J. Basic. Appl. Sci. Res., 2(12)12056-12065, 2012
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ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

Benchmarking Inefficient Decision Making Units in DEA

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ABSTRACT

Data envelopment analysis (DEA) is a non-parametric approach in operations research for assessing the relative efficiencies of a set of peer units called decision making units (DMUs) with multiple inputs and multiple outputs. DEA provides a fair benchmarking tool that includes a technical efficiency score for each DMU, a technical efficiency reference set with peer DMUs, a target for inefficient DMU, and information detailing by how much inputs can be decreased or outputs can be increased to the improve performance of DMUs. In this paper, we compare DEA models to benchmark inefficient DMUs and prove that popular models like the slack-based measure (SBM) and Charnes, Cooper and Rhodes (CCR) may not give the acceptable results for benchmarking inefficient DMUs as strong as the weighted additive (ADD) model. The study also warns applying those conventional DEA models for most of applications and suggests using the Kourosh and Arash Method to assess the performance evaluation of DMUs.

KEYWORDS: Data envelopment analysis, Benchmarking, Technical efficiency, Inefficiency, Arash method.

INTRODUCTION

Improving the performance of an organization is the most important responsibility of many managers. Possible inspections and detailed analysis of DMUs to understand the production process and extract useful information are necessary in order to improve on their efficiency. Efficiency is the ability to produce the outputs or services with a minimum resource level required, that is, to do the job right. Fortunately, DEA provides feasible simple methods for managers and economists in order to high performance in their firms and organizations. In fact, DEA does not require many assumptions, and it provides a number of additional opportunities in many different kinds of entities, activities and contexts. DEA was developed by Charnes et al. [1] based on the earlier work of Farrell [2]. It estimates the relative efficiencies through linear programming and considers continuous multiple inputs and multiple outputs of DMUs. In fact, Charnes et al. [1] described DEA as a mathematical programming model applied to observational data by providing a new way of obtaining empirical frontier of the production function which has become the cornerstones of modern economies. Production function is used in order to evaluate the performances of DMUs for producing maximum output for every combination of inputs.

DEA also provides a fair benchmarking tool that includes a technical efficiency score for each DMU, a technical efficiency reference set with peer DMUs, a target for inefficient DMU, and information detailing by how much inputs can be decreased or outputs can be increased to improve its performance. Indeed, a DMU is to be rated as fully (100%) technical efficient on the basis of available evidence in DEA (Pareto-Koopmans definition) if and only if the performances of other DMUs do not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs. Unfortunately, the definition of technical efficiency is wrongly interpreted as efficiency in DEA. Recently, Khezrimotlagh et al [3] identified the shortcomings of Pareto-Koopmans definition to call a DMU as efficient and proposed that an efficient DMU is a technical efficient DMU which the ratio of its output to its input (i.e., output/input) does not much change if a little error happen in its data. Moreover, the technical efficient reference set is composed by technical efficient DMUs which are used to construct the target or benchmarking standard for inefficient DMUs.

There are many DEA models, and each model has its own unique capabilities and properties. Full details on the description of some DEA techniques and the short history of DEA in three previous decades can be found in Cooper et al. [4] and Cook and Seiford [5], respectively. This paper is organized into four sections. In Section 2, we review some popular DEA models and demonstrate some of their strengths and weaknesses for benchmarking inefficient DMUs. In Section 3, we clearly demonstrate how DEA models find the reference sets and the targets for inefficient DMUs through some simple examples. The examples are very significant in exposing some important shortcomings of using conventional DEA models to benchmark inefficient DMUs. The study also warns using those

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conventional DEA models to assess the performance evaluation of decision making units and suggests the Kourosh and Arash Method in DEA for variety purposes and aims of evaluating DMUs. The paper is concluded in Section 4.

2. Conventional DEA models and their properties

In this section, we review some of the popular DEA models including CCR, BCC, ADD, SBM and ERM. In addition, we discuss some of their properties for benchmarking inefficient DMUs. Since there is no engineering standard and there is no available production function for defining efficient and effective performances, a production possibility set (PPS) is considered and its frontier is chosen for approximating the production function. The production possibility set is given by $T = \{(x, y): x \text{ can produce } y\}$, where $x \geq 0$ and $y \geq 0$. The following notations are also used in this paper:

n	number of DMUs,
m	number of inputs,
i	index of DMUs,
j	index of inputs,
k	index of outputs,
l	index of specific DMU whose efficiency is being assessed,
x_{ij}	observed amount of input j of DMU_i,
y_{ik}	observed amount of output k of DMU_i,
λ_i	multipliers used for computing linear combinations of DMUs' inputs and outputs,
s_{ij}^-	non-negative slack or potential reduction of input j of DMU_i,
s_{ik}^+	non-negative slack or potential increase of output k of DMU_i,
w_{ij}^-	positive specified weight or price for input j of DMU_i,
w_{ik}^+	positive specified weight or price for output k of DMU_i,
θ^*	the optimal technical efficiency score of a DMU in input-oriented approach,
φ^*	the optimal technical efficiency score of a DMU in output-oriented approach,
ρ^*	the optimal technical efficiency score of a DMU by SBM,
R_e^*	the optimal technical efficiency score of a DMU by ERM,
λ_i^*	optimal multipliers to identify the reference sets for a DMU, $i = 1,2, \ldots, n$,
s_{ij}^{-*}	optimal slack to identify an excess utilization of input j of DMU_i,
s_{ik}^{+*}	optimal slack to identify a shortage utilization of output k of DMU_i,
x_{ij}^*	target of input j of DMU_i after evaluation,
y_{ik}^*	target of output k of DMU_i after evaluation,

