

Choose the Best Way of Measuring Robustness in Resource Constrained Project Scheduling Problem by TOPSIS

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ABSTRACT

One of the practical problems that recently get attention of researchers is searching about uncertainly that may happen in data of project scheduling. In this paper we deal with uncertainly of activity duration. Practically some uncontrolled things, such as unpredictable increases in processing times caused by rework or supplier delays. As a result, it's very hard for project managers to meet the promised completion date. In 2008 He'di Chtourou and Mohamed Haouari proposed 12 alternative robustness predictive indicators formulated for the maximization of robustness. Finally, we have illustrated our TOPSIS using a numerical example. As a result we find out the best way of measuring robustness.

KEY WORDS: Resource constrained project scheduling problem, Robust Scheduling, TOPSIS

1. INTRODUCTION

The resource constrained project scheduling problem (RCPSP) is one of the most challenging problems in construction scheduling applications, in which optimal solutions are of great value of project planners. For over four decades, the RCPSP has received the attention of many researchers for this reason even though it is classical problem in construction planning and scheduling [1]. It is concerned with single-item or small batch production where scarce resources have to be met when scheduling dependent activities over time [2]. RCPSP is one of the most important problems in project scheduling [3]. Usually it is a hard work to estimate time and resources, namely manpower, machines, equipment, and capital budget needed to perform activities of real projects[4]. There are different branches, some of which depending on different available types of resources (renewable or nonrenewable) and others on several alternatives available for doing activities, which are called modes [5,6,7]. In this paper we focus on RCPSP with one renewable resource without different modes.

The problem assumptions are as follows:

- When project start, all of the activities are completed without any break
- First and last activities (1 & N) are somehow considered dummy that only the first activity is number 1 activity and also only the last activity is "N" activity.
- Only the renewable resources are considered and its value is constant during the project implementation.
- Activities are implemented in only one mode
- All data are deterministic

The considerable effort devoted hitherto to modeling and solving RCPSP has been almost extensively focusing on three optimization criteria. These criteria are: makespan minimization, defined as the total time elapsed between the start and the end of the project, net present value maximization, appropriate for capturing the monetary aspects of project management when important levels of cash flows (expenditures and/or payments) are available, and cost minimization that includes the case where activities may be performed in several modes resulting in different costs [7,8]. Al-Fawzen developed a multi objective Tabu search heuristic for solving a bi-objective RCPSP.

The issue of achieving a trade-off between quality robustness and solution robustness for resource unconstrained project scheduling is addressed by Van de Vonder et al [9].

In 2008 Chtourou proposed 12 alternatives to measure the robustness of scheduling that was very complete [10].

In this paper, we introduce the concept of robust scheduling and we investigate a best alternative of measuring robustness in RCPSP by TOPSIS. The rest of the paper is organized as follows. In section 2, we present notation and definitions. In section 3, we present TOPSIS. In

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section 4, we introduce the numerical example. In the final section, we present our conclusions and discuss future research.

2. Notation and Definitions

A resource constrained project scheduling problem consists of a set of activities, and set of finite capacity resources and each activity needs some demand on the resources. Totally RCPSP introduce as below:

- There are number of activities to do
- There are number of constrained resources that use to perform activities
- There are number of various constrain on activities and resources that should be observe at planning
- And there are number of objectives function that they are criteria for efficient measurement of scheduled plans

In this paper, one performance criteria is considered: maximization of robust scheduling.

Maximize $Z = RM$

subject to:

$RM_1 = \sum_{i=1}^N s_i(1)$

$RM_2 = \sum_{i=1}^N s_i \cdot NSucc_i(2)$

$RM_3 = \sum_{i=1}^N (s_i \cdot \sum_{k=1}^K r_{ik})(3)$

$RM_4 = \sum_{i=1}^N (s_i \cdot NSucc_i \cdot \sum_{k=1}^K r_{ik})(4)$

$RM_5 = \sum_{i=1}^N \alpha_i(5)$

$RM_6 = \sum_{i=1}^N \alpha_i \cdot NSucc_i(6)$

$RM_7 = \sum_{i=1}^N (\alpha_i \cdot \sum_{k=1}^K r_{ik})(7)$

$RM_8 = \sum_{i=1}^N (\alpha_i \cdot NSucc_i \cdot \sum_{k=1}^K r_{ik})(8)$

$RM_9 = \sum_{i=1}^N \min(s_i, frac.p_i) (9)$

$RM_{10} = \sum_{i=1}^N \min(s_i, frac.p_i) \cdot NSucc_i(10)$

$RM_{11} = \sum_{i=1}^N (\min(s_i, frac.p_i) \cdot \sum_{k=1}^K r_{ik}) (11)$

$RM_{12} = \sum_{i=1}^N (\min(s_i, frac.p_i) \cdot NSucc_i \cdot \sum_{k=1}^K r_{ik}) (12)$

