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Application of a Probabilistic Neural Network in Radial Velocity Curve Analysis of the Spectroscopic Systems PT Vel, MU Cas, V459 Cas, δ Ori Aa and δ Cir A

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ABSTRACT

Using measured radial velocity data of five double-lined spectroscopic binary systems PT Vel, MU Cas, V459 Cas, δ Ori Aa and δ Cir A, we find corresponding orbital and spectroscopic elements via a Probabilistic Neural Networks (PNNs). Our numerical results are in good agreement with those obtained by others using more traditional methods.
KEY WORDS: Stars: binaries: eclipsing -- Stars: binaries: spectroscopic.

1. INTRODUCTION

Analysis of both light and radial velocity (hereafter V_R) curves of binary systems helps us to determine the masses and radii of individual stars. One historically well-known method to analyze the V_R curve is that of Lehmann-Filhés [1]. Other methods were also introduced by Sterne [2] and Petrie [3]. The different methods of the V_R curve analysis have been reviewed in ample detail by Karami & Teimoorinia [4]. Karami V_R Teimoorinia [4] also proposed a new non-linear least squares velocity curve analysis technique for spectroscopic binary stars. They showed the validity of their new method to a wide range of different types of binary See Karami & Mohebi [5-7] and Karami et al. [8].

Probabilistic Neural Networks (PNNs) is a new tool to derive the orbital parameters of the spectroscopic binary stars. In this method the time consumed is considerably less than the method of Lehmann-Filhés and even less than the non-linear regression method proposed by Karami & Teimoorinia [4].

In the present paper we use a Probabilistic Neural Networks (PNNs) to find the optimum match to the four parameters of the V_R curves of the five double-lined spectroscopic binary systems: PT Vel, MU Cas, V459 Cas, δ Ori Aa and δ Cir A. Our aim is to show the validity of our new method to a wide range of different types of binary.

PT Vel is a eccentric binary system and consists of two main-sequence stars. The spectral type is A1V and A6V for the primary and the secondary stars, respectively, and the orbital period is P =1.8020075 days [9]. MU Cas is a double-lined spectroscopic binary and consists of the hotter, but smaller, less massive and less luminous photometric primary and the cooler, larger, more massive and more luminous photometric secondary. The spectral type is B5 V and the orbital period is P =9.652929 days [10]. V459 Cas is a double-lined spectroscopic binary and consists of the hotter, larger, more massive and more luminous photometric primary and the cooler, smaller, less massive and less luminous photometric secondary. The spectral type is A1 and the orbital period is P =8.45825381 days [11]. δ Ori Aa is an eclipsing spectroscopic and central binary in the massive triple HD 36486 (δ Orionis A). The spectral type is O9.5 II and B0.5 III for the primary and secondary stars and the orbital period is P =5.732503 days [12]. δ Cir A is a central close binary in the massive triple HD 135240 (δ Circini). The spectral type is O7 III-V and O9.5 V for the primary and secondary stars and the orbital period is P =3.902476 days [13].

This paper is organized as follows. In Sect. 2, we introduce a Probabilistic Neural Networks (PNNs) to estimate the four parameters of the V_R curve. In Sect. 3, the numerical results are reported, while the conclusions are given in Sect. 4.

2. V_R curve parameters estimation by the Probabilistic Neural Networks (PNNs)

Following Smart [14], the V_R of a star in a binary system is defined as follows

$$V_R = \gamma + K[\cos(\theta + \omega) + e\cos\omega] \qquad (1)$$

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where γ is the V_R of the center of mass of system with respect to the sun. Also K is the amplitude of the V_R of the star with respect to the center of mass of the binary. Furthermore θ, ω and e are the angular polar coordinate (true anomaly), the longitude of periastron and the eccentricity, respectively.

Here we apply the PNN method to estimate the four orbital parameters, γ, K, e and ω of the V_R curve in Eq. (1). In this work, for the identification of the observational V_R curves, the input vector is the fitted V_R curve of a star. The PNN is first trained to classify V_R curves corresponding to all the possible combinations of γ, K, e and ω. For this one can synthetically generate V_R curves given by Eq. (1) for each combination of the parameters:

- $-100 \leq \gamma \leq 100$ in steps of 1;
- $1 \leq K \leq 300$ in steps of 1;
- $0 \leq e \leq 1$ in steps of 0.001;
- $0 \leq \omega \leq 360^{\circ}$ in steps of 5;

This gives a very big set of k pattern groups, where k denotes the number of different V_R classes, one class for each combination of γ, K, e and ω. Since this very big number of different V_R classes leads to some computational limitations, hence one can first start with the big step sizes. Note that from Petrie [3], one can guess γ, K and e from a V_R curve. This enable one to limit the range of parameters around their initial guesses. When the preliminary orbit was derived after several stages, then one can use the above small step sizes to obtain the final orbit. The PNN has four layers including input, pattern, summation, and output layers, respectively (see Fig. 5 in Bazarghan et al. [15]). When an input vector is presented, the pattern layer computes distances from the input vector to the training input vectors and produces a vector whose elements indicate how close the input is to a training input. The summation layer sums these contributions for each class of inputs to produce as its net output a vector of probabilities. Finally, a competitive transfer function on the output layer picks the maximum of these probabilities, and produces a 1 for that class and a 0 for the other classes [16,17]. Thus, the PNN classifies the input vector into a specific k class labeled by the four parameters γ, K, e and ω because that class has the maximum probability of being correct.

