J. Basic. Appl. Sci. Res., 1(7) 617-621, 2011

ISSN 2090-424X
Journal of Basic and Applied Scientific Research
www.textroad.com

Order Reduction of Multivariable System Using Clustering, ISE Minimization and Dominant Pole Technique

S. K. Agrawal[1], D. Chandra[2], and I. A. Khan[3]

[1]Lucknow Institute of Technology and Management, Lucknow
[2]Motilal Nehru Institute of Technology, Allahabad
[3]Integral University, Lucknow

ABSTRACT

A mixed method is proposed for finding stable reduced order models of multivariable systems. This method reduces the order of a system by retaining the dominant pole in the denominator and evaluating rest of denominator coefficients by clustering technique. All the numerator coefficients are treated as free parameters for optimization using integral square error (ISE) minimization techniques. This method guarantees stability of the reduced order model when the original high order system is stable. The methodology of the proposed method is illustrated with the help of an example from literature.

KEY WORDS: Clustering technique, Order reduction, ISE, dominant pole, Multivariable system.

INTRODUCTION

The modeling of complex dynamic systems is one of the most important subjects in engineering. Moreover, a model is often too complicated to be used in real problems, so approximation procedures based on physical considerations or using mathematical approaches must be used to achieve simple models than original ones. In the analysis and synthesis of a high order multivariable system, it is often necessary to obtain a low order model so that it may be used for an analogue or digital simulation of the system. A large number of publications on model order reduction have been published. Various authors such as [Chen, 1974], [Genesio and Milanese, 1976], [Elrazaz and Sinha, 1981], [Pal and Prasad, 1990], [Prasad Pal and Pant, 1995], [Prasad Pal and Pant, 1998] and many others have studied extensively different methods of model reduction.

The proposed method is a mixed method for order reduction of multivariable systems which combine the clustering technique, integral square error (ISE) minimization techniques and retains one or more dominant poles. The proposed method for finding stable low-order equivalents of high-order systems is computationally easy to program and conceptually simple.

STATEMENT OF PROBLEM

Consider an nth order linear time invariant dynamic multivariable system (q input - p output) described in frequency domain by the transfer matrix:

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \ldots\ldots + a_{n-1}s + a_n}{s^n + b_1 s^{n-1} + \ldots\ldots + b_{n-1}s + b_n} = \frac{\sum_{k=1}^{n} a_k s^{n-k}}{s^n + \sum_{k=0}^{n} b_k s^{n-k}} \tag{1}$$

Where $a_k = [A_{ij}^k]_{p\,x\,q}$
i= 1,2, ….,p
j= 1,2, ….,q

***Corresponding Author:** Sushil Kumar Agrawal, Lucknow Institute of Technology and Management, Lucknow.
Email: skagrawal10@rediffmail.com

k= 1, 2, …., n
are the matrices of order p X q
and b_k where k= 1,2……n are scalar constants.
The corresponding r th (r < n) order reduced model is of the form :-

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{\hat{a}_1 s^{r-1} + \hat{a}_2 s^{r-2} + \dots\dots + \hat{a}_{r-1} s + \hat{a}_r}{s^r + \hat{b}_1 s^{r-1} + \dots\dots + \hat{b}_{r-1} s + \hat{b}_r}$$

$$= \frac{\sum_{k=1}^{r-1} \hat{a}_k s^{r-k}}{s^r + \sum_{k=1}^{r} \hat{b}_k s^{r-k}} \tag{2}$$

Where $\hat{a}_k = [B_{ij}^k]_{p\,X\,q}$
i= 1,2, ….,p
j= 1,2, ….,q
k= 1, 2, …., r
are the matrices of order p X q
and $\hat{b}_k$, where, k = (0,1,2,…r-1, r) are scalar constants

In this paper, assuming the original system described by (1), our problem is to find a reduced order model of the form (2) such that the reduced model retains the important chamcteristics of the original system and approximates its response as closely as possible for the same type of input.

DESCRIPTION OF THE METHOD

The proposed method consists of the following three steps:
Step-I: Determination of the Denominator Polynomial of the reduced model by Retaining dominant pole in the reduced order model and determining the remaining poles using clustering technique [Sinha and Pal, 1990]

The response of the system largely depends upon the pole which is near the j-ω axis, these poles are called as dominant poles. Keeping these poles in the reduced order model facilitates better response. Therefore one or more dominant poles are retained in this reduction process.
The criterion for-grouping the poles in one particular cluster are based on relative distance between the poles and the desired order in the process of reduced order modeling.
Since each cluster may be finally replaced by a single (pair of) real (complex) pole, the following rules are used for clustering the poles to get the denominator polynomial for reduced order models.
(i) Separate clusters should be made for real poles and complex poles.
(ii) Poles on the jw-axis have to be retained in the reduced order model.
The cluster center can be formed using a simple method known as 'inverse distance measure', which is explained as follows:
Let, k real poles in one cluster are (p_1, p_2, p_3, p_4…..p_k) then the Inverse Distance Measure (IDM) criterion identifies the cluster center as

$$P_c = \{(\textstyle\sum_{i=1}^{r}(1/p_i)) \div z\}^{-1} \tag{3}$$

Where p_c is cluster center from z real poles of the original system.

