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Transient Heat Conduction in Functionally Graded Thick Hollow Cylinders under Non-Uniform Heat Generation by Homotopy Perturbation Method

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ABSTRACT

In this paper, a general analysis of one-dimensional non-steady-state temperature distribution equation in a hollow thick cylinder made of functionally graded Material(FGM) with non-uniform heat generation is developed. The temperature distribution is assumed to be a function of radius and time, with general thermal and mechanical boundary conditions. The material properties are assumed to depend on variabler and they are expressed as power functions of r. The homotopy perturbation method (HPM) is used to solve the temperature distribution equation. In this method a nonlinear and complex and partial differential equation is transformed to series of linear and nonlinear and almost simpler differential equations. These set of equations are then solved secularly.
KEYWORDS: Homotopy Perturbation Method, Functionally graded material, Secular Terms, non-steady-state heat transfer, Nonlinear Differential Equation, non-uniformheat generation

List of symbols

r_1Inner radius
r_2Outer radius
$T(^0k)$ temperature distribution
$\rho_0 c_0$Material constant
D, E Constant coefficients
FConstant coefficients
C (kJ/kg K) Specific heat
h(W/m^2 K) Heat convection coefficients
k(W/m K) Heat conductivity
k_0 Material constant
n_1, n_2 Material constant
n_3 Generated heat constant
t(s)Time
r(m) Radius
HHomotopy operator
L Linear operator
NNonlinear operator
pHomotopy parameter (real number)
q^0 (W/m^3) Heat generation rate
$\boldsymbol{q^{\circ}_0}$(W/m^3) Heat generation constant

Greek symbols

ϕBoundary of the domain
θ Function describing the initial condition
ΩFunction describing the boundary condition

1. INTRODUCTION

FGM (functionally graded materials or functional gradient materials) are materials withnon-uniform microstructure, i.e. with (continuous or step wise) changes of (chemical and / or phase) composition and / or microstructure. In contrast to conventional coatings or joined materials these changes in FGM are more gradual in order to improve adhesion and avoid separation at the boundary (delamination) caused by the thermal stresses developing due to thermal expansion mismatch, the main disadvantage of coated or joined materials. For adhesion,

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the thermal expansion mismatch should not exceed 25 %. In FGM the sharp boundary is replaced by a smooth transition region.The FGM concept originated in Japan in 1984 during the space plane project, in the form of a proposed thermal barrier material capable of withstanding a surface temperature of 2000 K and a temperature gradient of 1000 K across a cross section <10 mm. Since 1984, FGM thin films have been comprehensively researched, and are almost a commercial reality. FGMs offer great promise in applications where the operating conditions are severe. For example, wear-resistant linings for handling large heavy abrasive ore particles, rocket heat shields, heat exchanger tubes, thermoelectric generators, heat-engine components, plasma facings for fusion reactors, and electrically insulating metal/ceramic joints. They are also ideal for minimizingthermo mechanical mismatch in metal-ceramic bonding.[1]

