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Spherically Symmetric Convection in a Perfect Gas

Bedier B. EL-Naggar and Ismail A. Kholif

Department of Engineering Mathematics and Physics, Cairo University, Giza, Egypt

ABSTRACT

In this paper, the concentration distribution of fine particles suspended in the atmosphere is obtained in spherically symmetric time dependent convection case. The ambient atmosphere consists of a perfect gas. A source in the origin emits particles in all directions; the radial velocity is obtained together with the resulting pressure in the isothermal case. The three conservation laws i. e., the continuity, momentum and energy are solved by introducing a similarity variable. An extra relation between the properties is needed since the conservation equations consist of three first order homogeneous equations.

KEY WORDS: Convection, Concentration, Continuity, Momentum, Energy.

INTRODUCTION

In studies of air pollution contamination of air with particles of pollutant is encountered. Sources of contaminated air spread in all directions. Two mechanisms are generally encountered; convection and diffusion. The general convective diffusion equation is sufficient to describe the concentration distribution in space, this equation[1] is

$$\frac{\partial c}{\partial t} = D\,\nabla^2 c - \underline{\nabla}\cdot\left(c\,\underline{u}\right)$$

This equation represents the conservation of the pollutant particles. $\underline{u}$ is the velocity vector of the atmosphere, c is the concentration such that $1 \geq c \geq 0$ and D is the constant diffusion coefficient. The D term consists of the diffusion part and the $\underline{u}$ term represents the convection contribution. The diffusion constant is directly proportional to the medium temperature and inversely proportional to the product μa, μ is the medium viscosity and a is the particle diameter. Although D is relatively large in gases; it is generally with small contribution to the solution w.r.t. relatively high $\underline{u}$. The purely convective equation is therefore acceptable. The convection equation is $\frac{\partial c}{\partial t} = -\underline{\nabla}\cdot\left(c\,\underline{u}\right)$. In spherically symmetric case, this equation reduces to $\frac{\partial c}{\partial t} = -\frac{\partial}{\partial r}(cu)$, u is the radial positive velocity, t is the time and r is the radial position.

To list some of the literature about the subject, we refer to the work by Sun, et al[2] about the determination of effective diffusivities and convective coefficients of pure gases in single pellets, and the work of Banas[3] on the convergence of steady state solutions for stabilized finite elements simulation of compressible flows. We refer also to the paper : Rapid cylindrically and spherically symmetric strong compression of a perfect gas, by Kraiko[4]. We also refer to the paper by Godin [5] on the global existence of a class of smooth 3D spherically symmetric flows a chaplygin gases with variable entropy. Godin [6] also presented the paper on the life span of spherically symmetric solutions of compressible Euler equations outside an impermeable sphere. We also refer to the paper by Zhang [7] et al. on a note on spherically symmetric isentropic compressible flows with density dependent viscosity coefficients. Finally, we refer to the work by Valiyev et al. [8], on cylindrically and spherically symmetric rapid intense compression of an ideal perfect gas with adiabatic exponents from 1.001 to 3.

THE FLOW FIELD

Perfect gases are characterized by the equation of the state $P = \rho RT$; P is the pressure, ρ is the density, T is the absolute temperature and R is the gas constant. In spherically symmetric flow, the behaviour of the gas is described by the following three conservation equations, namely, the continuity, the momentum and the energy equation, listed below successively as [9]

Continuity: $$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial r} + u\frac{\partial \rho}{\partial r} = 0 \qquad (1)$$

Momentum: $$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial r} \qquad (2)$$

Energy : $$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r}\right) = p\frac{\partial u}{\partial r} \qquad (3)$$

u is the radial velocity and c_p is the specific heat at constant pressure. Here r is the radial position and t is the time. If ρRT is substituted for p in equation (2), the set can be written in the matrix form

***Corresponding Author:** Bedier B. EL-Naggar, Cairo University, Giza, Egypt,
E mail: bbnaggar@hotmail.com.

$$\begin{pmatrix} u & \rho & 0 \\ \frac{RT}{\rho} & u & R \\ 0 & -\frac{RT}{c_v} & u \end{pmatrix} \begin{pmatrix} \frac{\partial \rho}{\partial r} \\ \frac{\partial u}{\partial r} \\ \frac{\partial T}{\partial r} \end{pmatrix} + \begin{pmatrix} \frac{\partial \rho}{\partial t} \\ \frac{\partial u}{\partial t} \\ \frac{\partial T}{\partial t} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

This set admits similarity variable $\eta = \frac{u^* t}{r}$. This similarity variable is non-dimensional since u^* is the initial velocity. Substituting for $\frac{\partial}{\partial t} = \frac{d}{d\eta}\frac{\partial \eta}{\partial t} = \frac{u^*}{r}\frac{d}{d\eta}$ and $\frac{\partial}{\partial r} = \frac{d}{d\eta}\frac{\partial \eta}{\partial r} = -\frac{u^* t}{r^2}\frac{d}{d\eta} = -\frac{\eta}{r}\frac{d}{d\eta}$. Substituting in (4)

$$\begin{pmatrix} 1-\frac{\eta u}{u^*} & -\frac{\eta \rho}{u^*} & 0 \\ -\eta\frac{RT}{u^*\rho} & 1-\frac{\eta u}{u^*} & -\eta\frac{R}{u^*} \\ 0 & \frac{RT\eta}{c_p u^*} & 1-\eta\frac{u}{u^*} \end{pmatrix} \begin{pmatrix} d\rho \\ du \\ dT \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