3. Numerical Results

Here, we use the PNN to derive the orbital elements for the five different double-lined spectroscopic systems PT Vel, MU Cas, V459 Cas, δ Ori Aa and δ Cir A. Using measured V_R data of the two components of these systems obtained by Bakıs et al. [9] for PT Vel, Sandberg Lacy et al. [10] for MU Cas, Sandberg Lacy et al. [11] for V459 Cas, Harvin et al. [12] for δ Ori Aa and Penny et al. [13] for δ Cir A, the fitted velocity curves are plotted in terms of the phase in Figs. 1 to 5. The orbital parameters obtaining from the PNN for PT Vel, MU Cas, V459 Cas, δ Ori Aa and δ Cir A are tabulated in Tables 1, 3, 5, 7 and 9, respectively. Tables show that the results are in good accordance with the those obtained by Bakıs et al. [9] for PT Vel, Sandberg Lacy et al. [10] for MU Cas, Sandberg Lacy et al. [11] for V459 Cas, Harvin et al. [12] for δ Ori Aa and Penny et al. [13] for δ Cir A.

Note that the Gaussian errors of the orbital parameters in Tables 1, 3, 5, 7 and 9 are the same selected steps for generating V_R curves, i.e. $\Delta\gamma = 1, \Delta K = 1, \Delta e = 0.001$ and $\Delta\omega = 5$. These are close to the observational errors reported in the literature. Regarding the estimated errors, following Specht [17], the error of the decision boundaries depends on the accuracy with which the underlying Probability Density Functions (PDFs) are estimated. Parzen [18] proved that the expected error gets smaller as the estimate is based on a large data set. This definition of consistency is particularly important since it means that the true distribution will be approached in a smooth manner. Specht [17] showed that a very large value of the smoothing parameter would cause the estimated errors to be Gaussian regardless of the true underlying distribution and the misclassification rate is stable and does not change dramatically with small changes in the smoothing parameter.

The combined spectroscopic elements including $m_p \sin^3 i$, $m_s \sin^3 i$, $(m_p + m_s)\sin^3 i$, $(a_p + a_s)\sin i$ and $\frac{m_s}{m_p}$ are calculated by substituting the estimated parameters K, e and ω into Eqs. (3), (15) and (16) in Karami and Teimoorinia [4]. The results obtained for the five systems are tabulated in Tables 2, 4, 6, 8 and 10 show that our results are in good agreement with the those obtained by Bakıs et al. [9] for PT Vel, Sandberg Lacy et al. [10] for MU Cas, Sandberg Lacy et al. [11] for V459 Cas, Harvin et al. [12] for δ Ori Aa and Penny et al. [13] for δ Cir A, respectively. Here the errors of the combined spectroscopic elements in Tables 2, 4, 6, 8 and 10 are obtained by the help of orbital parameters errors. See again Eqs. (3), (15) and (16) in Karami and Teimoorinia [4].

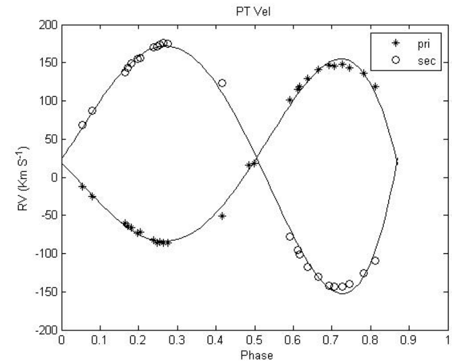

Figure 1. Radial velocities of the primary and secondary components of PT Vel plotted against the phase. The observational data have been measured Bakıs et al. [9]

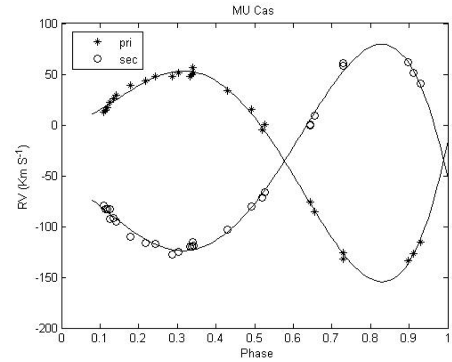

Figure 2. Radial velocities of the primary and secondary components of MU Cas plotted against the phase. The observational data have been measured by Sandberg Lacy et al. [10]