Let, m pair of complex conjugate poles in a cluster be $[(\alpha_1 \pm j\,\beta_1),\ (\alpha_2 \pm j\,\beta_2), \dots\dots\dots\dots (\alpha_m \pm\ j\,\beta_m)]$ then the IDM criterion identifies the complex cluster center in the form of $A_c \pm\ jB_c$.

$$A_c = \left\{\left(\sum_{i=1}^{m}(1/\alpha_1)\right) \div m\right\}^{-1} \text{ and}$$

$$B_c = \left\{\left(\sum_{i=1}^{m}(1/\beta_1)\right) \div m\right\}^{-1}$$

(4)

For getting the denominator polynomial for rth order reduced model, one of the following cases may occur.

Case 1: If all cluster centers are real, then denominator polynomial for rth order reduced model with retained dominant pole can be obtained as

$$D_r(s) = (s - p_d)(s - p_{c1})(s - p_{c2}) \dots (s - p_{cr-1}) \quad (5)$$

where $p_{c1}, p_{c2}, \dots \dots p_{cr}$ are $I^{st}, 2^{nd}, \dots \dots, r - 1^{th}$ cluster center and p_d is the dominant pole

Case 2: If $(r - 3)$ cluster centers are real and one pair of cluster center is complex conjugate, then $D_r(s)$ can be obtained as

$$D_r(s) = (s - p_d)(s - p_{c1})(s - p_{c2}) \dots \dots \dots (s - p_{c(r-3)})(s - \dot{p}_{c1})(s - \check{p}_{c1}) \quad (6)$$

where $\dot{p}_{c1}$ and $\check{p}_{c1}$ are complex conjugate cluster centers or = $\dot{p}_{c1} = A_c, + jB_c$ and $\check{P}c_1 = A_c - jB_c$.

Case 3: If all the cluster centers are complex conjugate, then reduced denominator polynomial can be taken as

$$D_r(s) = (s - \dot{p}_{d1})(s - \check{p}_{d1})(s - \dot{p}_{c1})(s - \check{p}_{c1})(s - \dot{p}_{c2})(s - \check{p}_{c2}) \dots (s - \dot{p}_{c\frac{(r-1)}{2}})(s - \check{p}_{c\frac{(r-1)}{2}}) \quad (7)$$

Step-II: *Determination of the numerator of rth order reduced model using Integral square error [Singh, Chandra and Kar, 2004], [Luss and Jakola, 1973].*

Once the reduced order denominator of the stable nth order linear time invariant dynamic multivariable system is obtained, we utilize the denominator coefficients as fixed parameters and all the coefficients of the numerator are treated as free parameters in the process of optimization based on Integral Square Error Minimization technique.

NUMERICAL EXAMPLES

Example-I: Consider a fourth order system described by the transfer function

$$G(s) = \begin{bmatrix} \dfrac{(s+20)}{(s+1)(s+10)} \\ \dfrac{(s+10)}{(s+2)(s+5)} \end{bmatrix}$$

$$= \frac{\begin{bmatrix} g_{11}(s) \\ g_{21}(s) \end{bmatrix}}{D(s)}$$

$$G(s) = \frac{\begin{bmatrix} g_{11}(s) \\ g_{21}(s) \end{bmatrix}}{(s+1)(s+2)(s+5)(s+10)}$$

Using step-1, two cluster centers from the real poles -1, -2 and -3, -4 can be formed from equation (3) as

$$P_{c1} = \left[\left(\frac{1}{-1} + \frac{1}{-2}\right) \div 2\right]^{-1} = -1.3333$$

$$P_{c2} = \left[\left(\frac{1}{-5} + \frac{1}{-10}\right) \div 2\right]^{-1} = -6.667$$

Therefore, denominator $D_2(s)$ can be synthesized using equation (5) and is given by

$D_2(s) = (s+1.3333)(s+6.667)$

$= 8.8871 + 8s + s^2$

The reduced second order model can be taken in the form of

$$G_2(s) = \frac{\begin{bmatrix} g_{11}(s) \\ g_{21}(s) \end{bmatrix}}{s^2 + 8s + 8.8871}$$

Now on using step 2, the model takes the following form:

$$G_2(s) = \frac{\begin{bmatrix} -1.003749s + 17.7742 \\ 2.214090s + 8.8871 \end{bmatrix}}{s^2 + 8s + 8.8871}$$

