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A Multi-Attribute Decision-Making and Mathematical Model for University Examination Timetabling

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ABSTRACT

Examination timetabling problem (ETP) is one of the most important issues in universities. An improper timetable may result in students' dissatisfaction as it may not let them study enough between two sequential exams. In this paper, a Multi-Attribute Decision-Making (MADM) technique is applied to rank exams and the output of this step is used in a mathematical model to generate examination timetable. At first, courses are ranked using Linear Assignment technique in order to find which courses are hard for students. Then, a mathematical model is formulated including some hard constraints as well as a soft one. The soft constraint is formulated in such a way that students have one free day to study for hard exams. The objective function is set to minimize the deviation of soft constraint from its full satisfaction. It ensures that the best timetable is reached by meeting all hard constraints and satisfying the soft one as much as possible. Finally, the model is applied in a real case and is solved by GAMS.
Keywords: Multi-Attribute Decision-Making, Linear Assignment Technique, Binary Programming, Mathematical Modeling, Examination Timetabling Problem

1. INTRODUCTION

Timetabling is a common problem in universities. It is divided in two categories: course and examination timetabling. Examination timetabling problem (ETP) is a hard, complex and time consuming task. It would be very confusing if it is developed experimentally. ETP is to determine the exact date and time on which each exam should be held. Such a problem can be formulated using mathematical models.

In this paper, ETP is formulated and solved in two phases. At first, hard courses are distinguished from the others. This is done using Linear Assignment technique. Then, a mathematical model is used to schedule the exams. In the model, some hard constraints are formulated. Also, one soft constraint is defined. The objective function minimizes the deviation from the soft constraint dissatisfaction. Finally, a numerical example is solved for illustration.

ETP has been considered in different researches. Here, some previous researches are briefly presented. A comprehensive survey on exam timetabling problem was performed by Qu *et al.* [1]. They considered both theoretical and practical researches done in a ten years period. Some researches focused on room assignment to exams in order to minimize total movement of students between rooms during two consecutive exams [2, 3].

Ayob *et al.* solved a model with the aim of improving the quality of the timetable. It tries to minimize the number of students with two consecutive exams in a day [4]. Burke *et al.* defined a model including seven objectives. They grouped the objectives in such a way that each group satisfied the specific party (students, markers, invigilators and estates) [5]. MirHassani developed a model based on a predefined exam timetable in order to maximize paper spread [6]. Cheong *et al.* developed a model to minimize the length of timetable and also prevented students to take exams in consecutive exams as much as possible. They formulated a multi-objective model and solved it using an evolutionary algorithm [7]. Sagir and Ozturk formulated invigilator assignment to exams as a multi-objective model and calculated the weights of objectives using Analytic Network Process (ANP) [8]. Kahar and Kendall applied a model in a real case regarding some new constraints such as distance between rooms. At the end, they used a heuristic to solve the problem [9]. McCollum *et al.* considered an integer programming model with a cost penalty objective function. It tried to satisfy soft constraints as much as possible and if it failed, a penalty is accounted [10].

The structure of the paper is as follows:

The methods, techniques and mathematical model applied in this research are presented in Section 2. The model is illustrated using a numerical example in Section 3. Conclusion is presented in Section 4.

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2. MATERIALS AND METHODS

Naturally, the difficulty of courses is not the same and students need more time to study for a hard exam. In order to distinguish hard courses from the other ones, Linear Assignment technique is used. For this purpose, courses are ranked regarding three criteria: *A*, *B* and *C*. Each criterion has a weight indicating its importance (W_A, W_B and W_C). Then, *weight assignment matrix* is calculated. Courses and ranks are rows and columns of the matrix. The value of the matrix (a_{er}) is summation of the weights of criteria that assign rank *r* to course *e*. At the next step, a classical assignment model is used to assign a rank to each course as follows:

$$Max\ \sum_e \sum_r a_{er} * T_{er} \tag{1}$$

Subject to:

$$\sum_e T_{er} = 1 \quad \forall r \tag{2}$$

$$\sum_r T_{er} = 1 \quad \forall e \tag{3}$$

where T_{er} is a binary variable that equals one if rank *r* is assigned to course *e*. The first *k* ranked courses are assumed as hard ones.

At the next phase, a mathematical model is presented to formulate ETP. The sets, indices, parameters and variables are as follows:

Indices and sets

E	set of exams (courses)
e	index of exam ($e \in E$)
D	set of available days for exams
d	index of day ($d \in D$)
T	set of time slots
t	index of time slot ($t \in T$)
E_h	set of hard courses
T_d	Set of time slots available for day *d*

Variables

X_{edt}	A binary variable that is one if exam *e* is scheduled on day *d* and time slot *t*, otherwise it is zero.
Y_{dt}	A binary variable that is one if an exam is scheduled on day *d* and time slot *t*, otherwise it is zero.
U_{ed}	An integer variable that shows the degree of soft constraint satisfaction. If it is greater than zero, the soft constraint is satisfied.
U'_{ed}	An integer variable that shows the deviation from soft constraint satisfaction. If it is zero, the soft constraint is satisfied.