Analyticaland computational studies of appointing stresses and displacements in cylindrical shell made of FGMhave been carried out by some of researchers as following.Temperature and stress distributions were determined in a stress-relief-type plate of FGMs with steady state and transient temperature distributions by Awaji[2].The analytical solution for the stresses of FGMs in theone-dimensional case for spheres and cylinders are given by Lutz and Zimmerman [3,4]. A multi-layered material model was employed to solve the transient temperature field in an FGM stripwith continuous and piecewise differentiable material properties by Jin [5].He obtained a closed form asymptotic solution of the temperature field for short times, by using an asymptotic analysis and an integration technique and the Laplace transform. A general analysis of one dimensional steady state thermal stresses in a thick hollow cylinder under axisymmetric and non-axisymmetric loads was developed by Jobberies al. [6, 7].These authors consider the nonhomogeneous material properties as linear function. The thermal and mechanical stress analyses of these types of structures are sometimes carried out using the theory oflaminatedcomposites [8–13].A local boundary integral equation method with the moving least squares approximation of physical fields was applied to transient heat conduction analysis in functionally graded materials by Sladek et al. [14]. They solved the initial boundary value problem in the Laplace transform domain with a subsequent numerical Laplace inversion to obtain time-dependent solutions. MohammadSetareh and Mohammad Reza Isvandzibaei have studied FGM Cylindrical Shell with Clamped-Simply Support Boundary Conditions [15].Tarn et al. [16] have studied the end effects of steady state heat conduction in a hollow or solid circular cylinder of FGM under 2D thermal loads with arbitrary end conditions. They evaluated the decay length that characterizes the end effects on thermal field by using matrix algebra and Eigen function expansion. Thesensitivity analysis of heat conduction for functionally graded materials and the steady state, transient problem treated with the direct method and the adjoin method were presented by Chen et al. [17]. The same authors used perturbation techniques to derive the thermal stress equations ofthick hollow spheres and plates made of FGMs under different assumptions of temperature distributions .Theprecise time integration method is employed to solve the transient problem by them. Transient temperature field and associated thermal stresses in functionally gradedmaterials have been determined by using Finitedifference method (FEM/FDM) by Wanget[18].Thermal shock fracture of a FGM plate andthe thermal shock resistances of FGMs wereanalyzedby them. A finite element/finite difference method(FEM/FDM) was developed also to solve the timedependenttemperature field in non-homogeneous materials such as functionally graded materials by Wang et al. [19].Methods that allow finding solutions of any type of nonlinear physical and technical problems have been utilized in many application, recently,these methods include, among the others: the Adomiandecomposition method [20–22], variational iteration method [23–25], andhomotopy perturbation method [26–31]. In general, a mathematical formulation ofthose methods makes it possible to solve nonlinear operator equations. In this type of methods a sequence or functional series is constructed,whose limit is a function which is the solution of discussed problem (with appropriate assumptions). Usually, due to a quite fast convergence of appropriate sequences or series, determining of their few first components leads to a very good approximation of solution searchedThehomotopy perturbation is an effective solution method for abroad class of problems. Applications of this method for solving nonlinear ordinary differential equations with boundary conditions or similarproblems are presented in papers [32, 33].Perturbation method is one of the well-known methods to solve the nonlinear equations which was studied by a large number of researchers such as Bellman [34], Cole [35], andO'Malley [36].In the literature one can also find its applications in wave and diffusion equations [37–39], inverse problem of diffusion equation [40], Laplace equation [41] and hyperbolic partial differential equation [42]. Ganjiin many works [43–47] have dealt with homotopy perturbation method used in solution of various problems connected with heat transfer processes.Slota [48, 49] applied the method for determination of exact (or approximate) solution of one- or two-phase inverse Stefan problem.Another work [50] shows a utilization of the method mentioned forfindingtemperature distribution in the cast-mouldheterogeneous Most of works are devoted to steady or transient heat conduction models. Huang and Wang [51] estimate the unknown surface heat fluxes in the solid. Huang and Tsai [52] employed an inverse method to determine the time-dependent local heat transfer coefficients for a plate fin.N. Moallemiet al. [53] show the application of Homotopy Perturbation Technique to Analysis non-Newtonian Fluid Flow in Collector.

The purpose of this study is to solving the nonlinear equation of conduction heat transfer with the variable Physical properties in FGM thick hollow cylindersbyHPM.many methods have been developed for approximating or numerical solutions.Perturbation method is based on the existence of small parameters.in order to overcome the problems associated with finding the small parameter ,different new methods have been proposed to eliminate the small parameter ,for example homotopy perturbation method(HPM),variational iteration method (VIM),Adomian's decomposition method (ADM)and differential transformation method (DTM),but The HPM has the merits of simplicity and easy execution. Unlike the traditional numerical methods, the HPM does not needdiscretization and linearization. The HPM can overcome the difficulties arising in calculation of Adomian's polynomials in Adomian's decomposition method.TheHPM yields very rapid convergence of the solution series in most cases and usually only a few iterations leading to high accuracy solutions. ThusHPM is a universal one which can solve various kinds of nonlinear equations.

This paper is arranged in the following manner, in Section 2, we present Problem formulation. In Section 3 we solve the problem and determine non-steady state heat transfer equation in FGM thick hollowcylinders with the variable Physical properties. In Section 4, we define the Initial and boundary conditions.In Section 5, we present the standard homotopy perturbation method. In Section 6, we present the modification technique of homotopy perturbation method for solving heat transfer equation. In Section 7, we present test example. In the end the conclusion and results are presented in section 8.

2. Problem formulation

Consider a long solid tube, insulated at the outer radius ***r2***, and cooled at the inner radius***r1***, with non-uniform heat generation q^0(W/m3) within the solid.