In order to illustrate DEA models, let there be n decision making units DMU_l, for $l = 1,2, \ldots, n$, such that each DMU consumes m nonnegative inputs x_{ij}, for $j = 1,2, \ldots, m$ and p nonnegative outputs y_{ik}, for $k = 1,2, \ldots, p$. Assume that each DMU has at least one positive input and one positive output value. The production possibility set (PPS) called T_C is the set of $(x, y) \in \mathbb{R}_{\geq 0}^{m+p}$ such that $\sum_{i=1}^{n} \lambda_i x_{ij} \leq x_j$, for $j = 1,2, \ldots, m$ and $\sum_{i=1}^{n} \lambda_i y_{ik} \geq y_k$, for $k = 1,2, \ldots, p$, where $\lambda \in \mathbb{R}_{\geq 0}^{n}$ [4]. Besides, the constant returns to scale (CRS) technology yields $(\lambda x, \lambda y) \in T_C$ if $(x, y) \in T_C$. The frontier of T_C is defined as an approximation of the production function called Farrell frontier. A DMU on Farrell frontier is called technical efficient and otherwise it is inefficient. The radial and non-radial DEA models reflect inefficient DMUs to Farrell frontier to benchmark them (see Figures 1 to 9). Furthermore, by adding the convexity constraint to T_C, or $\sum_{i=1}^{n} \lambda_i = 1$, it implies the variable returns to scale (VRS) PPS which suggests T_V [6].

Table 1 shows some previous popular DEA models in CRS. The CCR model in Table 1 becomes BCC [6] by replacing VRS with CRS. This means by adding the convexity constraint, or $\sum_{i=1}^{n} \lambda_i = 1$, to CCR model, it becomes BCC. Also, by adding the convexity constraint to other CRS models, they become VRS models. CCR and BCC are radial projection constructs for characterizing the technical efficiencies and inefficiencies. This means they decrease the additional input usage (increase the shortages in the output production) along the radius with the same scale. In addition, the models in input-oriented consider only possible input that decreases while keeping at least the present output levels. It is also in output-oriented which maximizes the output amounts under at most the present input consumption. Besides, the CCR and BCC models are invariant to the units of measurement and they describe a technical efficiency score of between 0 and 1. The unit invariance property means the technical efficiency scores of DMUs are independent of the units in which the inputs and outputs are measured provided these units are the same in every DMU. It can also be defined by replacing $(\alpha_j x_{ij}, \beta_k y_{ik})$ with (x_{ij}, y_{ik}) for inputs and outputs of DMUs, where the technical efficiency score of DMUs is not changed, for $\alpha_j > 0$, $\beta_k > 0$, $i = 1,2, \dots, n$, $j = 1,2, \dots, m$ and $k = 1,2, \dots, p$.

Table 1: Some of the previous common DEA Models in CRS case.