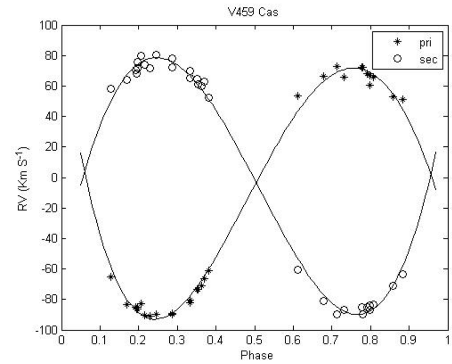

Figure 3. Radial velocities of the primary and secondary components of V459 Cas plotted against the phase. The observational data have been measured by Sandberg Lacy et al. [11]

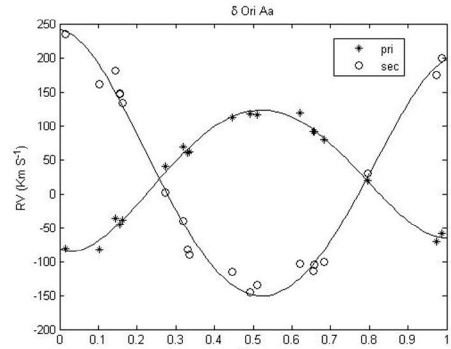

Figure 4. Radial velocities of the primary and secondary components of δ Ori Aa plotted against the phase. The observational data have been measured by Harvin et al. [12]

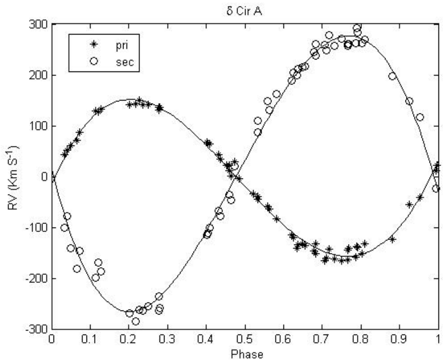

Figure 5. Radial velocities of the primary and secondary components of δ Cir A plotted against the phase. The observational data have been measured by Penny et al. [13]

Table 1. Orbital parameters of PT Vel

	This Paper	Bakıs et al. [9]
γ(km/s)	24 ± 1	24.4 ± 0.5
K_p(km/s)	117 ± 1	117.2 ± 0.2
K_s(km/s)	159± 1	158.5 ± 0.5
e	0.130 ± 0.001	0.131 ± 0.004
$\omega(^\circ)$	285 ± 5	289.91 ± 0.02

Table 2. Combined spectroscopic elements of PT Vel

Parameter	This Paper	Bakıs et al. [9]
$m_p \sin^3 i / M_\odot$	2.2042 ± 0.0467	2.195± 0.016
$m_s \sin^3 i / M_\odot$	1.6220 ± 0.0380	1.623± 0.009
$(m_p + m_s)\sin^3 i / M_\odot$	3.8262± 0.0847	—
$a_p \sin i / R_\odot$	4.1302 ± 0.0358	—
$a_s \sin i / R_\odot$	5.6128 ± 0.0360	—
$(a_p + a_s)\sin i / R_\odot$	9.7429 ± 0.0719	9.736± 0.020
m_s / m_p	0 .7358 ± 0.0111	0.739± 0.003

Table 3. Orbital parameters of MU Cas

	This Paper	Sandberg Lacy et al. [10]
γ_p(km/s)	-35± 1	− 35.7± 0.7
γ_s(km/s)	-35± 1	− 35.4± 0.6
K_p(km/s)	108± 1	107.7± 1.0
K_s(km/s)	106± 1	105.8±0.9
e	0.194± 0.001	0.1930± 0.0003
$\omega(^\circ)$	10± 5	13.4±0.4

Table 4. Combined spectroscopic elements of MU Cas

Parameter	This Paper	Sandberg Lacy et al. [10]
$m_p \sin^3 i / M_\odot$	4.5834± 0.1317	4.555± 0.092
$m_s \sin^3 i / M_\odot$	4.6699± 0.1334	4.639± 0.099
$(m_p + m_s)\sin^3 i / M_\odot$	9.2533± 0.2650	—
$a_p \sin i / R_\odot$	20.2059± 0.1912	—
$a_s \sin i / R_\odot$	19.8317± 0.1911	—
$(a_p + a_s)\sin i / R_\odot$	40.0375± 0.3823	39.95± 0.19
m_s / m_p	1.0189± 0.0195	1.019± 0.013

