Optimal Portfolios in a Data Envelopment Analysis Framework
(A Case Study of Tehran Stock Exchange)

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ABSTRACT

In this study, optimal portfolio was selected in Tehran Stock Exchange (TSE) by using the Data Envelopment Analysis (DEA) method. Data for trading prices of 82 active stocks in the TSE, for the period of early 1388 (March 2009) to the end of the same year, were obtained from Tadbir Pardaz software. Results revealed that, in the study period of this time, the companies of Kemase, Fejam, Makbafgh and Fesorb are selected as the main priorities and four superior companies. A reference set was determined for each 82 company in two cases of input and output orientation. Also, the results showed that, in the present conditions of the TSE, stocks that are prized more than about 13,000 Rials and the stocks with a mean expected return of about less than 60% are not considered as optimal stocks.

KEYWORDS: Optimal Portfolio Selection, Data Envelopment Analysis, Tehran Stock Exchange, Iran.

INTRODUCTION

Today, the economic activities are associated with various risks. Changes in different factors such as price levels, economic laws and factors affecting market supply and demand, are among the main reasons for uncertainty in economic trades. Along with development of economic activities and increase in bankruptcy of different financial institutions, the risk management, utilizing appropriate instruments for measurement and control of market risks has gotten a special importance. Paying attention to using such instruments in organized and legalized markets such as stock and commodity exchanges that are basically places for transfer and hedge of market risks has a double importance (3,9).

In Iran, the development in capital market in recent years and paying attention to the roles of different markets such as stock exchange has raised serious attention to risk management. In TSE, which has numerous regional trading floors in other cities as well, trades and investments are made based on cash contracts. These types of investment are usually made for the purpose of gaining profit through fluctuation of stock prices as well as receiving dividends after their tax deduction1. That is while the stock price fluctuation being increasing or decreasing. Therefore, trading in the stock exchanges for using the price fluctuations is an activity associated with risk and, therefore, investors should use the risk management instruments when making for purchasing portfolios.

Due to the important role of risk management in the capital markets, introduction and use of new measures and indices in these markets have been done rapidly. Measures such as Delta (Δ), Gamma (Γ), Vega (ν), Theta (Θ) and Rho (ρ) are usually calculated by financial institutions for describing different aspects of risk in capital markets (5). These risk measures provide valuable information for the traders who are responsible for managing various components of financial institution portfolio. However, they are of limited use in management (5).

By the necessity for paying attention to introducing the new instruments for risk management and selecting optimal portfolios in the stock exchange, in this study we have tried to examine the application of DEA method, which mainly is used for measuring efficiency and productivity, for choosing the optimal portfolio. DEA method has a high capability in selection of the optimal portfolio in the stock exchange for being a non-parametric method in determining efficiency in mathematical programming and is able to rank the units under study and introducing reference sets for them.

1 A major Part of trades made in the world stock and commodity exchanges related to derivative instruments such as futures and options. However, these instruments have not been used yet in Iran.

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Andes et al. (2002) had compared 115 medical services units using the DEA method. In this study, the variable of gross income is considered as an output and variables of the numbers of administrative, medical and technical personnel and the measurement of these units in proportion to square foot are considered as inputs. The result of this study indicates that the largeness of these units doesn’t increase the efficiency and the increasing of efficiency of these units is obtainable only by better management of resources.

Many studies have focused on application of DEA method in measuring efficiency and productivity of different units with different operation and different goals (such as Tankersley et al. (1996), Sinuany et al. (2000), Hammond (2003) and Shim (2003)). However, the performance of DEA method has not been yet tested in the portfolio selection field. So, the main aim of this study is to represent the use of this method in determining the optimal portfolio in the case of TSE.

**MATERIALS AND METHODS**

DEA method is the non-parametric mathematical programming approach to frontier estimation of efficiency and productivity. This method changes the multiple products and multiple production factors to the simple form of one factor and one product innovatively. Using the information about k production factors and M products for each N firms, the calculation process will be as follows (2):

Max \( \frac{U^\top Y}{V^\top X} \)

Subject to:

\[
\frac{U^\top Y}{V^\top X} \leq 1 \quad j = 1, 2, \ldots, N \\
U \geq 0, \ V \geq 0
\]

Where, \( U \) is an M*1 vector which includes product weights and \( V \) is a K*1 vector which includes production factor weights. \( V^\top \) and \( U^\top \) are the transposition of \( V \) and \( U \). \( X \) matrix is a K*N matrix of production factors and \( Y \) matrix is an M*N matrix of products. These two matrixes will be the indicator of all information which is related to N firms. In above model, the objective is obtaining the optimum amount of \( U \) and \( V \). The total ratio of the sum of products weight relative to the sum of production factors weight (the measuring of efficiency of each firm) should be maximized on the condition that the measurement the efficiency of each firm should be smaller than or equal to unit. The above fraction model has so many optimum answers. For this reason, the considered model is solved by maximizing the total weights of product in the condition of normalized the sum of weights of production factors. Notice that other limits are kept (2):