Equating the determinant to zero and solving for $\frac{u}{u^*}$ we get

$$\frac{u}{u^*} = \frac{1}{\eta} - \sqrt{\frac{RT}{u^{*2}}} \tag{6}$$

This relation is correct for the case of temperature variation. Where we also note that $u = 0$ at $\eta_0 = \sqrt{\frac{u^{*2}}{RT}}$ and in this case reverse flow occurs and this is not discussed here because turbulence starts. We point out that to avoid singularity; a small sphere is taken around the origin which is isolated. To complete the set of equations (5), an external relation is needed, and this is the nature of the process of convection. In the isothermal condition T is constant and in the adiabatic condition $\frac{P}{\rho^\gamma} = \text{constant}$. For low flow parameters u, T, P the isothermal condition is satisfied, i. e. $T = T^*$ a constant. $dT = 0$ and the third equation is trivial. A solution for ρ by integrating the first equation to give

$$\frac{\rho}{\rho^*} = e^{\frac{1}{k}\left(\frac{1}{\eta} - \frac{1}{\eta^*}\right)} \tag{7}$$

$k = \sqrt{\frac{RT^*}{u^{*2}}}$, and the pressure P is given by

$$P = RT^*\rho^* \, e^{\frac{1}{k}\left(\frac{1}{\eta} - \frac{1}{\eta^*}\right)}$$

For $P^* = RT^*\rho^*$,

$$\frac{P}{P^*} = e^{\frac{1}{k}\left(\frac{1}{\eta} - \frac{1}{\eta^*}\right)} \tag{8}$$

The star refers to initial state.

The convective equation for concentration

A fine dust is spread in space and assumes a concentration distribution, the equation for distribution of particle concentration c in the radially symmetric case is:

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial r}(cu) = 0 \tag{9}$$

Using the non-dimensional similarity variable $\eta = \frac{u^* t}{r}$ we get

$$u^* dc - \eta d(uc) = 0$$

Integrating to obtain

$$\frac{c}{c^*} = e^{\frac{1}{k}\left(\frac{1}{\eta} - \frac{1}{\eta^*}\right)} \tag{10}$$

The resulting distributions of u, P, c, ρ are plotted graphically in figures (1)-(2) for $\frac{\sqrt{RT^*}}{u^*} = 1$, so that $k = \eta_0 = 1$, $\eta^* = 0.50$.

Conclusion

The similarity solution used succeeded in reducing the set of differential equations describing the flow field to a first order set of a total differentials. Since this set is homogeneous, an additional condition is needed for full solutions. In this work we added the constant temperature in isothermal process and full solutions are obtained. The fact that the equation describing ρ, p and c is the same is that the differential operator in all three cases is the same.

Fig. (1):Shows the concentration, density and pressure versus time t and radial position r

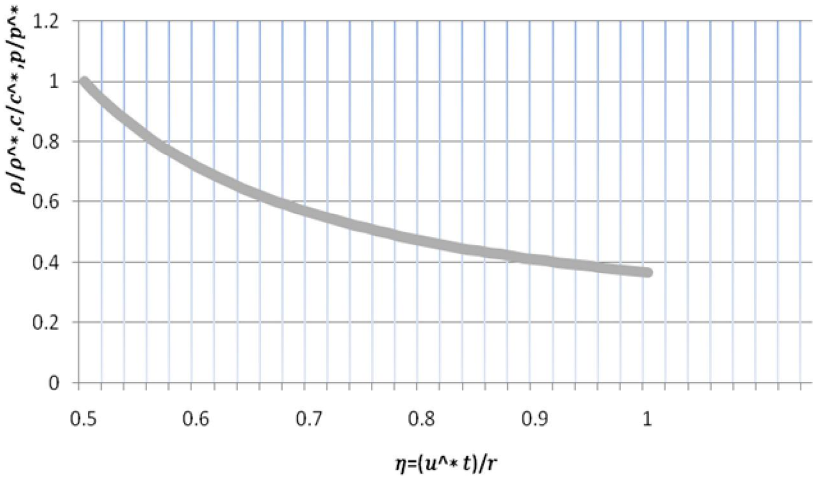
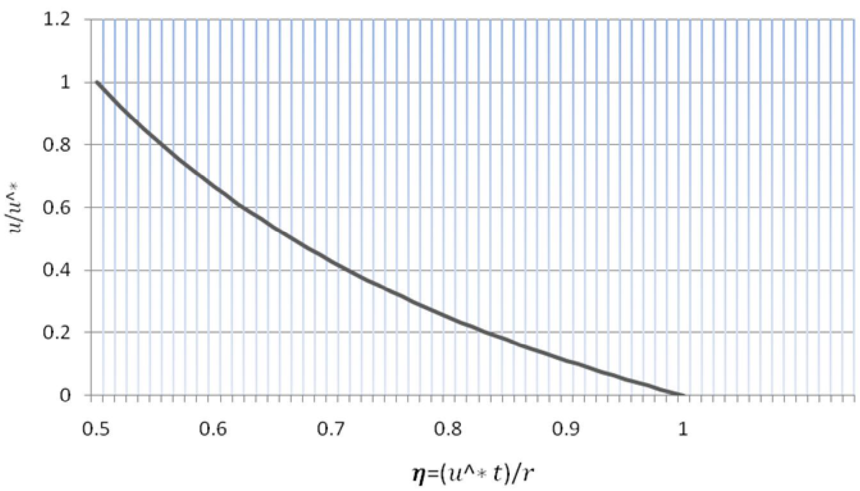


Fig. (2):Shows the velocity versus time *t* and radial position *r*



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