	Models	Targets
CCR Input Oriented	$\theta^* = \min \theta$, Subject to $\sum_{i=1}^{n} \lambda_i x_{ij} \le \theta x_{lj}$, for $j = 1,2, \dots, m$, $\sum_{i=1}^{n} \lambda_i y_{ik} \ge y_{lk}$, for $k = 1,2, \dots, p$, $\lambda_i \ge 0$, for $i = 1,2, \dots, n$.	$\begin{cases} x_{lj}^* = \theta^* x_{lj}, \text{for } j = 1,2, \dots, m, \\ y_{lk}^* = y_{lk}, \text{for } k = 1,2, \dots, p. \end{cases}$ If $\theta^* = 1$, DMU_l is CCR technical efficient.
CCR Output Oriented	$\varphi^* = \max \varphi$, Subject to $\sum_{i=1}^{n} \lambda_i x_{ij} \le x_{lj}$, for $j = 1,2, \dots, m$, $\sum_{i=1}^{n} \lambda_i y_{ik} \ge \varphi y_{lk}$, for $k = 1,2, \dots, p$, $\lambda_i \ge 0$, for $i = 1,2, \dots, n$.	$\begin{cases} x_{lj}^* = x_{lj}, \text{for } j = 1,2, \dots, m, \\ y_{lk}^* = \varphi^* y_{lk}, \text{for } k = 1,2, \dots, p. \end{cases}$ If $\varphi^* = 1$, DMU_l is CCR technical efficient.
ADD CRS	$\max \sum_{j=1}^{m} s_j^- + \sum_{k=1}^{p} s_k^+$, Subject to $\sum_{i=1}^{n} \lambda_i x_{ij} + s_j^- = x_{lj}$, for $j = 1,2, \dots, m$, $\sum_{i=1}^{n} \lambda_i y_{ik} - s_k^+ = y_{lk}$, for $k = 1,2, \dots, p$, $\lambda_i \ge 0$, for $i = 1,2, \dots, n$, $s_j^- \ge 0$, for $j = 1,2, \dots, m$, $s_k^+ \ge 0$, for $k = 1,2, \dots, p$.	$\begin{cases} x_{lj}^* = x_{lj} - s_j^{-*}, \text{for } j = 1,2, \dots, m, \\ y_{lk}^* = y_{lk} + s_k^{+*}, \text{for } k = 1,2, \dots, p. \end{cases}$ If $s_j^{-*} = 0, \forall j$ and $s_k^{+*} = 0, \forall k$, DMU_l is ADD technical efficient.
SBM CRS	$\rho^* = \min \frac{1-(1/m)\sum_{j=1}^{m}(s_j^-/x_{lj})}{1+(1/s)\sum_{k=1}^{p}(s_k^+/y_{lk})}$, Subject to $\sum_{i=1}^{n} \lambda_i x_{ij} + s_j^- = x_{lj}$, for $j = 1,2, \dots, m$, $\sum_{i=1}^{n} \lambda_i y_{ik} - s_k^+ = y_{lk}$, for $k = 1,2, \dots, p$, $\lambda_i \ge 0$, for $i = 1,2, \dots, n$, $s_j^- \ge 0$, for $j = 1,2, \dots, m$, $s_k^+ \ge 0$, for $k = 1,2, \dots, p$.	$\begin{cases} x_{lj}^* = x_{lj} - s_j^{-*}, \text{for } j = 1,2, \dots, m, \\ y_{lk}^* = y_{lk} + s_k^{+*}, \text{for } k = 1,2, \dots, p. \end{cases}$ If $\rho^* = 1$, DMU_l is SBM technical efficient. In the model, the terms s_j^-/x_{lj} and s_k^+/y_{lk} are deleted where $x_{lj} = 0$ and $y_{lk} = 0$, respectively, and m and p are reduced by 1.
ERM CRS	$R_e^* = \min \frac{\sum_{j=1}^{m}(\theta_j/m)}{\sum_{k=1}^{p}(\varphi_k/s)}$, Subject to $\sum_{i=1}^{n} \lambda_i x_{ij} \le \theta x_{lj}$, for $j = 1,2, \dots, m$, $\sum_{i=1}^{n} \lambda_i y_{ik} \ge \varphi y_{lk}$, for $k = 1,2, \dots, p$, $0 \le \theta_j \le 1$, for $j = 1,2, \dots, m$, $\varphi_k \ge 1$, for $k = 1,2, \dots, p$, $\lambda_i \ge 0$, for $i = 1,2, \dots, n$.	$\begin{cases} x_{lj}^* = \theta_j^* x_{lj}, \text{for } j = 1,2, \dots, m, \\ y_{lk}^* = \varphi_k^* y_{lk}, \text{for } k = 1,2, \dots, p. \end{cases}$ If $R_e^* = 1$, DMU_l is ERM technical efficient.

In contrast to CCR model (also applies to BCC model), the non-radial model ADD which was introduced by Charnes et al. [7] considers the possibility of simultaneous input decreases and output increases. There is no weak technical efficiency in the targets of the ADD model, whereas the previous models may reflect weak technical efficiency on the Farrell frontier. The weak technical efficiency means that there are some non-zero optimal slacks for a DMU whereas the models show that the DMU is technical efficient. However, the ADD model does not have the property of units' invariance. The model also does not give a technical efficiency score of between 0 and 1. In addition, if there is no weak technical efficiency in the targets of BCC model, a DMU is VRS ADD-technical efficient if and only if it is BCC-technical efficient [8]. This is also the case for CCR-technical efficient in relation to CRS ADD-technical efficient.

In order to restrain the shortcomings of ADD model, Tone [9] proposed the slack-based measure (SBM) model which is a non-radial model. The SBM model also gives a technical efficiency score between 0 and 1, and it has the units' invariance property. Besides, the optimal SBM technical efficiency score is not greater than the optimal CCR technical efficiency score and a DMU is CCR-technical efficient if and only if it is SBM-technical efficient [9]. In addition, if the input-oriented CCR and SBM scores of DMU_l be θ^* and ρ_{in}^*, respectively, the mix technical efficiency is defined by ρ_{in}^*/θ^*. Besides, the equality $\rho_{in}^* = \theta^*$ holds if and only if the input-oriented CCR model has zero input-slacks for every optimal solution [4].