Max \( \mu Y_i \)

ST:

\[
\psi X_i = 1 \\
\mu Y_j - \psi X_j < 0 \\
j = 1, 2, 3, \ldots, N \\
\mu \geq 0 \quad \psi \geq 0
\]

\( \psi, \mu \) symbols have been used instead of \( V \) and \( U \) in order to liner transfer. The recent problem can be solved by using the current techniques of linear programming and the advantages of changing into dual. The dual model need for lower limits relative to primary method (because \( k+M<N+1 \)). For the same reason the dual form is better for solving above problem. Solving this problem is become easier if generally lower limits impose on liner programming. It is more interesting if we know that the dual form present the measurement of technical efficiency (\( \theta_i \)) for every firm separately (2):

Min \( 0 \)

ST:

\[
-Y_j + \psi Y \lambda \geq 0 \\
0X_j - \psi X \lambda \geq 0 \\
\lambda \geq 0
\]
where $\lambda$ is a N*1 vector which includes fixed numbers and it shows the weights of reference set. The obtained scalar quantities represent the efficiency ($\theta$) of firms. These quantities satisfy the condition of $\theta \leq 1$. In the above model, the first constrain declares that "could the real amount of product, which is produced by ith firm, be more than that by using the usable production factors?" Second one expresses that the production factors are used by $i^{th}$ firm should at least be equal to production factors which are used by reference set.

It is necessary for liner programming model to be solved N times and for each firm in each time. Therefore, the measurement of efficiency ($\theta$) will be obtained for every firms. If $\theta =1$, it will show a point on the Iso quant curve or on the frontier production function and consequently the relative efficiency of that firm becomes one hundred percent or one.

When some firms can produce a determined amount of different products by minimum quantity of production factors, other firms of this industry will be efficient just if they able to act in the same way. All efficient firms are on the Iso quant curve or frontier production curve. An efficient firm or a combination of two or several efficient firms is introduced as a reference set in DEA method. Since this aggregate corporation (Combination of two or several efficient firms) will not exist necessarily in the industry, it is known as an efficient figurative firm. On the other word, the reference set can be a real firm or totally a figurative firm for an inefficient firm. Therefore finding the best efficient figurative firm for each real firm (efficient or inefficient) is an advantage of DEA method and if a firm has been efficient, its reference set (efficient figurative firm) will be this firm. It is declarable that the share of each efficient firm in formation of efficient figurative firm (reference set) depends on weight of $\lambda$ ($\lambda$ 1, $\lambda$ 2, ..., $\lambda$ n) for an efficient firm which is presented and calculated by DEA method for each efficient firm.

Data required for this study was extracted from Tadbir Pardaz software. It contains the most recent daily trading stock prices of 82 active companies in the TSE from early 1388 up to the end of the same year. It is noteworthy that since the financial year of most of these companies coincides with one Iranian year, the date was extracted from early 1388 (March 2009).

RESULTS AND DISCUSSION

At first, considering the fact that the purpose of realistic investors in the stocks exchange is maximizing the expected return of the stock purchased, the daily return of stocks were computed and then their expected return was computed using the means of these values. Also, since minimizing the variability of the expected return of the stock purchased is also being paid attention by the same investors, the standard deviation of the expected return value of the selected stock was computed as well.

As already mentioned, the DEA method usually is used to determine the efficiency and productivity of units. However, since our purpose in this study is using the method for determination of the optimal portfolio, we must first consider quantities as input and output values of the companies under study. Considering the purpose of realistic investors, in this study, the expected return of each company was taken as its output and the most recent market price as well as the standard deviation of expected return of the companies stocks were taken as their inputs.

Then we ranked the companies under study and determined reference sets for them in the form of two viewpoints: Input orientation and output orientation. We used the DEAP software, which is the specialized software for DEA efficiency determination for the purpose.

In the input orientation viewpoint, the objective of investors is in fact choosing the stock of companies that for achieving a certain amount of expected return, have the minimum market value as well as the minimum expected standard deviation of the return.

In the output orientation viewpoint, investors choose the stocks of companies that at a certain level of price and standard deviation of the expected return, get the maximum expected return for them.

