \sigma_2 \end{matrix} \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 1 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

And to obtain C'_{22} we delay the columns α_3 and α_5 and hence we have the following form

$$C'_{22} = \begin{matrix} \\ \sigma_1 \\ \sigma_2 \end{matrix} \begin{bmatrix} \alpha_6 & \alpha_7 & \alpha_8 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Then we construct a zero a matrix C'_{12} of order 2×3 and we call the columns $\alpha_6, \alpha_7, \alpha_8$ i.e.;

$$C'_{12} = \begin{bmatrix} \alpha_6 & \alpha_7 & \alpha_8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Finally we construct C'_{21} matrix of order 2×5 with zero elements except that of extending edges (α_3 and α_5) is the same as that of C'_{11} but with opposite sign i.e.;

$$C'_{21} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

By collecting these four matrices in one matrix we obtain the 1- incidence matrix I'_2 of order 4×8 of the following form.

$$I'_2 = \begin{bmatrix} & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \square & \alpha_6 & \alpha_7 & \alpha_8 \\ \sigma_1 & 1 & 1 & 1 & 0 & 0 & \square & 0 & 0 & 0 \\ \sigma_2 & -1 & 0 & 0 & 1 & 0 & \square & 0 & 0 & 0 \\ & = & = & = & = & = & = & = & = & = \\ \sigma_3 & 0 & 0 & -1 & 0 & 0 & \square & 1 & 1 & 0 \\ \sigma_4 & 0 & 0 & 0 & 0 & -1 & \square & -1 & 0 & 1 \end{bmatrix}$$

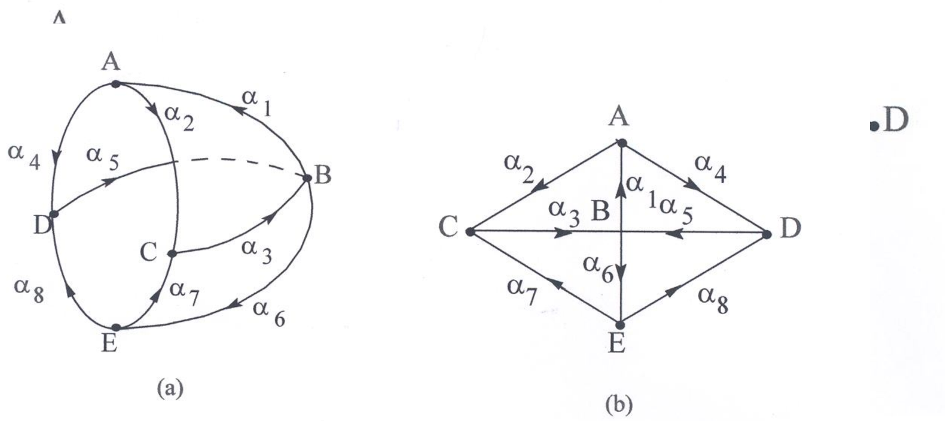


Fig (5)

The figure has the following numbers $\alpha=5, \beta=8, \gamma=4, n=3$ and $k=4$. These numbers does not satisfy most of relations. Thus figure (5) can not represent stratifications of closed surface which simplicially folded over a triangle.

Thus to obtain a closed surface we unfold again the last surface along the four edges $\alpha_2, \alpha_4, \alpha_7$ and α_8, the boundary of the simplicial complex in Fig (5), and hence construct the matrices as follows.

$$C''_{11} = \begin{bmatrix} & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 \\ \sigma_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \sigma_2 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \sigma_3 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ \sigma_4 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix},$$

$$C''_{22} = \begin{matrix} \\ \sigma_5 \\ \sigma_6 \\ \sigma_7 \\ \sigma_8 \end{matrix} \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

Where we have delayed the columns $\alpha_2, \alpha_4, \alpha_7$ and α_8 of C''_{11}
Also

$$C''_{12} = \begin{bmatrix} \alpha_9 & \alpha_{10} & \alpha_{11} & \alpha_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

is a matrix of order 4×4 with columns say $\alpha_9, \alpha_{10}, \alpha_{11}$ and α_{12}.
Finally

$$C''_{21} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_7 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

is a matrix of order 4×8 with zero elements except that of the extending edges is the same as of C''_{11} but with opposite sign.
By collecting last four matrices in one matrix we obtain a matrix I''_2 of order 8×12 of the following form.

$$I''_2 = \begin{matrix} \\ \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \\ \sigma_5 \\ \sigma_6 \\ \sigma_7 \\ \sigma_8 \end{matrix} \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & & \alpha_9 & \alpha_{10} & \alpha_{11} & \alpha_{12} \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & \square & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \square & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & \square & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & \square & 0 & 0 & 0 & 0 \\ = & = & = & = & = & = & = & = & = & = & = & = & = \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \square & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & \square & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \square & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \square & 0 & 0 & -1 & -1 \end{bmatrix}$$

The surface corresponding to I''_2 is shown in Fig (6).