1. Obtain the general solution for the temperature distribution in the tube.

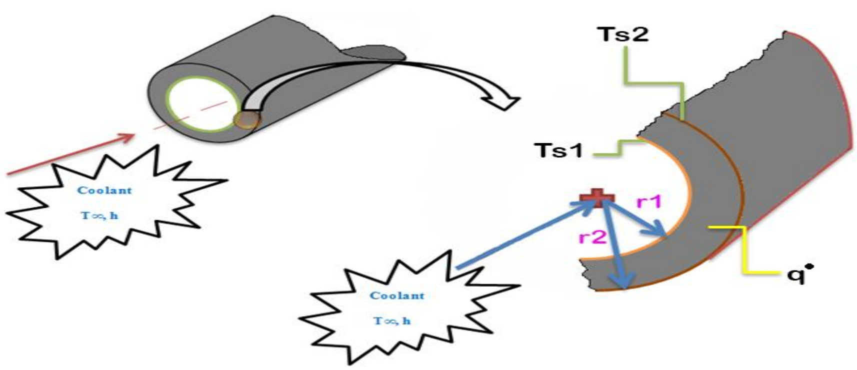

Fig1.three- dimentional form of problem

Assumptions:

1. Non-Steady state conditions.
2. One-dimensional radial conduction.
3. Non-Constant properties. (Functionally graded material)
4. Non-Uniform volumetric heat generation.

3. Solving:

The heat transfer equation in functionally graded hollow cylinder with Uniform volumetricheat generation. Forplane strain and axisymmetric case is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k \times r \times \frac{\partial T}{\partial r}\right) + q^{\circ} = \alpha\left(\frac{\partial T}{\partial r}\right) \quad (1)$$

α=ρc

It is assumed that the thermal conductivity **k** and**ρc** arethe power functions of r as:

k=$k_0 r^{n_1}$ (2)
ρc= $\rho_0 c_0 r^{n_2}$ (3)
q°=$q_0{}^{\circ} r^{n_3}$(4)
With**i**ntroducing Eqs.(2),(3**)**and(4) inEq.(1) the followingequation is obtained:

$$k_0(n_1+1)\times r^{\zeta-1}\times\left(\frac{\partial T}{\partial r}\right)+k_0\times r^{\zeta}\left(\frac{\partial^2 T}{\partial r^2}\right)+q_0{}^{\circ}\times r^{\eta-1} = \rho_0 c_0\left(\frac{\partial T}{\partial t}\right)(5)$$

Where
ζ=n_1-n_2 (6)
$\eta = n_3$-n_2(7)

Let's start with a formulation of a mathematical model of the considered Problem.

4. Initial and boundary conditions

ϕI=[(r,0),r∈[r_1,r_2]]
ϕII=[(r_1,t),t∈[0,t]] (8)
ϕIII=[(r_2,0),t∈[0,t]]

On the boundary the initialCondition ϕI is defined:
V(r,0)= θ(r) r∈[r_1, r_2](9)
On boundary ϕIIthe Dirichlet boundary condition is assumed:
V(r_1,t)=Ω(t)(10)

5. Basic idea of homotopy perturbation method

The homotopy perturbation method iscombination of the classical perturbation technique and homotopy technique. To explainthe basic idea of the HPM forsolvingnonlineardifferentialequations, weconsiderthefollowing nonlinear differential equation:

$A(u) - f(r) = 0,\ r\in\Omega,$ (11)

Subject to boundary condition

$B(u, \partial u/\partial n) = 0,\ r\in\Gamma,$(12)

Where***A***is a general differential operator, ***B***a boundary operator, ***f (r)***is a knownanalyticalfunction,***Γ***is the boundary of domain***Ω***and ***∂u/∂n***denotes differentiation along the normal drawn outwards from***Ω***. The operator ***A***can, generally speaking, be dividedinto two parts: a linear part ***L***and a nonlinear part ***N***. Eq. (12) therefore can be rewritten as follows:

$L(u) + N(u) - f(r) = 0.$(13)

In case the nonlinear Eq (11) has no “small parameter”,
We can construct the followinghomotopy,
$H(v,p) = L(v) - L(u_0) + pL(u_0) + p\,(N(v) - f(r)) = 0$(14)

Where ***p***is called homotopyparameter. According to the homotopy perturbation method, theapproximation solution of Eq. (4) canbe expressed as a series of the power of *p*, i.e.,

$v = p^0 v_0 + p^1 v_1 + p^2 v_2 + \cdots,$ (15)
$v = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots.$ (16)

When Eq. (14) corresponds to Eq. (11) and Eq. (16) becomes the approximate solution of Eq. (11).