$St_j - St_i \geq p_i + sl_i \forall j \in Su(i) ; i = 1, \dots, N(13)$

$St_N \leq DD(14)$

$\sum_{i=1}^N r_{ik} \leq R_k ; \forall i \in A(t) \quad K = 1, \dots, k ; t = 1, \dots, DD(15)$

$s_i \geq 0 ; i=1, 2, \dots, N(16)$

In continuous of this study instead of quality-robustness, we use “robustness”. The robustness is the ability to deal with small increase in time’s period of functions. Furthermore, Nobuffer insertions supported to improve the robustness of programs. According to the Vonder et al (2005), s_i (i.e. activity free slack) is defined as the time that an activity like i ($i=1, 2, \dots, N$) can have an error without making difference in start of its important successors while Maintaining Resources validations and Free slack is computed by $s_i = LS_i - ES_i$ that ES_i (LS_i) is the earliest (latest) start time of activity i as defined by the standard forward backward method [9]. The definition of latest start time for each activity is the latest time that the activity could start without delaying any of its successors earliest start time. The processes of computing these measures are as above [10].

Model parameters are defined as follows:

RM: Robust Measurement

S_i : free float of Activity “i”

St_i : start time of Activity “i”

P_i : duration time of Activity “i”

DD: Project completion date

K: The number of renewable resources

R_k : The number of available units of resource “k” per unit time

r_{ik} : Resource “k” consumption for activity “i”

Su (i): Collection of successors activities of activity “i”

A (t): Collection of Activities in the work flow at time “t”

NSucc_i : Number of immediate successors of activity i ; $i=1, \dots, N$

$\alpha_i=1$ if $s_i>0$ and $\alpha_i=0$ if $s_i=0$; $i=1, \dots, N$

$0 < frac < 1$

Equation (13) is showing minimum variance of activity “j” (activity “j” is the successor of activity “i”) and activity “i” start time should be equal to total float time and duration time of activity “i”.

Equation (14) is indicating maximum start time of dummy activity “N” should be equal to Project completion date that has specified by project owner. Equation (15) is showing at any moment of

project planning horizon, resource “K” consumption by the activity “i” should not be more than its amount available and finally limitation (16) is indicating start time for each activity “i” should be positive.

3. TOPSIS

Activity 1

Establish a decision matrix for ranking. The structure of the matrix can be expressed as follows:

$$D = \begin{bmatrix} f_{11} & \dots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{m1} & \dots & f_{mn} \end{bmatrix}$$

Where A_i denotes the alternatives $i, i = 1, \dots, m; F_j$ represent j^{th} attribute or criterion, $j = 1, \dots, n$, related to i^{th} alternative; and f_{ij} is a crisp value indicating the performance rating of each alternative A_i with respect to each criterion F_j .

Activity 2

Calculate the normalized decision matrix $R(=[r_{ij}])$. The normalized value r_{ij} is calculated as:

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{j=1}^n f_{ij}^2}}$$

Where $j=1, \dots, n; i=1, \dots, m$.

Activity 3

Calculate the weighted normalized decision matrix by multiplying the normalized decision matrix by its associated weights. The weighted normalized value V_{ij} is calculated as:

$$V_{ij} = W_j \cdot r_{ij}$$

Where W_j represent the weight of the j^{th} attribute or criterion.

Activity 4

Determine the PIS and NIS, respectively:

$$V^+ = \{v_1^+, \dots, v_n^+\} = \{(\text{Max } v_{ij} | j \in J), (\text{Min } v_{ij} | j \in J')\}$$

$$V^- = \{v_1^-, \dots, v_n^-\} = \{(\text{Min } v_{ij} | j \in J), (\text{Max } v_{ij} | j \in J')\}$$

Where J is associated with the positive criteria J' is associated with the negative criteria.

Activity 5

Calculate the separation measure, using the dimensional Euclidean distance. The separation measure D_i^+ of each alternative from the PIS is given as:

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}, i = 1, \dots, m$$

Similarly, the separation measure D_i^- of each alternative from the NIS is as follows:

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, i = 1, \dots, m$$

Activity 6

Calculate the relative closeness to the idea solution and rank the alternatives in descending order. The relative closeness of the alternative A_i with respect to PIS V^+ can be expressed as:

$$\bar{C}_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

Where the index value of \bar{C}_i lies between 0 and 1. The larger the index value, the better the performance of the alternatives [11,12,13].