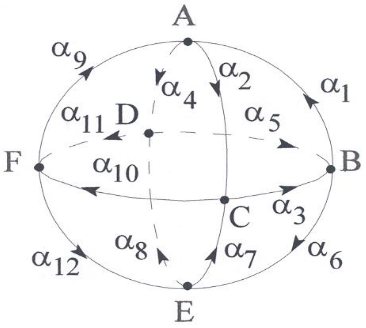


Fig (6)

Figure (6) has quintuplet $(4,3,6,12)$ these numbers satisfies all relations. Thus figure (6) represent a closed surface which can be simplicially folded onto P_3 .In fact it is a sphere.

2- Unfolding a triangle to a tours

Suppose P_3 is a triangle, and we want to unfold it symmetrically along one of its edges say $\alpha_3 = AC$ to have a tours. First $1-$incidence matrix of P_3 is given by

$$I_2 = \begin{array}{c} \\ \sigma_1 \end{array}\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 1 & 1 & 1 \end{bmatrix}$$

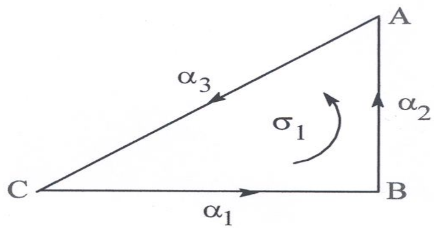
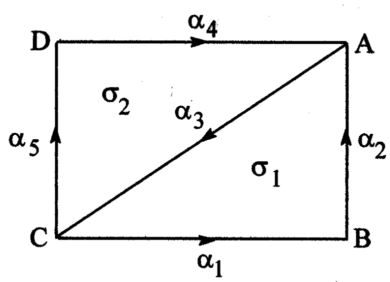


Fig (7)

After unfolding Fig(7) along α_3 we have a simplicial complex with $1-$incidence matrix I_2' of the form

$$I_2' = \begin{array}{c} \\ \sigma_1 \\ \\ \sigma_2 \end{array}\left[\begin{array}{cccccc} \alpha_1 & \alpha_2 & \alpha_3 & | & \alpha_4 & \alpha_5 \\ 1 & 1 & 1 & | & 0 & 0 \\ - & - & - & - & - & - \\ 0 & 0 & -1 & | & 1 & 1 \end{array}\right]$$

Which can be realized as given Fig (8).



Fig(8)

The surface can be unfolded again along the edges α_5 or $\alpha_1, \alpha_2, \alpha_4$ with:

$$C'_{11} = \begin{matrix} \\ \sigma_1 \\ \sigma_2 \end{matrix} \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}$$

$$C'_{22} = \begin{matrix} \\ \sigma_3 \\ \sigma_4 \end{matrix} \begin{bmatrix} \alpha_6 & \alpha_7 & \alpha_8 & \alpha_8 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$C'_{12} = \begin{matrix} \\ \sigma_1 \\ \sigma_2 \end{matrix} \begin{bmatrix} \alpha_6 & \alpha_7 & \alpha_8 & \alpha_9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{And,} C'_{21} = \begin{matrix} \\ \sigma_3 \\ \sigma_4 \end{matrix} \begin{bmatrix} \alpha_6 & \alpha_7 & \alpha_8 & \alpha_9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

By collecting these four matrices in one matrix we obtain the 1- incidence matrix I'_2 of order 4×8 of the following form.

$$I''_2 = \begin{matrix} \\ \sigma_1 \\ \sigma_2 \\ \\ \sigma_3 \\ \sigma_4 \end{matrix} \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & & \alpha_6 & \alpha_7 & \alpha_8 & \alpha_9 \\ 1 & 1 & 1 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & \vdots & 0 & 0 & 0 & 0 \\ = & = & = & = & = & = & = & = & = & = \\ 0 & 0 & 0 & 0 & 0 & \vdots & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & \vdots & 0 & 0 & -1 & -1 \end{bmatrix}$$

The simplicial complex corresponding I''_2 is shown in Fig (9).

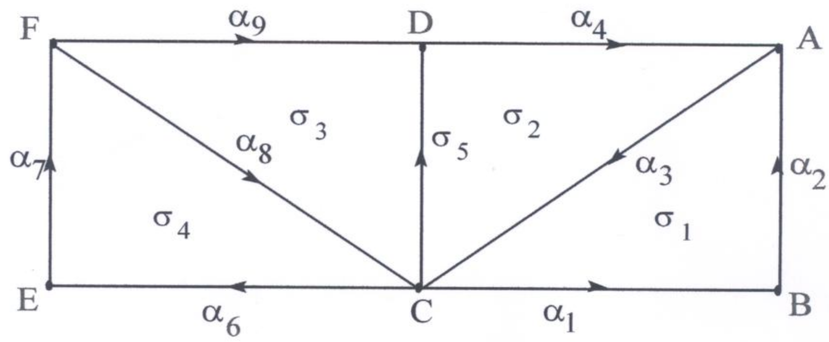


Fig (9).