Likewise, a non-radial model called Enhanced Russell Measure Model (ERM) was further developed [10,11] which escapes from the limitations in the radial measure. ERM and SBM are equivalent in their λ_i values where the optimality in one also results in the optimality in the other [4]. However, as it is also illustrated in this paper, for multiple optimal solutions the reference sets are not unique and they may sometime yield the worse reference sets for inefficient DMUs. Indeed, Khezrimotlagh et al. [3] proved that the meaning of technical efficiency should not be wrongly interpreted as efficiency similar to economics. An efficient DMU although is a technical efficient DMU, a technical efficient DMU may not be efficient. In order to remove this important shortcoming in DEA, Khezrimotlagh et al [12] proposed that an efficient DMU is a technical efficient DMU which the ratio of its output to its input (i.e., output/input) does not much change where a little error happens in its data. They also proposed a significant method to estimate the performance evaluation of decision making units [3, 12-14]. The proposed Arash Model based on the weighted additive model is as following where DMU_l is evaluated for $l = 1,2,\dots,n$.

$\boldsymbol{\varepsilon}$-AM:

$$\max \sum_{j=1}^{m} w_j^- s_j^- + \sum_{k=1}^{p} w_k^+ s_k^+,$$

Subject to

$$\sum_{i=1}^{n} \lambda_i x_{ij} + s_j^- = x_{lj} + \varepsilon_j / w_j^-, \forall j,$$
$$\sum_{i=1}^{n} \lambda_i y_{ik} - s_k^+ = y_{lk}, \ \forall k,$$
$$\lambda_i \ge 0, \ \forall i,$$
$$s_j^- \ge 0, \ \forall j,$$
$$s_k^+ \ge 0, \ \forall k.$$

Targets:

$$\begin{cases} x_{lj}^* = x_{lj} + \varepsilon_j / w_j^- - s_j^{-*}, \forall j, \\ y_{lk}^* = y_{lk} + s_k^{+*}, \forall k, \end{cases}$$

Score:

$$A_\varepsilon^* = \frac{\sum_{k=1}^{p} w_k^+ y_{lk} \,/\, \sum_{j=1}^{m} w_j^- x_{lj}}{\sum_{k=1}^{p} w_k^+ y_{lk}^* \,/\, \sum_{j=1}^{m} w_j^- x_{lj}^*}.$$

In Arash Model, w_j^-, for $j = 1,2,\dots,m$, and w_k^+, for $k = 1,2,\dots,p$, allow the summations in the objective be meaningful and they can be the user specified weights obtained through values judgment, prices or cost information. In addition, the epsilon in the **$\boldsymbol{\varepsilon}$-AM** is defined as $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_\mathrm{m})$, $\varepsilon_j \ge 0$, for $j = 1,2,\dots,m$. Moreover, if the weights w_j^- and w_k^+ are unknown they can be defined as $1/x_{lj}$ and $1/y_{lk}$ where $x_{lj} \ne 0$ and $y_{lk} \ne 0$, respectively, and N_j and M_k where $x_{lj} = 0$ and $y_{lk} = 0$, respectively, for $j = 1,2,\dots,m$ and $k = 1,2,\dots,p$. The N_j and M_k can be nonnegative real numbers regarding to the goals of each DMU.

In other words, **AM** is able to consider the variety weights w_j^- and w_k^+, where they are available and otherwise they can be defined with diversity scale such as $1/x_{lj}$ and $1/y_{lk}$ or $1/\min\{x_{ij}: x_{ij} \ne 0, i = 1,2,\dots,n\}$, $1/\max\{x_{ij}: x_{ij} \ne 0\}$, $1/\text{average}\{x_{ij}: x_{ij} \ne 0\}$ and so on and similarly for outputs, too. Indeed, **AM** is exactly flexible with varieties of weights corresponding to the aims of estimating the performance evaluation of decision making units. Furthermore, it is generally defined that $\boldsymbol{\varepsilon} = (\varepsilon, \varepsilon, \dots, \varepsilon)$, and when $\varepsilon > 0$ and $A_\varepsilon^* < 1$ for a DMU, **$\boldsymbol{\varepsilon}$-AM** proposes the DMU to change its data to the new **$\boldsymbol{\varepsilon}$-AM** target and otherwise i.e., when $A_\varepsilon^* \ge 1$, **$\boldsymbol{\varepsilon}$-AM** warns that the DMU should not change its data, because it may decrease its efficiency score.

3. Comparing DEA models to benchmark inefficient DMUs

In this section, we discuss how DEA models benchmark inefficient DMUs with two simple examples. Although, our examples are quite specific, the weaknesses can be generalized to other cases as well. Moreover, simulations have been performed with Microsoft Excel Solver and DEA-Solver software.

First, consider Table 2 and Figure 1 which show 18 DMUs labeled as A1, A2, …, A18. Each DMU has two inputs and a single constant output. Assume that the inputs have the same weights and scale, for example, in dollars.