4. Numerical Example

Kolish et al [6] developed the parameter driven project generator ProGen which thereafter has been widely used as a tool for the evaluation of algorithms proposed for resource constrained project scheduling. Meanwhile test sets with 10, 20 and 30 activity test set was also used to choose the best alternative of measuring Robustness.

In this section we work out a numerical example to choose the best way of robust measurement among all proposed way. Suppose that we have twelve alternatives among which decision making have to choose and, also, twelve benefit criteria are identified as the evaluation criteria for these alternatives.

Table 1
The decision matrix

	RM1	RM2	RM3	RM4	RM5	RM6	RM7	RM8	RM9	RM10	RM11	RM12	W
Max Z=RM1	0.96	0.90	0.89	0.81	0.76	0.72	0.68	0.64	0.73	0.69	0.66	0.62	$\frac{1}{12}$
Max Z=RM2	0.93	0.96	0.87	0.90	0.80	0.80	0.72	0.72	0.77	0.78	0.68	0.70	$\frac{1}{12}$
Max Z=RM3	0.90	0.85	0.95	0.90	0.81	0.78	0.78	0.76	0.76	0.73	0.74	0.72	$\frac{1}{12}$
Max Z=RM4	0.87	0.88	0.91	0.95	0.80	0.81	0.78	0.79	0.77	0.79	0.75	0.78	$\frac{1}{12}$
Max Z=RM5	0.64	0.63	0.64	0.63	0.95	0.93	0.90	0.89	0.84	0.81	0.79	0.76	$\frac{1}{12}$
Max Z=RM6	0.64	0.65	0.62	0.64	0.95	0.95	0.90	0.92	0.81	0.82	0.75	0.77	$\frac{1}{12}$
Max Z=RM7	0.63	0.61	0.65	0.63	0.94	0.92	0.94	0.91	0.82	0.79	0.81	0.78	$\frac{1}{12}$
Max Z=RM8	0.62	0.63	0.64	0.65	0.95	0.96	0.94	0.96	0.82	0.82	0.81	0.81	$\frac{1}{12}$
Max Z=RM9	0.66	0.66	0.66	0.67	0.89	0.87	0.84	0.82	0.94	0.91	0.90	0.87	$\frac{1}{12}$
MaxZ=RM10	0.66	0.68	0.66	0.69	0.90	0.91	0.85	0.86	0.94	0.95	0.89	0.91	$\frac{1}{12}$
MaxZ=RM11	0.68	0.67	0.70	0.69	0.91	0.87	0.90	0.86	0.93	0.88	0.95	0.89	$\frac{1}{12}$
MaxZ=RM12	0.68	0.70	0.70	0.73	0.91	0.91	0.89	0.90	0.93	0.93	0.94	0.95	$\frac{1}{12}$

Normalized decision matrix and weighted normalized decision matrix are given in Tables 2 and 3, respectively.

Table 2
The normalized decision matrix

	RM1	RM2	RM3	RM4	RM5	RM6	RM7	RM8	RM9	RM10	RM11	RM12
Max Z=RM1	0.37	0.35	0.34	0.31	0.25	0.24	0.23	0.22	0.25	0.24	0.24	0.22
Max Z=RM2	0.36	0.37	0.33	0.35	0.26	0.26	0.24	0.25	0.26	0.27	0.24	0.25
Max Z=RM3	0.35	0.33	0.37	0.35	0.26	0.26	0.26	0.26	0.26	0.25	0.26	0.26
Max Z=RM4	0.33	0.34	0.35	0.37	0.26	0.27	0.27	0.27	0.27	0.27	0.27	0.28
Max Z=RM5	0.25	0.24	0.25	0.24	0.31	0.31	0.31	0.31	0.29	0.28	0.28	0.27
Max Z=RM6	0.24	0.25	0.24	0.25	0.31	0.31	0.31	0.31	0.28	0.28	0.27	0.28
Max Z=RM7	0.24	0.24	0.25	0.24	0.31	0.30	0.32	0.31	0.28	0.28	0.29	0.28
Max Z=RM8	0.24	0.24	0.24	0.25	0.31	0.32	0.32	0.33	0.28	0.29	0.29	0.29
Max Z=RM9	0.25	0.26	0.26	0.26	0.29	0.29	0.29	0.28	0.32	0.32	0.32	0.32
MaxZ=RM10	0.25	0.26	0.26	0.27	0.30	0.30	0.29	0.30	0.32	0.33	0.32	0.33
MaxZ=RM11	0.26	0.26	0.27	0.27	0.30	0.29	0.31	0.30	0.32	0.31	0.34	0.32
MaxZ=RM12	0.26	0.27	0.27	0.28	0.30	0.30	0.30	0.31	0.32	0.33	0.33	0.34