Table 5. Orbital parameters of V459 Cas

	This Paper	Sandberg Lacy et al. [11]
$\gamma_p(km/s)$	-7± 1	− 7.9± 0.6
$\gamma_s(km/s)$	-7± 1	− 6.4± 0.6
$K_p(km/s)$	82± 1	81.7± 0.6
$K_s(km/s)$	84± 1	83.9± 0.6
e	0.029± 0.001	0.0243
$\omega(^{\circ})$	245± 5	240.1

Table 6. Combined spectroscopic elements of V459 Cas

Parameter	This Paper	Sandberg Lacy et al. [11]
$m_p \sin^3 i / M_{\odot}$	2.0259± 0.0731	—
$m_s \sin^3 i / M_{\odot}$	1.9777± 0.0719	—
$(m_p + m_s)\sin^3 i / M_{\odot}$	4.0035± 0.1451	—
$a_p \sin\ i / R_{\odot}$	13.6973± 0.1674	—
$a_s \sin\ i / R_{\odot}$	14.0314± 0.1674	—
$(a_p + a_s)\sin\ i / R_{\odot}$	27.7288± 0.3349	—
m_s / m_p	0 .9762± 0.0236	0.974

Table 7. Orbital parameters of δ Ori Aa

	This Paper	Harvin et al. [12]
$\gamma_p(km/s)$	23± 1	24(3)
$\gamma_s(km/s)$	23± 1	20(4)
$K_p(km/s)$	104± 1	105(4)
$K_s(km/s)$	187± 1	186(7)
e	0.076± 0.001	0.075
$\omega(^{\circ})$	175± 5	173(23)

Table 8. Combined spectroscopic elements of δ Ori Aa

Parameter	This Paper	Harvin et al. [12]
$m_p \sin^3 i / M_{\odot}$	9.3236± 0.1802	9.3(10)
$m_s \sin^3 i / M_{\odot}$	5.1853± 0.1223	5.3(7)
$(m_p + m_s)\sin^3 i / M_{\odot}$	14.5090± 0.3025	—
$a_p \sin\ i / R_{\odot}$	11.7448± 0.1138	—
$a_s \sin\ i / R_{\odot}$	21.1180± 0.1145	—
$(a_p + a_s)\sin\ i / R_{\odot}$	32.8628± 0.2284	32.9(9)
m_s / m_p	0.5561± 0.0084	—

Table 9. Orbital parameters of δ Cir A

	This Paper	Penny et al. [13]
$\gamma_p(km/s)$	4± 1	-4.5(1.1)
$\gamma_s(km/s)$	4± 1	7.2(2.2)
$K_p(km/s)$	154± 1	153.0(1.4)
$K_s(km/s)$	268± 1	268.2(2.8)
e	0.051± 0.001	0.051(8)
$\omega(^{\circ})$	280± 5	276.0(8.5)

Table 10. Combined spectroscopic elements of δ Cir A

Parameter	This Paper	Penny et al. [13]
$m_p \sin^3 i / M_\odot$	19.2216± 0.2569	19.2(6)
$m_s \sin^3 i / M_\odot$	11.0453± 0.1781	11.0(3)
$(m_p + m_s)\sin^3 i / M_\odot$	30.2669± 0.4350	—
$a_p \sin i / R_\odot$	11.8582± 0.0776	11.78(11)
$a_s \sin i / R_\odot$	20.6364± 0.0781	20.65(22)
$(a_p + a_s)\sin i / R_\odot$	32.4946± 0.1557	—
m_s / m_p	0.5746± 0.0059	—

4. Conclusions

A Probabilistic Neural Networks to derive the orbital elements of spectroscopic binary stars was applied. PNNs are used in both regression (including parameter estimation) and classification problems. However, one can discretize a continuous regression problem to such a degree that it can be represented as a classification problems [16,17], as we did in this work.

Using the measured V_R data of PT Vel, MU Cas, V459 Cas, δ Ori Aa and δ Cir A obtained by Bakıs et al. [9], Sandberg Lacy et al. [10], Sandberg Lacy et al. [11], Harvin et al. [12] and Penny et al. [13], respectively, we find the orbital elements of these systems by the PNN. Our numerical results show that the results obtained for the orbital and spectroscopic parameters agree well with those obtained by others using traditional methods.

This method is applicable to orbits of all eccentricities and inclination angles. In this method the time consumed is considerably less than the method of Lehmann-Filhés. It is also more accurate as the orbital elements are deduced from all points of the velocity curve instead of four in the method of Lehmann-Filhés. The present method enables one to vary all of the unknown parameters γ, K, e and ω simultaneously instead of one or two of them at a time. It is possible to make adjustments in the elements before the final result is obtained. There are some cases, for which the geometrical methods are inapplicable, and in these cases the present one may be found useful. One such case would occur when observations are incomplete because certain phases could have not been observed. Another case in which this method is useful is that of a star attended by two dark companions with commensurable periods. In this case the resultant velocity curve may have several unequal maxima and the geometrical methods fail altogether.

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