Constraints

Constraints of the model are as follows:

1) All exams should be scheduled.

$$\sum_d \sum_{t \in T_d} X_{edt} = 1 \quad \forall e \tag{4}$$

2) In each time slot, at most one exam is scheduled.

$$\sum_e X_{edt} \le 1 \quad \forall d, t \in T_d \tag{5}$$

3) Hard exams should not be scheduled on the first day of planning period. This gives students more time to study for hard exams.

$$\sum_{e \in E_h} \sum_{t \in T_d} X_{edt} \le 0 \quad d = 1 \tag{6}$$

4) Hard exams are not scheduled on two successive days.

$$\sum_{e \in E_h} \sum_{t \in T_d} (X_{edt} + X_{ed+1t}) \le 1 \quad \forall\, d < 10 \tag{7}$$

5) If a hard exam is scheduled on a day, no other exam is scheduled on that day.

$$\sum_{e', e' \neq e} \sum_{t \in T_l} X_{e'd\,t} \le (1 - \sum_{t \in T_d} X_{edt})M \quad \forall e \in E_h, d \tag{8}$$

6) At most three exams can be scheduled on a single day.

$$\sum_e \sum_{t \in T_d} X_{edt} \le 3 \quad \forall\, d \tag{9}$$

7) If three exams are scheduled on a single day, they should not be held at three successive time slots.

$$\sum_{t=t'}^{t'+2} Y_{dt} \le 2 \quad \forall\, d\,, t' = 1, \ldots, t-2 \tag{10}$$

8) A constraint is required to relate decision variables. It is written as follows:

$$\sum_e X_{edt} \le M Y_{dt} \quad \forall d, t \in T_d \tag{11}$$

9) It is suitable for students to have one free day before taking a hard exam. This is shown in relation (12).

$$\sum_{e', e' \neq e} \sum_{t \in T_l} X_{e'd-1\,t} \le (1 - \sum_{t \in T_d} X_{edt})M \quad \forall e \in E_h, d > 1 \tag{12}$$

In this model, this is assumed as a soft constraint, not a hard one. Although a soft constraint is preferably met, the model will not be infeasible if it violates. In order to ensure that soft constraint is satisfied as much as possible, the deviation from its satisfaction is minimized in the objective function. So, constraint (12) is rewritten in the following form:

$$\sum_{e', e' \neq e} \sum_{t \in T_l} X_{e'd-1\,t} + U^+_{ed} - U^-_{ed} = (1 - \sum_{t \in T_d} X_{edt})M \quad \forall e \in E_h, d > 1 \tag{13}$$

Objective Function

The objective is written with the aim of soft constraint satisfaction.

$$Min \sum_{e\in E_h} \sum_d W_e * U_{ed}^{-} \qquad (14)$$

Where W_e is calculated using the result of assignment model (T_{er}). After distinguishing hard exams from the others, T_{er} is converted to score value and W_e is obtained by normalizing the score values. The model is illustrated using a numerical example.

3. NUMERICAL EXAMPLE AND RESULTS

Assume a department offers 15 courses and the exams are to be scheduled in a 10 days period. At first, we are going to rank the exams in order to find which ones are hard. For this purpose, three criteria (A, B and C) are defined and courses are ranked regarding them. The results are shown in Table 1.

Table 1. Decision making matrix

	A	B	C
E1	15	4	7
E2	2	3	2
E3	7	1	6
E4	12	11	11
E5	1	15	15
E6	3	8	3
E7	11	5	9
E8	10	9	1
E9	9	10	5
E10	6	13	14
E11	14	2	13
E12	5	7	8
E13	8	12	4
E14	4	14	10
E15	13	6	12

The weights of criteria are assumed 0.2, 0.3 and 0.5 respectively. So, weight assignment matrix is obtained (Table 2).

Table 2. Weight assignment matrix

	Ranks														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
E1	0	0	0	0.3	0	0	0.5	0	0	0	0	0	0	0	0.2
E2	0	0.7	0.3	0	0	0	0	0	0	0	0	0	0	0	0
E3	0.3	0	0	0	0	0.5	0.2	0	0	0	0	0	0	0	0
E4	0	0	0	0	0	0	0	0	0	0	0.8	0.2	0	0	0
E5	0.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8
E6	0	0	0.7	0	0	0	0	0.3	0	0	0	0	0	0	0
E7	0	0	0	0	0.3	0	0	0	0.5	0	0.2	0	0	0	0
E8	0.5	0	0	0	0	0	0	0	0.3	0.2	0	0	0	0	0
E9	0	0	0	0	0.5	0	0	0	0.2	0.3	0	0	0	0	0
E10	0	0	0	0	0	0.2	0	0	0	0	0	0	0.3	0.5	0
E11	0	0.3	0	0	0	0	0	0	0	0	0	0	0.5	0.2	0
E12	0	0	0	0	0.2	0	0.3	0.5	0	0	0	0	0	0	0
E13	0	0	0	0.5	0	0	0	0.2	0	0	0	0.3	0	0	0
E14	0	0	0	0.2	0	0	0	0	0	0.5	0	0	0	0.3	0
E15	0	0	0	0	0	0.3	0	0	0	0	0	0.5	0.2	0	0

There are 15 courses and 15 ranks are to be assigned them. Using the classic assignment model, the ranks are obtained (Table 3).