6. In our case, for heat transfer Eq. (5) **we obtain:**

$$\left(\frac{\partial^2 V}{\partial r^2}\right)-1/\Gamma \times r^{-\zeta}\left(\frac{\partial V}{\partial t}\right)+1/r\times\lambda/\Gamma\times\left(\frac{\partial V}{\partial r}\right) = -\frac{\beta}{\Gamma}\times r^{\psi-1}(17)$$

Where:

$\Gamma = k_0 / \rho_0 c_0$(18)

$\lambda = k_0(n_1+1)/ \rho_0 c_0$(19)

$\beta = q_0^{\circ} / \rho_0 c_0$(20)

$\psi = \eta - \zeta$(21)

$L(V) = \left(\frac{\partial^2 V}{\partial r^2}\right)$(22)

$N(V) = -\frac{1}{\Gamma} \times r^{-\zeta} \left(\frac{\partial V}{\partial t}\right) \quad + \frac{1}{r} \times \frac{\lambda}{\Gamma} \times \left(\frac{\partial V}{\partial r}\right)$(23)

$f(r) = -\frac{\beta}{\Gamma} \times r^{\psi-1}$(24)

Therefore:

$H(V,P) = \left(\frac{\partial^2 V}{\partial r^2}\right) - \left(\frac{\partial^2 u0}{\partial r^2}\right) + p\left(\frac{\partial^2 u0}{\partial r^2} - \frac{1}{\Gamma} \times r^{-\zeta} \left(\frac{\partial V}{\partial t}\right) \quad + \frac{1}{r} \times \frac{\lambda}{\Gamma} \times \left(\frac{\partial V}{\partial r}\right) + \frac{\beta}{\Gamma} \times r^{\psi-1}\right)$(25)

Changingfrom$u_0$to u*(r)*. We consider *v*, as the following:

$V = p^0 v_0 + p^1 v_1 + p^2 v_2 + p^3 v_3 + \cdots$(26)

And the best approximation for the solution is:

$u = \lim_{p \to 1} (V)$

$V = v_0 + v_1 + v_2 + v_3 + \cdots$(27)

Comparison of the expressions with the same powers of the parameter**p** gives the following equations:

$\boldsymbol{p^1}$: $\left(\frac{\partial^2 V1}{\partial r^2}\right) + \left(\frac{\partial^2 u0}{\partial r^2}\right) - \frac{1}{\Gamma} \times r^{-\zeta} \left(\frac{\partial u0}{\partial t}\right) + \frac{1}{r} \times \frac{\lambda}{\Gamma} \times \left(\frac{\partial u0}{\partial r}\right) + \frac{\beta}{\Gamma} \times r^{\psi-1}$(28)

$\boldsymbol{p^2}$: $\left(\frac{\partial^2 V2}{\partial r^2}\right) - \frac{1}{\Gamma} \times r^{-\zeta} \left(\frac{\partial v1}{\partial t}\right) + \frac{1}{r} \times \frac{\lambda}{\Gamma} \times \left(\frac{\partial v1}{\partial r}\right)$(29)

$\boldsymbol{p^3}$: $\left(\frac{\partial^2 V3}{\partial r^2}\right) - \frac{1}{\Gamma} \times r^{-\zeta} \left(\frac{\partial v2}{\partial t}\right) + \frac{1}{r} \times \frac{\lambda}{\Gamma} \times \left(\frac{\partial v2}{\partial r}\right)$(30)

In the complete form:

$\boldsymbol{p^i}$: $\left(\frac{\partial^2 Vi}{\partial r^2}\right) - \frac{1}{\Gamma} \times r^{-\zeta} \left(\frac{\partial v(i-1)}{\partial t}\right) + \frac{1}{r} \times \frac{\lambda}{\Gamma} \times \left(\frac{\partial v(i-1)}{\partial r}\right)$ **i≥2**(31)

The above partial differential equations must be supplemented byconditions ensuring a uniqueness of the solution. For Eq. (28) we assumethe following conditions:

$v_0(r_1, t) + v_1(r_1, t) = \Omega(t)$

(32)

$v_0(r_d, t) + v_1(r_1, t) = \Omega d(t)$

Whilefor Eq. (31) conditions are in the form (for vi that i≥2):

$v_i(r_1, t) = 0$(33)

$v_i(r_d, t) = 0$

In this way, the solution of problem was brought to the sequenceof easy to solve partialdifferential equations. Looking for the solution of the above problem, we need to define an

Initialapproximation v0, which we can assume as the function determining the initial condition:

$v_0(r,t) = \theta(r)$(34)

7. Test problem:

The application of the homotopy perturbation method for the functionally graded thick hollowCylinder problems

With non-uniformheat generationthatdescribed in the fig (2) will be illustrated in the example in whichsuppose that the inner surface is made ofalumina (ceramic).The alumina specifications are

k_0 = 46 W/m.K

c_0 = 0.76kJ/kg.K

ρ_0 = 3800 kg/m3

Inner radius r_2 is 30.5cm, r_d=30cm, inner radius r_1 is 25cm

n_1=1.3

n_2=0.2

n_3=2

q_0°=1000 watt/m^3

Initial and boundary condition:

$$\begin{cases} \theta(r) = 800r^2 + 1000r \\ \Omega(t) = \frac{1}{4}t^2 + 300 \\ \Omega d(t) = 372 + \frac{1}{4}t^2 + \frac{1}{6}t \end{cases}$$

We can determine Γ, λ, β, ψ by replacing k_0, c_0, ρ_0, n_1, n_2, n_3, q_0° at equations (6),(7),(18),(19),(20),(21).

As the initial approximation v_0 the function that satisfies the initialcondition is taken:

v_0(r,t)= $\theta(r) = 800r^2 + 1000r$

Solving now Eq. (28)with boundary and initial conditions Eq.**(32) that $\theta(r)$ and $\Omega(t)$ and $\Omega d(t)$ are given above:**

$v_1(r,t) = -800 \times r^2 - 1840.0954 \times r^2 - 383.3532 \times (r \times (\ln r) - r) - 18.2693 \times r^{1.7} + 3.33333 \times r \times t + 969.1971 \times r + .25 \times t^2 - .8333 \times t - 304.26102$

For the functions vi(x, t), i≥2 by solving the Eq. **(31) for i=2**with boundary condition Eq.(33). Finally weobtain:

$v_2(r,t) = 1840.0954 \times r^2 + 4232.4035 \times r^2 + 2645.252 \times ((r \times (\ln r)^2) - (2 \times r \times \ln r) + 2 \times r) + 60.0302 \times r^{1.7} - 7.6669 \times (r \times \ln r - r) \times t - 2229.2502 \times (r \times \ln r - r) + 122389.5426 \times r^{1.9} + 18358.61458 \times r^{1.9} \times t - 30597.5692 \times r^{1.9} - 65251.34 \times r - 10922.744 \times r \times t + 3580.5304 + 1408.0878 \times t$

And For**the functions vi(x, t), i≥2 by solving the Eq. (31) for i=3 with boundary condition Eq. (33). Finally we obtain:**

$546 \times r^2 - 2028.114 \times ((r \times lnr)^3 - (3 \times r \times (\ln r)^2) + (6 \times r \times \ln r) - 6r) - 197.25 \times r^{1.7} + 8.8173 \times (r \times (lnr)^2 - (2 \times r \times \ln r) + 2r) \times t + 2563.7491 \times (r \times (lnr)^2 - 2 \times r \times (\ln r) + 2 \times r) - 312786.8743 \times r^{1.9} - 4692.183 \times r^{1.9} \times t + 78197.1877 \times r^{1.9} + 150084.6 \times (r\ln r - r) + 25123.472 \times (r \times$

$v_3(r,t) = -13967.3\ln r - r) \times t - 281507.3298 \times ((r^{1.9} \times \ln r) - ((\frac{1}{1.9}) \times r^{1.9}) - ((\frac{1}{.9}) \times r^{1.9}) - r^{1.9}) + 2.286904762 \times 10^8 \times r^{2.8} - 4.010526316 \times 10^8 \times r^{1.9} - 9.822222222 \times 10^8 \times r^{.9} + 1181335832 \times r + 35243.156 \times r \times t + 1.08234848 \times 10^7 + 6499.3231 \times t$