Certainly, the above two viewpoints are two faces of one, and the same coin. Therefore, they are expected to give the same results for ranking of companies. Table (1) shows these results. As you see in the table, the companies under study have been ordered from the strongest to the weakest. Note that the rank compute for these companies ranges from 0 to 100, 0 for the weakest and 100 for the strongest one.

Also, the names specified for the companies represent them in the TSE. As you see, the companies kemase, fejam, makbafgh and fesorb are selected as main priorities and four superior companies. Certainly, just these four companies are not elements constituting the portfolio of TSE investors. Companies with a rank less than 100 as well can be chosen by the investors. However a company such as kegaz that has a rank of 85.8 has more priority than another company such as sefarood that has a rank of 58.6. The advantage of this method compared to other methods for determination of portfolio is that here the investment of the investor is not the case here. In other words, different investors with different
investment amounts can choose their optimal portfolio, paying attention to the priorities specified and considering the computations made for reference sets to be stated later.

Table 1. The results of ranking the companies

<table>
<thead>
<tr>
<th>Name of company</th>
<th>Rank</th>
<th>Name of company</th>
<th>Rank</th>
<th>Name of company</th>
<th>Rank</th>
<th>Name of company</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kemase</td>
<td>100</td>
<td>shetooly</td>
<td>45.3</td>
<td>vanvin</td>
<td>24</td>
<td>dezahravi</td>
<td>9.7</td>
</tr>
<tr>
<td>Fejam</td>
<td>100</td>
<td>Vasana</td>
<td>45.3</td>
<td>fehaz</td>
<td>23.7</td>
<td>keghazvi</td>
<td>8.3</td>
</tr>
<tr>
<td>makhafgh</td>
<td>100</td>
<td>kabgach</td>
<td>45</td>
<td>madaran</td>
<td>23</td>
<td>dekimi</td>
<td>8.2</td>
</tr>
<tr>
<td>Fesorb</td>
<td>100</td>
<td>tepompi</td>
<td>43.7</td>
<td>Sharak</td>
<td>22.5</td>
<td>shenaft</td>
<td>7.9</td>
</tr>
<tr>
<td>Kegaz</td>
<td>85.8</td>
<td>Kerooy</td>
<td>43.3</td>
<td>khtoogha</td>
<td>21.2</td>
<td>beterans</td>
<td>7.7</td>
</tr>
<tr>
<td>Sefars</td>
<td>81.8</td>
<td>chebaspa</td>
<td>40.2</td>
<td>Vakar</td>
<td>20.5</td>
<td>kheterek</td>
<td>6.7</td>
</tr>
<tr>
<td>Tekno</td>
<td>81.5</td>
<td>Deler</td>
<td>39.1</td>
<td>vatoosem</td>
<td>20.2</td>
<td>shezang</td>
<td>6.7</td>
</tr>
<tr>
<td>kehmada</td>
<td>77.6</td>
<td>Vasakht</td>
<td>37.7</td>
<td>Shefara</td>
<td>19</td>
<td>tekomba</td>
<td>6.1</td>
</tr>
<tr>
<td>Sazari</td>
<td>71.8</td>
<td>Beniroo</td>
<td>36.1</td>
<td>vasah</td>
<td>19</td>
<td>hemoto</td>
<td>5</td>
</tr>
<tr>
<td>Kayta</td>
<td>67.4</td>
<td>Vanaft</td>
<td>34.9</td>
<td>semaskan</td>
<td>18.7</td>
<td>vabank</td>
<td>4</td>
</tr>
<tr>
<td>Vatoosa</td>
<td>67.1</td>
<td>Sepanta</td>
<td>33.4</td>
<td>vabooali</td>
<td>17.5</td>
<td>valime</td>
<td>3.9</td>
</tr>
<tr>
<td>Sakht</td>
<td>60.9</td>
<td>shekolor</td>
<td>33.3</td>
<td>shedoos</td>
<td>16.1</td>
<td>kemangenez</td>
<td>3.4</td>
</tr>
<tr>
<td>Bekab</td>
<td>60.6</td>
<td>Vapetro</td>
<td>33</td>
<td>vaghadir</td>
<td>15.9</td>
<td>shesfaha</td>
<td>3</td>
</tr>
<tr>
<td>Sefarood</td>
<td>58.6</td>
<td>ghamahra</td>
<td>32.9</td>
<td>shepetro</td>
<td>14.3</td>
<td>kesapa</td>
<td>2.6</td>
</tr>
<tr>
<td>chekaren</td>
<td>58.5</td>
<td>shepakxa</td>
<td>32.2</td>
<td>Kama</td>
<td>14</td>
<td>depars</td>
<td>2.3</td>
</tr>
<tr>
<td>Sayra</td>
<td>54.3</td>
<td>Vaalbar</td>
<td>32.1</td>
<td>Sheiran</td>
<td>13.3</td>
<td>kecharkhes</td>
<td>1.6</td>
</tr>
<tr>
<td>Saman</td>
<td>50.6</td>
<td>seshomal</td>
<td>32.1</td>
<td>febahonar</td>
<td>12.9</td>
<td>vatooshe</td>
<td>1.5</td>
</tr>
<tr>
<td>Fasmin</td>
<td>49.9</td>
<td>desohha</td>
<td>30.9</td>
<td>Deosve</td>
<td>11.6</td>
<td>kechad</td>
<td>1.3</td>
</tr>
<tr>
<td>Vatoos</td>
<td>47.2</td>
<td>shekhark</td>
<td>29.1</td>
<td>tesahand</td>
<td>11.4</td>
<td>pelooe</td>
<td>0</td>
</tr>
<tr>
<td>vamellat</td>
<td>45.9</td>
<td>Ketahas</td>
<td>27.5</td>
<td>vasanat</td>
<td>10.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vatooshe</td>
<td>45.4</td>
<td>Khosaz</td>
<td>25.4</td>
<td>ghedasht</td>
<td>9.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Introducing reference sets differ in the two viewpoints: Input and output orientation. As mentioned in the materials and methodology section, reference sets show that inefficient units must get their output or input level to what levels so to be regarded as an efficient unit. Therefore, in this study, the reference sets introduced for any of the companies under study in the input orientation viewpoint can determine the fact that with fixed expected return of each company, how much their purchase value must be reduced so they get a rank of 100? Also, the reference sets introduced in the output orientation viewpoint can specify the fact that with fixed purchase price as well as the standard deviation of the expected return of the stocks of any company, how much their expected return must be increases so we can regard them as the strongest companies and give them the rank 100. Paying attention to reference sets introduced can direct investors as regards the time of purchasing companies that have lower ranks. Considering the above explanations, white columns in the figures show the last prices in the stock market of any company and gray columns show their target Price of the stock, that is the price that, if realized, the company in question would be regarded as a powerful company with a rank of 100.