Table 2: Example of two inputs and one constant output.

DMUs	Input 1	Input 2	Output
A1	1	12	10
A2	2	15	10
A3	2	8	10
A4	3	12	10
A5	3	5	10
A6	4	16	10
A7	4	3	10
A8	5	6	10
A9	6	10	10
A10	6	2	10
A11	7	4	10
A12	9	1	10
A13	10	9	10
A14	12	4	10
A15	13	3	10
A16	13	0	10
A17	14	1	10
A18	15	2	10

Table 3 and Figures 2 to 4 show the results of applying input-oriented CCR, ADD, SBM and ERM models in both CRS and VRS cases. There is no any weak technical efficiency in the targets of these models. In addition, there is no any difference between these models with regard to CRS or VRS cases as they characterize technical efficient DMUs. Moreover, ADD, SBM and ERM models can only consider possible input reduction like CCR and BCC models, because our example has a single constant output. From here, a comparison can be made on the models based on how they benchmark the inefficient DMUs to determine their shortcomings and powers.

Table 3: The benchmarking for DMUs in Table 2 by conventional DEA models.

Models	CCR or BCC (Input Oriented)			ADD (CRS or VRS)			SBM or ERM (CRS or VRS)		
DMUs	Input1	Input2	Output	Input1	Input2	Output	Input1	Input2	Output
A1	1	12	10	1	12	10	1	12	10
A2	1.39	10.43	10	2	8	10	1	12	10
A3	2	8	10	2	8	10	2	8	10
A4	2	8	10	3	5	10	1	12	10
A5	3	5	10	3	5	10	3	5	10
A6	2	8	10	4	3	10	1	12	10
A7	4	3	10	4	3	10	4	3	10
A8	3.44	4.12	10	4	3	10	4	3	10
A9	3	5	10	4	3	10	4	3	10
A10	6	2	10	6	2	10	6	2	10
A11	4.67	2.67	10	4	3	10	4	3	10
A12	9	1	10	9	1	10	9	1	10
A13	3.79	3.41	10	4	3	10	4	3	10
A14	6	2	10	4	3	10	9	1	10
A15	7.09	1.64	10	4	3	10	13	0	10
A16	13	0	10	13	0	10	13	0	10
A17	10.11	0.72	10	9	1	10	13	0	10
A18	8.57	1.14	10	6	2	10	13	0	10

From Table 2, the technical efficient DMUs A1 and A16 are not more efficient than other technical efficient DMUs (the DMUs on Farrell frontier in Figure 1), especially DMUs A7, A5 and A10. In fact, according to the hypothesis of this example, for instance, the efficiency of A1 is 10/(1+12) i.e., 10/13 whereas, for instance, the efficiency of A7 is 10/7.

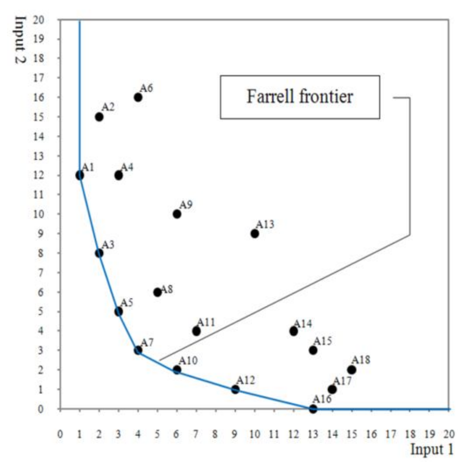

Figure 1: Example of two inputs and one constant output.

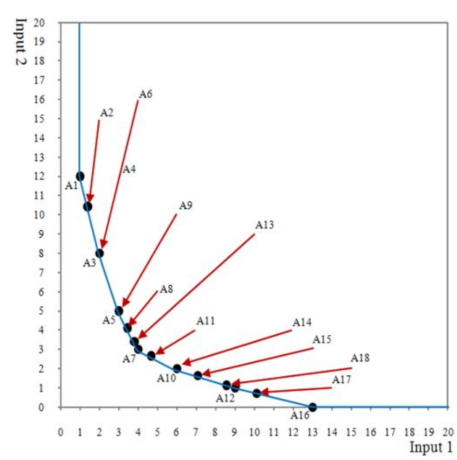

Figure 2: Benchmarking by Input-Oriented CCR or BCC model.

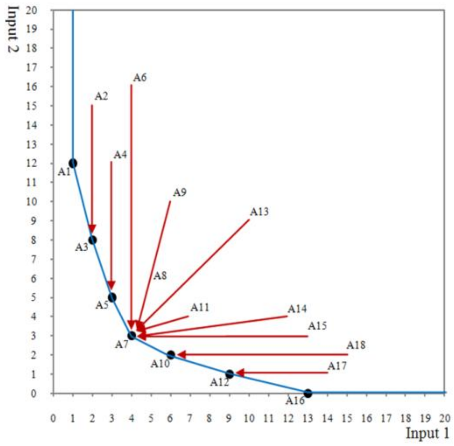

Figure 3: Benchmarking by ADD model (CRS or VRS).