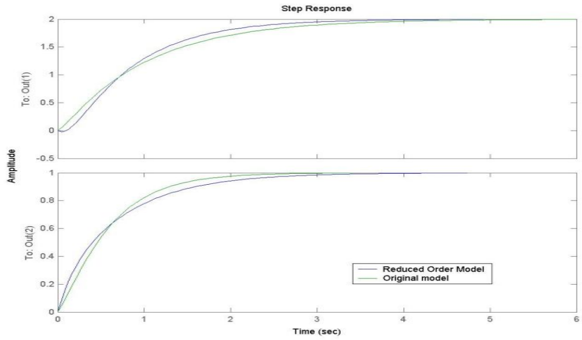

Figure 1: Comparison of step responses reduced order model

On solving the problem by dominant pole technique

Step l. The dominant pole (s+1) retained in the reduced order model and one cluster center from the real poles -2,-3 and -4 was formed from equation (3) as

$$P_{c1} = \left[\left(\frac{1}{-2} + \frac{1}{-5} + \frac{1}{-10}\right) \div 3\right]^{-1} = 3.75$$

Therefore, denominator $D_2(s)$ can be synthesized using equation (5) and is given by

$D_2(s) = (s+1)(s+3.75)$

$= s^2 + 4.75s + 3.75$

The reduced second order model can be taken in the form of

$$G_2(s) = \frac{\begin{bmatrix} g_{11}(s) \\ g_{21}(s) \end{bmatrix}}{s^2 + 4.75s + 3.75}$$

Now on using step 2, the model takes the following form:

$$G_2(s) = \frac{\begin{bmatrix} 1.611031s + 7.50 \\ 2.293924s + 3.75 \end{bmatrix}}{s^2 + 4.75s + 3.75}$$

This reduced model is stable and non-minimum phase.

The comparison of step responses of the reduced 2nd order models and the original fourth order system is shown in Figure 2.

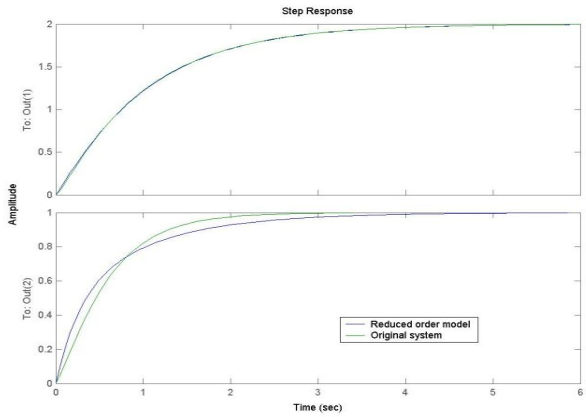

Figure 2: Comparison of step responses reduced order model with dominant pole

Conclusion

A mixed method for order reduction of MIMO stable system has been presented, based on dominant pole retention, Integral square error minimization and clustering technique. In this method, one or more dominant poles are retained and remaining poles are obtained by clustering technique. The coefficients of the numerator polynomial are obtained by Integral square error minimization using Luss-Jakola algorithm. The proposed method is illustrated with the help of an example and it has been observed from figure 1 and 2 that the response of the reduced order models has been found to be in agreement to the original system. The method is simple, efficient and takes little computational time.

References

1. C F Chen. 'Model Reduction of Multivariable Control Systems by Means of Matrix Continued Fraction.' International Journal of Control. vol 20. 1974. p225.

2. R Genesio and M Milanese. 'A Note on the Derivation and use of Reduced Order Models; IEEE Transactions on Automatic Control. vol ac-21. 1976. p 118.

3. Z Elrazaz and N K Sinha. 'Review of Some Model Reduction Techniques.' Canadian Electrical Engineering Journal, vol 6, 1981. p 34.

4. J Pal and R Prasad. 'Multivariable System Reduction using the Routh Hurwitz Array. Proceeding of International Conference on Control and Modelling, De¬partment of Electrical Engineering. University of Tehran. Iran. July 17-20. 1990. p895.

5. R Prasad. J Pal and A K Pant. 'Multivariable System Approximation using Polynomial Derivatives.' Journal of The Institution of Engineers (lndia), pt EL. vol 76. 1995. P 186.

6. R Prasad. J Pal and A K Pant. 'Multivariable System Reduction using Model Methods and Pade Type Approximation.' Journal of The Institution of Engineers (India), pt EL. vol 79. 1998, P 84.

7. A. K. Sinha, and J. Pal, Simulation based reduced order modeling using a clustering technique, *Computer and Electrical Engg.,* Vol. 16, No.3, pp. 159-169, 1990.

8. Singh, V, Chandra, Dinesh, and Kar, H., “Optimal routh approximants through integral squared error minimisation : Computer- aided approach, “ IEE Proc., Contr. Theory Appl., Vol. 151, pp. 53-58, 2004.

9. Luss, R., and jaakola, T.H.I.: ‘Direct search and systematic reduction of size of search region’, AIChE J., 1973, 19, pp. 760 – 766Y.