Table 3. Result of linear assignment model

Course	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15
Rank	7	2	6	11	15	3	9	1	5	14	13	8	4	10	12

The first four courses are assumed as hard ones. In order to calculate the weight of each hard course, the ranks are converted to scores. The ranks of E8, E2, E6 and E13 are 1, 2, 3 and 4 and the scores are 4, 3, 2 and 1 respectively. Normalizing scores leads to weights of hard courses: (W_{E8} =0.4, W_{E2} =0.3, W_{E6}=0.2 , W_{E13}=0.1).

These weights are used in the objective function of the model. The mathematical model presented in Section 3 is solved and the results are shown in Figure 1.

Time slots / Days	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Time slot 4	E3	E5		E2				E8	E14	
Time slot 3		E11								
Time slot 2	E1				E12				E7	E13
Time slot 1	E9	E15			E10	E6			E4	

Figure 1. Exam timetable

As shown in Figure 1, there is one free time for students to study hard exams E_2 and E_8 which are the first two hardest exams. It means that the soft constraint cannot be fully met and due to the high importance of E_2 and E_8 compare to E_6 and E_{13} , there is one free time for them in the timetable. It is clear that all hard constraints are met. All exams are scheduled, at most one exam is scheduled in a single time slot, hard exams are not scheduled on the Day 1 and also they are not scheduled on two successive days. Also, on days that a hard exam is scheduled, no other exam is scheduled. At most, there are three exams on a single day and they are not scheduled at three successive time slots.

4. Conclusions and future research

In this research, combination of Linear Assignment technique and mathematical modeling is applied to generate examination timetable for a department in Islamic Azad University. The results of Linear Assignment model show that four courses are considered as hard exams. At the next step, a mathematical model is presented to schedule exams. This model aims to give students one free day to study for hard exams if possible. This is considered as a soft constraint in the model. The results of the model show that students have one free day for only two hard exams while they have no free day for two other hard exams. The exams that students have one free day to study are the first two hardest ones. Also, hard exams have not been scheduled on two successive days. This is the best solution that meets hard constraints and satisfies the soft constraint as much as possible. Some new areas are suggested for future researches. One of them is that several departments can be considered simultaneously and the model is solved using a meta-heuristic approach. Also, room split has not been allowed in the paper and this assumption can be relaxed in future study. Moreover, some soft constraints, such as the number of students who have simultaneous exams, can be added to the model and also can be considered in the objective function. Finally, the problem is suggested to be considered as a multi-objective model in future research.

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REFERENCES

[1] Qu, R., E.K. Burke, B. McCollum, L.T.G. Merlot and S.Y. Lee, 2009. A Survey of Search Methodologies and Automated System Development for Examination Timetabling. *J. Sched.*, 12(1): 55-89.

[2] Dammak, A., A. Elloumi and H. Kamoun, 2006. Classroom Assignment for Exam Timetabling. Adv. Eng. Soft., 37(10): 659-666.

[3] Ayob, M. and A. Malik, 2011. A New Model for an Examination-Room Assignment Problem. Intl. J. Comp. Sci. Net. Sec., 11(10): 187-190.

[4] Ayob, M., S. Abdullah and A. Malik, 2007. A Practical Examination Timetabling Problem at the Universiti Kebangsaan Malaysia. *Intl J. Comp. Sci. Net. Sec.*, 7(9): 198-204.

[5] Burke, E. K., B. McCollum, P. McMullan and A.J. Parkes, 2008. Multi-Objective Aspects of the Examination Timetabling Competition Track. *Proc. of PATAT 2008*, Montreal, Canada.

[6] MirHassani, S. A., 2006. Improving Paper Spread in Examination Timetables Using Integer Programming. Appl. Math. Comp., 179(2): 702-706.

[7] Cheong, C.Y., K.C. Tan and B. Veeravalli, 2009. A Multi-Objective Evolutionary Algorithm for Examination Timetabling. *J. Sched.*, 12(2): 121-146.

[8] Sagir, M. and Z.K. Ozturk, 2010. Exam Scheduling: Mathematical Modeling and Parameter Estimation with the Analytic Network Process Approach. *Math. Comp. Modl*., 52(5-6): 930-941.

[9] Kahar, M.N.M. and G., Kendall, 2010. The Examination Timetabling Problem at University Malaysia Pahang: Comparison of a Constructive Heuristic with an Existing Software Solution. Eur. J. Oper. Res., 207(2): 557-565.

[10] McCollum, B., P. McMullan, A.J. Parkes, E.K. Burke and R. Qu, 2012. A New Model for Automated Examination Timetabling. Ann. Oper. Res., 194(1): 291-315.