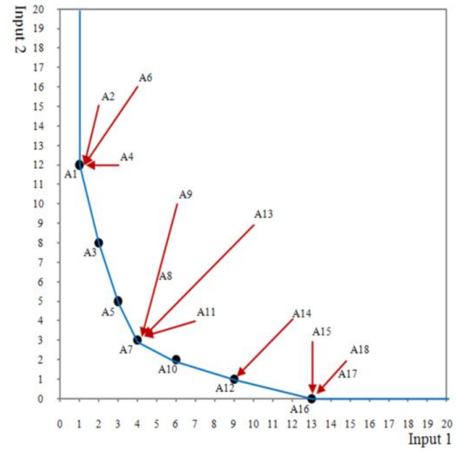

Figure 4: Benchmarking by SBM or ERM model (CRS or VRS).

As illustrated in Figure 4, SBM model (also ERM model) presents six inefficient DMUs, namely, A2, A4, A6, A15, A17 and A18 which are benchmarked to A1 and A16 (the worst technical efficient DMUs in comparison with other technical efficient DMUs), whereas none of the ADD, CCR and BCC models map to A1 or A16 for other inefficient DMUs (Figures 2 and 3). In fact, the SBM model shows that A1 is the reference set for A2, A4 and A6 with $\lambda_1^* = 1$, and A16 is the reference set for A15, A17 and A18 with $\lambda_{16}^* = 1$. However, ADD suggests A6 as the most efficient DMU (A7). This happens because of ADD gives the optimal slacks for inefficient DMUs, whereas SBM does not. In fact, SBM always maximizes the summation of ratios of s_{ij}^-/x_{ij}'s, while ADD maximizes the summation of s_{ij}^-'s where the weights are 1. For instance, Figure 3 suggests by applying the ADD model to A6, the slacks become $s_1^{-*} = 0$ and $s_2^{-*} = 13$, or $s_1^{-*} + s_2^{-*} = 13$. In comparison, SBM yields $s_1^{-*} = 3$ and $s_2^{-*} = 4$, or $s_1^{-*} + s_2^{-*} = 7$ (Figure 4). In other words, the amount of $3/4 + 4/16(= 1)$ is greater than the amount of $0/4 + 13/16\ (= 0.8125)$, and SBM cannot get the maximum summation of slacks.

Figure 5 demonstrates the differences between DEA models for benchmarking A6 and A15. In addition, the optimal slacks of applying DEA models are shown in Table 4. Furthermore, since the SBM and CCR models may not benchmark suitably the inefficient DMUs equivalent to DMUs with the same weights and scale for inputs and outputs, they may not be acceptable for the variety of weights and scale of DMUs.

On the other hand, we only consider one of the reference sets for A6 by applying SBM model. This is because for multiple solutions, the reference set is not unique. We can, however, choose any one for our purposes, as quoted on page 102 of Copper et al., [4]. This means, for example, if we assume the optimal slacks of the CCR model for A6 which are $s_1^{-*} = 2$ and $s_2^{-*} = 8$ (Figure 2) the result of $2/4 + 8/16$ is 1. Hence, SBM also suggests A3 is the reference set for A6. However, SBM cannot imply A7 (or even A5) for A6 as strong as ADD.

Obviously, the above example demonstrates the SBM or ERM models over all their abilities and their benefits may not comprehend all the inefficiencies where ADD can identify clearly. They may also suggest the worst reference sets for inefficient DMUs against ADD. However, ADD does not give an efficiency score for each DMU which is the most important part of assessing the performance evaluation of each decision making unit.

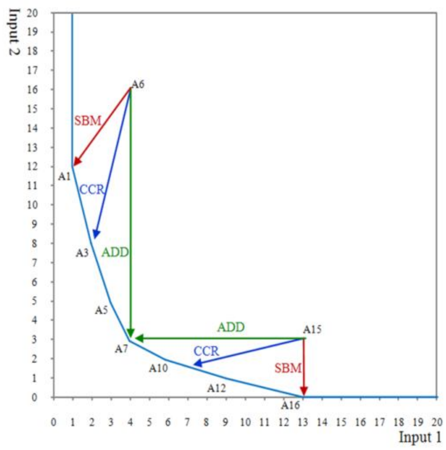

Figure 5: Benchmarking by DEA models (CRS or VRS).

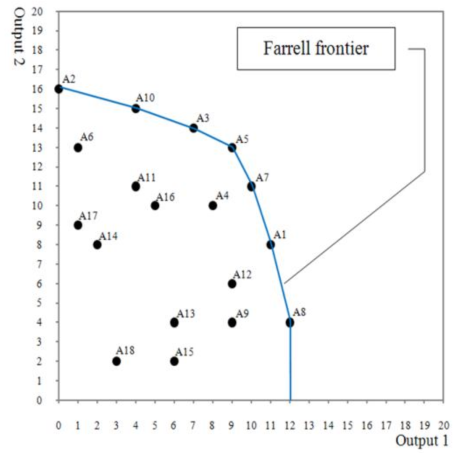

Figure 6: Example of two inputs and one constant output.