Table 3
The weighted normalized decision matrix

	RM1	RM2	RM3	RM4	RM5	RM6	RM7	RM8	RM9	RM10	RM11	RM12
MaxZ=RM1	0.031	0.029	0.029	0.026	0.021	0.020	0.019	0.018	0.021	0.020	0.020	0.019
MaxZ=RM2	0.030	0.031	0.028	0.029	0.022	0.022	0.020	0.021	0.022	0.023	0.020	0.021
MaxZ=RM3	0.029	0.027	0.030	0.029	0.022	0.022	0.022	0.022	0.022	0.021	0.022	0.022
MaxZ=RM4	0.028	0.029	0.029	0.030	0.022	0.022	0.022	0.023	0.022	0.023	0.022	0.023
MaxZ=RM5	0.021	0.020	0.021	0.020	0.026	0.026	0.026	0.025	0.024	0.023	0.023	0.023
MaxZ=RM6	0.020	0.021	0.020	0.021	0.026	0.026	0.026	0.026	0.023	0.024	0.022	0.023
MaxZ=RM7	0.020	0.020	0.021	0.020	0.026	0.025	0.027	0.026	0.023	0.023	0.024	0.023
MaxZ=RM8	0.020	0.020	0.020	0.021	0.026	0.027	0.027	0.027	0.023	0.024	0.024	0.024
MaxZ=RM9	0.021	0.021	0.021	0.021	0.024	0.024	0.024	0.024	0.027	0.026	0.027	0.026
MaxZ=RM10	0.021	0.022	0.021	0.022	0.025	0.025	0.024	0.025	0.027	0.028	0.026	0.027
MaxZ=RM11	0.022	0.022	0.022	0.022	0.025	0.024	0.026	0.025	0.027	0.026	0.028	0.027
MaxZ=RM12	0.022	0.023	0.022	0.023	0.025	0.025	0.025	0.026	0.027	0.027	0.028	0.029

The closeness coefficients, which are defined to determine the ranking order of all alternatives by calculating the distance to both the “positive-ideal solution” and the “negative-ideal solution” simultaneously, are given in Table 4.

Table 4
Closeness coefficients

	d_i^+	d_i^-
Max Z=RM1	0.022	0.018
Max Z=RM2	0.017	0.020
Max Z=RM3	0.017	0.019
Max Z=RM4	0.020	0.016
Max Z=RM5	0.023	0.014
Max Z=RM6	0.023	0.015
Max Z=RM7	0.023	0.015
Max Z=RM8	0.022	0.017
Max Z=RM9	0.02	0.017
Max Z=RM10	0.019	0.018
Max Z=RM11	0.018	0.018
Max Z=RM12	0.020	0.014

Now a preference order can be ranked according to the order of \tilde{R}_i . Therefore, the best alternative is the one with the shortest distance to the positive ideal solution and with the longest distance to the negative ideal solution. According to the closeness coefficient, ranking the preference order of these alternatives is as Table 5.

Table 5
Ranking

	d_i^-	d_i^+	cl_i^+
Max Z=RM1	0.0225	0.0177	0.4403
Max Z=RM2	0.0174	0.0198	0.5319
Max Z=RM3	0.0165	0.0191	0.5368
Max Z=RM4	0.0166	0.0203	0.5504
Max Z=RM5	0.0227	0.0144	0.3882
Max Z=RM6	0.0228	0.0147	0.3918
Max Z=RM7	0.0229	0.015	0.3959
Max Z=RM8	0.0222	0.0168	0.4309
Max Z=RM9	0.0199	0.0166	0.4554
Max Z=RM10	0.0189	0.0183	0.4914
Max Z=RM11	0.0183	0.0182	0.4993
Max Z=RM12	0.0143	0.0201	0.5846

5. Conclusion

In this paper, we review the all alternative of robust measurement. Authors proposed a best alternative to choose a best approach to measure the robustness of each resource constrained project scheduling problems by using TOPSIS. In this paper, as well as considering the distance of an alternative from the positive ideal solution, its alternative from the negative ideal solution is also considered. That is to say, the less the distance of the alternative under evaluation from the positive ideal solution and the more its distance from the negative solution, the better its ranking. So among all of this twelve way of robust measurement, RM12 is the best one. It means if you want to measure the robustness of RCPSP you could find it by RM12.

For future research, one issue is worth investigating. It would be interesting to find the best way of measuring Robustness by other MCDM methods.

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