The four-term expansion in Eq. (25) now becomes:

V=$v_0 + v_1 + v_2 + v_3$

Therefore:

$V(r,t) = 800r^2 + 1000r - 800 \times r^2 - 1840.0954 \times r^2 - 383.3532 \times (r \times (\ln r) - r) - 18.2693 \times r^{1.7} + 3.33333 \times r \times t + 969.1971 \times r + .25 \times t^2 - .8333 \times t - 304.26102 + 1840.0954 \times r^2 + 4232.4035 \times r^2 + 2645.252 \times ((r \times (\ln r)^2) - (2 \times r \times \ln r) + 2 \times r) + 60.0302 \times r^{1.7} - 7.6669 \times (r \times \ln r - r) \times t - 2229.2502 \times (r \times \ln r - r) + 122389.5426 \times r^{1.9} + 18358.61458 \times r^{1.9} \times t - 30597.5692 \times r^{1.9} - 65251.34 \times r - 10922.744 \times r \times t + 3580.5304 + 1408.0878 \times t - 13967.3546 \times r^2 - 2028.114 \times ((r \times lnr)^3 - (3 \times r \times (\ln r)^2) + (6 \times r \times \ln r) - 6r) - 197.25 \times r^{1.7} + 8.8173 \times (r \times (lnr)^2 - (2 \times r \times \ln r) + 2r) \times t + 2563.7491 \times (r \times (lnr)^2 - 2 \times r \times (\ln r) + 2 \times r) - 312786.8743 \times r^{1.9} - 4692.183 \times r^{1.9} \times t + 78197.1877 \times r^{1.9} + 150084.6 \times (r\ln r - r) + 25123.472 \times (r \times \ln r - r) \times t - 281507.3298 \times ((r^{1.9} \times \ln r) - ((\frac{1}{1.9}) \times r^{1.9}) - ((\frac{1}{.9}) \times r^{1.9}) - r^{1.9}) + 2.286904762 \times 10^8 \times r^{2.8} - 4.010526316 \times 10^8 \times r^{1.9} - 9.822222222 \times 10^8 \times r^{.9} + 1181335832 \times r + 35243.156 \times r \times t + 1.08234848 \times 10^7 + 6499.3231 \times t$

Heat transfer equation $T(r,t)$ *is*:

$\boldsymbol{T(r,t) = V(r,t)}$

8. RESULTS& DISCUSSION

In the figure 2, the Radial distribution of heat conductivity for n_1=1.3, k0 = .046 KW/m K, from inner radius r_1=.25 m to r_2=.305 m is shown. In the figure 3 the Radial distribution of temperature at inner radiusr_1=.25 m and outerradius r_2 =.305 m is shown separately in the time interval [0,10]; In the figure 4 Radial distribution of temperature in the t=0 for initial condition is presented.

Figure 5 shows the Temperature distribution for different time interval between inner radius r_1 = 25 cm and outer radius r_2 = 30.5 cm. This diagram is shown for functionally graded parameters n_1=1.3, n_2=.2, n_3=2, From this chart can be concluded that at low times effect of initial conditions is more than the heat generation parameter and with the time increasing the effect of heat generation surpass the initial condition and the concavity of the curve changes from downward to upward.