Once again we can unfold the simplicial complex shown in Fig (9) along the edges α_1 and α_2 .The incidence matrix can construct as follows:

$$C''_{11}=\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & \alpha_9 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & -1 \end{bmatrix}, C''_{12}=\begin{matrix} \\ \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{matrix}\begin{bmatrix} \alpha_{10} & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} & \alpha_{16} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C''_{21}=\begin{matrix} \\ \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{matrix}\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & \alpha_9 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & -1 \end{bmatrix}$$

$$C''_{22}=\begin{matrix} \\ \sigma_5 \\ \sigma_6 \\ \sigma_7 \\ \sigma_8 \end{matrix}\begin{bmatrix} \alpha_{10} & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} & \alpha_{16} \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix}$$

By collecting the last four matrices in one matrix we obtain the 1- incidence matrix I'''_2 of order 8×16 of the following form.

$$I'''_2=\begin{matrix} \\ \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \\ \sigma_5 \\ \sigma_6 \\ \sigma_7 \\ \sigma_8 \end{matrix}\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & \alpha_9 & & \alpha_{10} & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} & \alpha_{16} \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \square & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & \square & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & \square & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & -1 & \square & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ = & = & = & = & = & = & = & = & = & = & = & = & = & = & = & = & = \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \square & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \square & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \square & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \square & 0 & 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

The surface corresponding to I'''_2 is shown in Fig (10), which is homeomorphic to the tours.

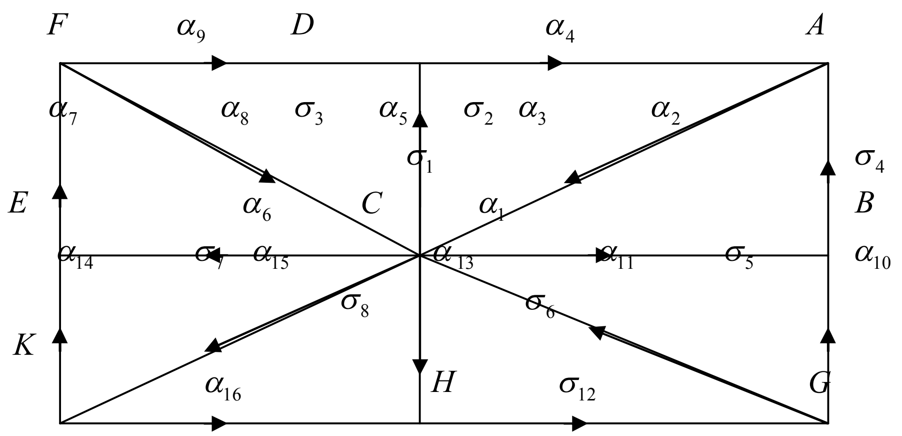
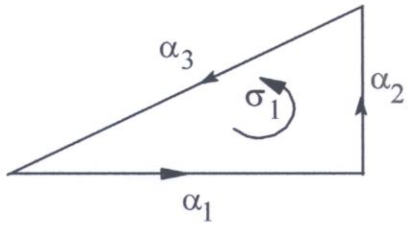
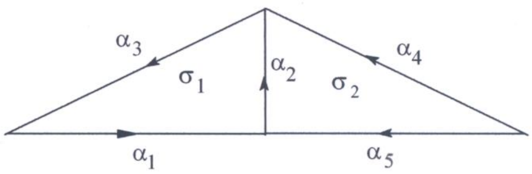


Fig (10),

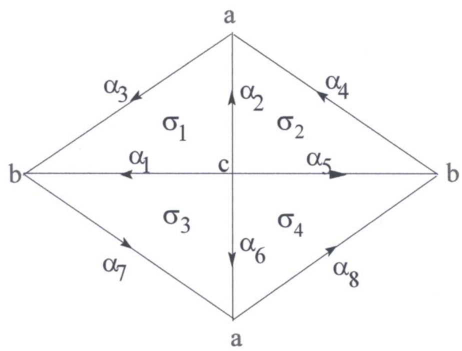
3- Unfolding a triangle to a projective plane

The projective plane can be obtained by unfolding a triangle (P_3) given in Fig(11)along the edges ,then unfolding again the surface given in Fig(12) along α_1 and α_5 to have a projective plane Fig(13)





Fig(12)



Fig(13)

The incidence matrices of three simplicial complexes are as follows

$$I_2 = \begin{array}{c} \\ \sigma_1 \end{array}\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 1 & 1 & 1 \end{bmatrix},$$

$$I_2' = \begin{array}{c} \\ \sigma_1 \\ \\ \sigma_2 \end{array}\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \vdots & \alpha_4 & \alpha_5 \\ 1 & 1 & 1 & \vdots & 0 & 0 \\ - & - & - & - & - & - \\ 0 & -1 & 0 & \vdots & -1 & 1 \end{bmatrix}$$

$$I_2'' = \begin{array}{c} \\ \sigma_1 \\ \sigma_2 \\ \\ \sigma_3 \\ \sigma_4 \end{array}\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \vdots & \alpha_6 & \alpha_7 & \alpha_8 \\ 1 & 1 & 1 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & \vdots & 0 & 0 & 0 \\ = & = & = & = & = & = & = & = & = \\ -1 & 0 & 0 & 0 & 0 & \vdots & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & \vdots & -1 & 0 & 1 \end{bmatrix}$$

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