Table 4: The slacks of benchmarking DMUs in Table 2.

Models	**CCR or BCC (Input Oriented)**		**ADD (CRS or VRS)**		**SBM or ERM (CRS or VRS)**	
DMUs	s_1^{-*}	s_2^{-*}	s_1^{-*}	s_2^{-*}	s_1^{-*}	s_2^{-*}
A1	0	0	0	0	0	0
A2	0.61	4.57	0	7	1	3
A3	0	0	0	0	0	0
A4	1	4	0	7	2	0
A5	0	0	0	0	0	0
A6	2	8	0	13	3	4
A7	0	0	0	0	0	0
A8	1.56	1.88	1	3	1	3
A9	3	5	2	7	2	7
A10	0	0	0	0	0	0
A11	2.33	1.33	3	1	3	1
A12	0	0	0	0	0	0
A13	6.21	5.59	6	6	6	6
A14	6	2	8	1	6	2
A15	5.91	1.36	9	0	0	3
A16	0	0	0	0	0	0
A17	3.89	0.28	5	0	1	1
A18	6.43	0.86	9	0	2	2

In addition, CCR model requires more caution in terms of benchmarking inefficient DMUs. For instance, consider Table 5 and Figure 6 which show 18 DMUs labeled A1, A2, …, A18 with a single constant input and two outputs for each DMU. Assume also that the outputs have the same weights and scale, for instance, in dollars. The results by applying CCR output-oriented, ADD, SBM in CRS and VRS cases are demonstrated in Figures 7, 8 and 9, as well as Table 6.

From the figures there are seven technical efficient DMUs A1, A2, A3, A5, A7, A8 and A10, which all models are able to characterize them. However, the benchmarking of inefficient DMUs is different.

Table 5: Example of one constant input and two outputs.

DMUs	Input	Output1	Output2
A1	10	11	8
A2	10	0	16
A3	10	7	14
A4	10	8	10
A5	10	9	13
A6	10	1	13
A7	10	10	11

A8	10	12	4
A9	10	9	4
A10	10	5	15
A11	10	4	11
A12	10	9	6
A13	10	6	4
A14	10	2	8
A15	10	6	2
A16	10	5	10
A17	10	1	9
A18	10	3	2

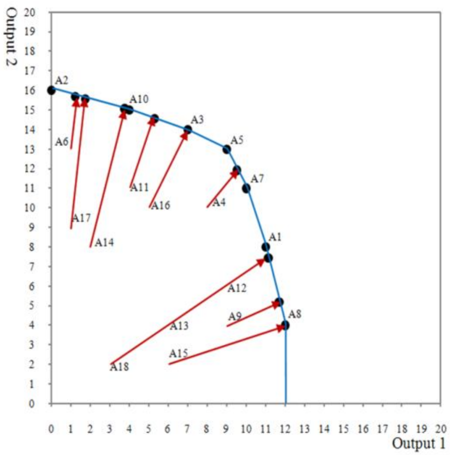

Figure 7: Benchmarking by Input-Oriented CCR or BCC model.

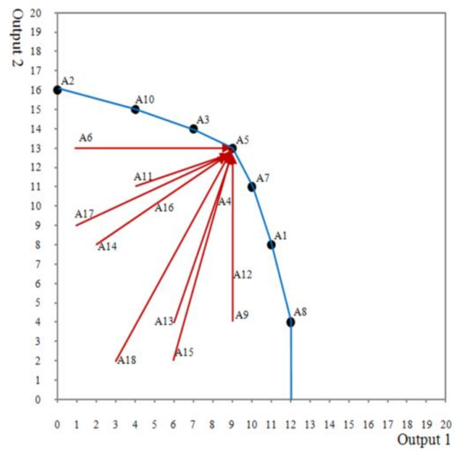

Figure 8: Benchmarking by ADD model (CRS or VRS).

For instance, Figure 7 demonstrates the results of applying CCR model (radial model) for benchmarking inefficient DMUs such as A18, A13, and A12 to a virtual efficient DMU next to A1. As it can be seen, A12 uses the same input in comparison with A5 and they have also the same quantity for output 1. This obviously suggests that A12 requires only increasing the quantity of output 2 to be the most efficient DMU in comparison with other DMUs, whereas CCR model benchmarks a worse efficient virtual DMU for it. In fact, the efficiency score of A5 is $(9+13)/10=2.2$, whereas the efficiency score of that virtual DMU is $(11.14+7.43)/10=1.86$. This is also for A9, that is, A9 needs to increase its output2, but CCR suggests it to the point with efficiency score of 16.9. These outcomes exactly warn users to apply those conventional DEA models in benchmarking inefficient DMUs and suggest only using ADD. However, ADD does not give and efficiency score for each DMU and it is not able to distinguish between technical efficient DMUs, too.