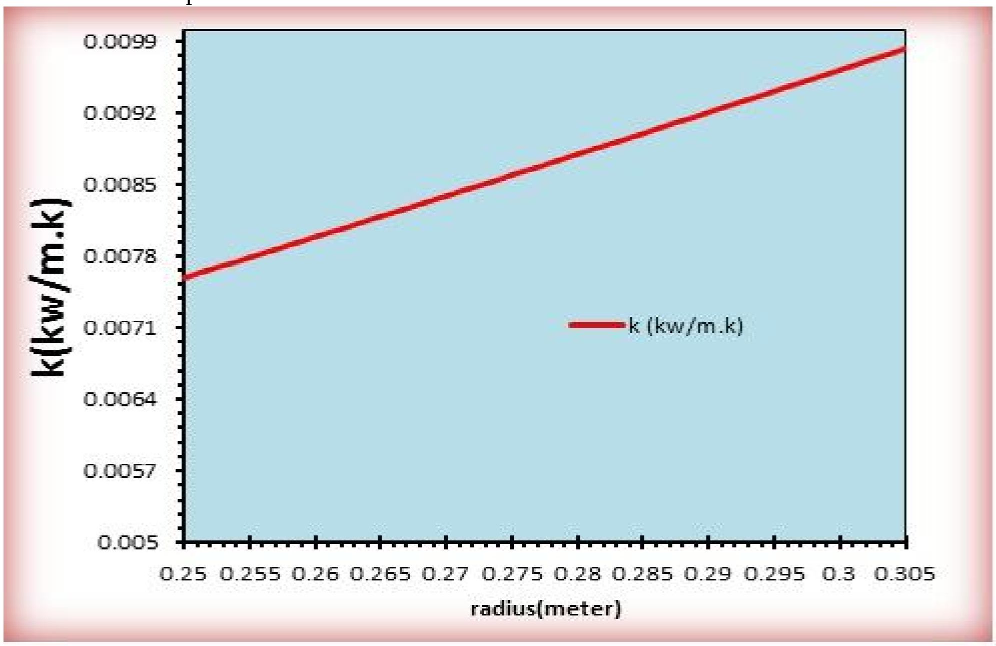

Fig. 2: Radial distribution of heat conductivity for n_1=1.3, k_0 = .046 KW/m. K

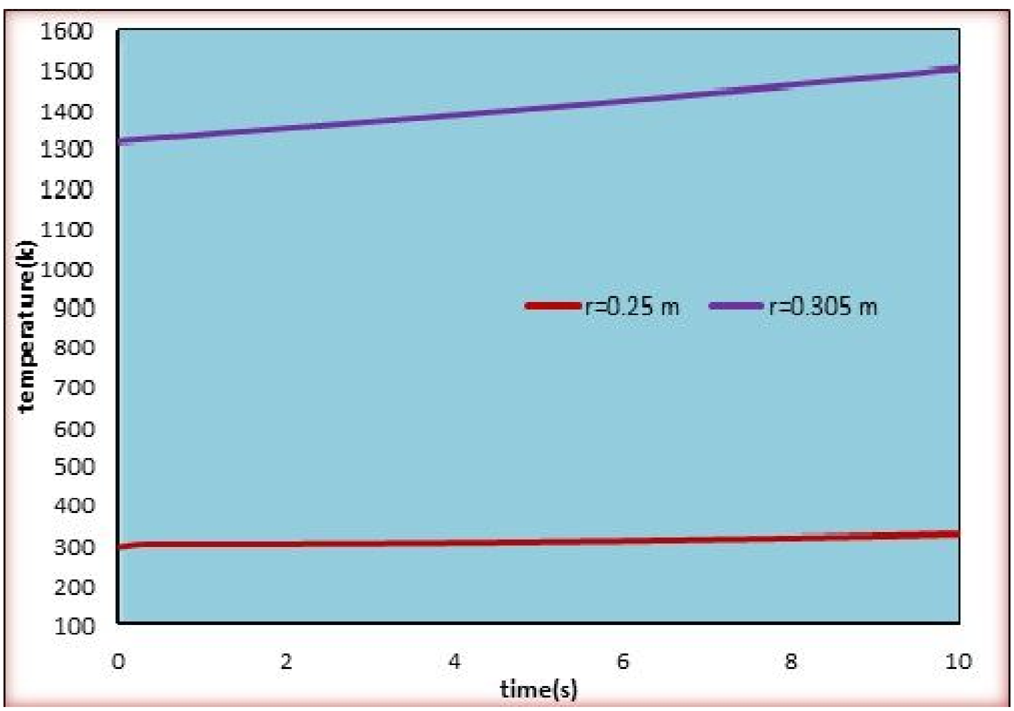

Fig 3: Radial distribution of temperature for 0≤t≤10 at r=25 cm &r=30.5 cm

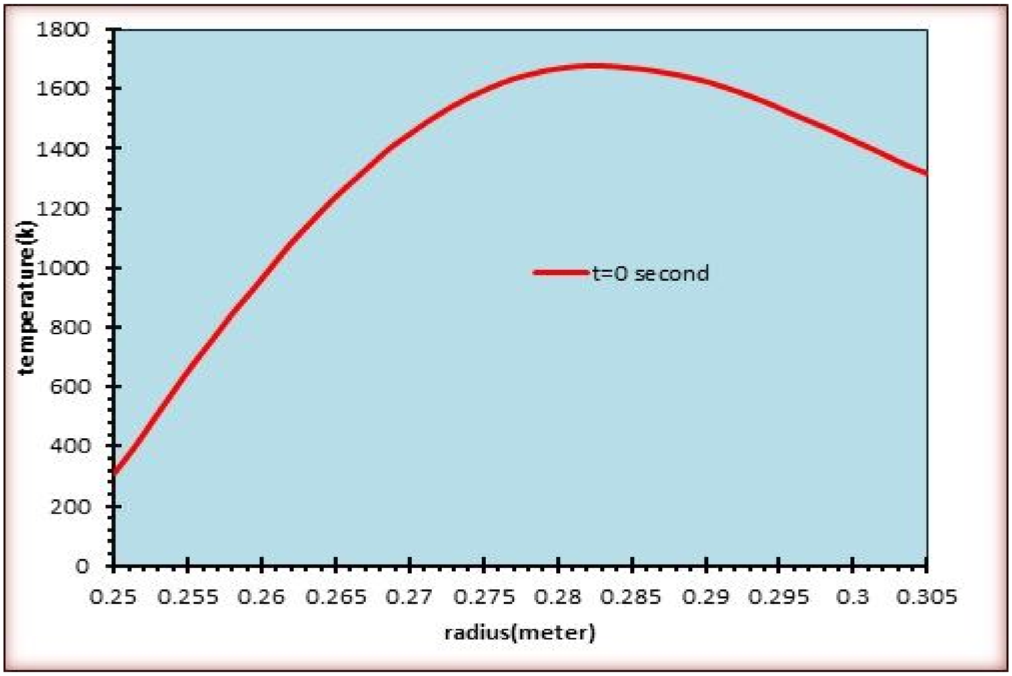