Figure 1. The results of introducing reference sets in the input orientation case (Rials)
Figure 2. The results of introducing reference sets in the input orientation case (Rials)

As you see, for the four superior companies, these two columns show the same value that is their last price in the market. However, it must be lowered for other companies. The difference between the values of these two columns for each company, can be regarded as their price disadvantage related to choice in the portfolio. Figures (3) and (4), too, show the results for introduction of reference sets in the output orientation case. In these figures as well, white columns show the expected return of any company and gray columns show the target expected returns. In other words, assuming other conditions remaining fixed, investors can regard the companies as strong, if the expected return of companies reach those value in the gray column, and attempt to purchase them.

Figure 3. The results of introducing reference sets in the output orientation case (Return)
In these figures as well, the four superior companies have the same values in the two columns, which shows their being superior. As regards other companies, the column relating to the expected target return shows a value more than that existing expected return and in fact, their difference shows the disadvantage of the stock return of any of the selected companies.

Another point that can be mentioned in considering figures (1) and (2) is that in the present conditions of the TSE, stocks that are priced more than about 13,000 Rials, are not considered as optimal stocks. The stocks at the bottom of table (1) as well as a large number of stocks in table (2) are such stocks. This situation supports the point considering the fact that the company shown are ordered by their priority from left to right.

Also, as regards figures (3) and (4) one can say that a stock with a mean expected return of about less than 60% can be regarded as weak stock. This is quite apparent as regards the stock shown at the bottom of table (3) as well as the whole stocks shown in table (4).

Conclusions

This study looks at optimal portfolio selection by using the Data Envelopment Analysis (DEA) approach in the case of Tehran Stock Exchange (TSE). DEA approach usually is used to determine the efficiency and productivity of units. This method has a high capability in selection of the optimal portfolio in the stock exchange for being a non-parametric method in mathematical programming and is able to rank the units under study and introducing reference sets for them. One of the most important advantages of this method compared to other methods for determination of portfolio is that here the investment of the investor is not the case here. In other words, different investors with different investment amounts can choose their optimal portfolio, paying attention to the priorities specified and considering the computations made for reference sets. The results revealed that considering the trading data in the period of time from early 1388 up to the end of this year, the companies kemase, fejam, makhafgh and fesorb are selected as main priorities and four superior companies. Either, for each considered company was determined a reference set in two cases of input and output orientation. Also, the results showed that in the present conditions of the TSE, stocks that are priced more than about 13,000 Rials and the stocks with a mean expected return of about less than 60% are not considered as optimal stocks.

REFERENCES