Figure 10 also illustrates the differences between those models for benchmarking A17 and A15 (the inefficient DMUs).

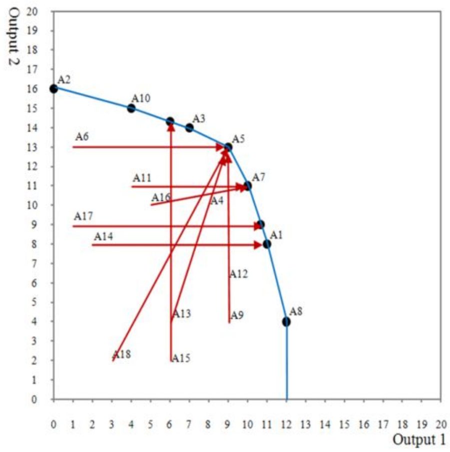

Figure 9: Benchmarking by SBM or ERM model (CRS or VRS).

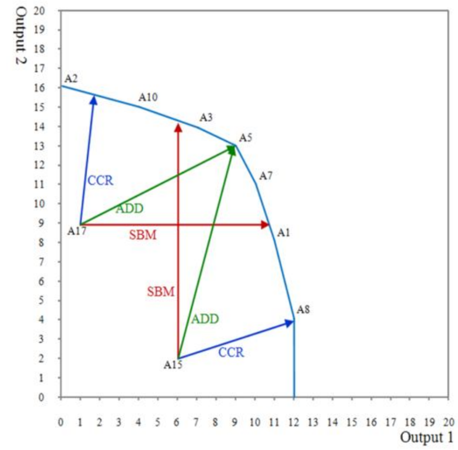

Figure 10: Benchmarking by DEA models (CRS or VRS).

Table 6: The benchmarking for DMUs in Table 5.

Models	CCR or BCC (Output Oriented)			ADD (CRS or VRS)			SBM or ERM (CRS or VRS)		
DMUs	**Input**	**Output1**	**Output2**	**Input**	**Output1**	**Output2**	**Input**	**Output1**	**Output2**
A1	10	11	8	10	11	8	10	11	8
A2	10	0	16	10	0	16	10	0	16
A3	10	7	14	10	7	14	10	7	14
A4	10	9.54	11.92	10	9	13	10	9	13
A5	10	9	13	10	9	13	10	9	13
A6	10	1.21	15.70	10	9	13	10	9	13
A7	10	10	11	10	10	11	10	10	11
A8	10	12	4	10	12	4	10	12	4
A9	10	11.7	5.2	10	9	13	10	9	13
A10	10	4	15	10	4	15	10	4	15
A11	10	5.3	14.57	10	9	13	10	10	11
A12	10	11.14	7.43	10	9	13	10	9	13
A13	10	11.14	7.43	10	9	13	10	9	13
A14	10	3.76	15.06	10	9	13	10	11	8
A15	10	12	4	10	9	13	10	6	14.33
A16	10	7	14	10	9	13	10	10	11
A17	10	1.73	15.57	10	9	13	10	10.67	9
A18	10	11.14	7.43	10	9	13	10	9	13

From the above illustrations the following proposition is proved.

Proposition: The Charnes, Cooper and Rhodes (CCR), Banker, Charnes and Cooper (BCC), Slack-Based Measure (SBM) and Enhanced Russell Measure (ERM) models may not give the acceptable results for benchmarking inefficient DMUs as strong as additive model (ADD).

Therefore, it is quite obvious that additive model is more significant than other DEA models to benchmark inefficient DMUs. In order to remove the shortcomings of ADD to give efficiency scores and distinguish between technical efficient DMUs, Khezrimotlagh et al. [3] proposed a significant technique called Kourosh and Arash Method which is exactly flexible in any purposes in DEA. It not only gives an efficiency score for each DMU, but also it is able to distinguish between technical efficient DMUs which none of the CCR, BCC, ADD, SBM and ERM is able to do it. From their proposed methods, all those DMUs in Tables 2 and 5 are benchmarked to A7 and A5, respectively. The rank of all DMUs is also characterized as the same as the rank by definition of efficiency, i.e., output/input.

4. Conclusion

This paper illustrates that the previous data envelopment analysis (DEA) models are not a complete benchmarking tool for assessing the performance evaluation of decision making units (DMUs). In fact, the common models such as CCR, BCC, SBM and ERM models for some particular examples may not benchmark inefficient DMUs to the acceptable level like what we can logically imagine. They may not also produce good results when there are many inputs and outputs for DMUs with different weights and scales. The ADD model does not also produce an efficiency score between 0 and 1. None of those models are able to distinguish between the technical efficient DMUs. Therefore, the paper suggests applying Kourosh and Arash Method for any purposes in assessing the performance evaluation of DMUs with diversity capabilities in selecting weights and scales.

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