Fig4: Radial distribution of temperature for initial condition

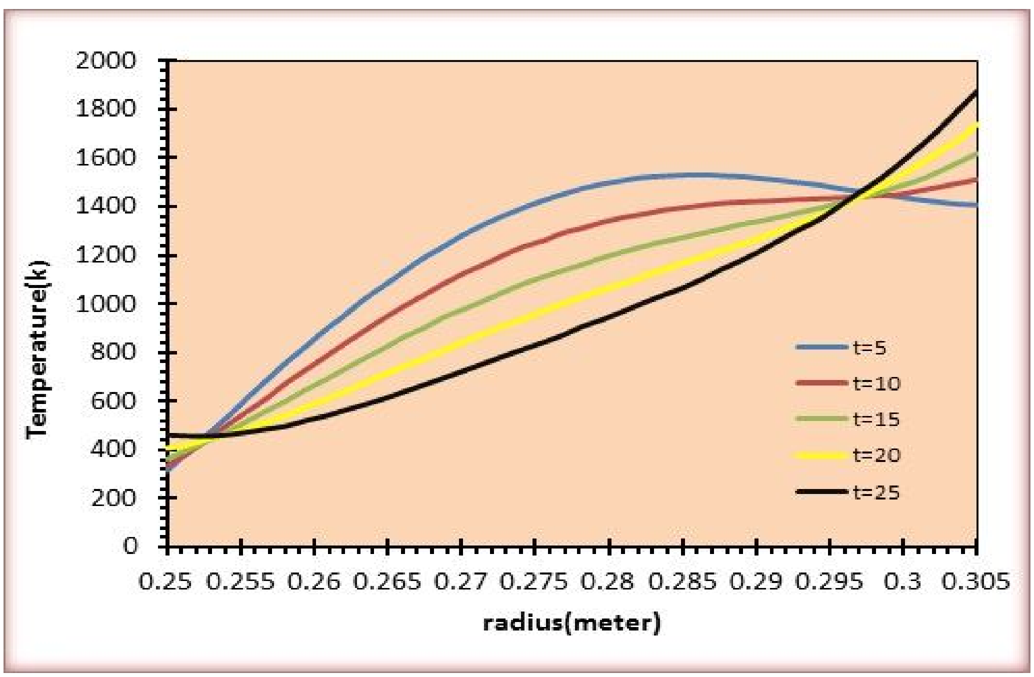

Fig.5 History of temperature radial distribution forn_1=1.3, n_2=0.2, n_3=2,r_1=25 cm , r_2=30.5 cm

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