J. Basic. Appl. Sci. Res., 2(3)2607-2618, 2012
© 2012, TextRoad Publication
ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

Linear Quadratic Regulator Time-Delay Controller for Hydraulic Actuator

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ABSTRACT

A novel algorithm based on LQR approach is presented to optimally tune the gains of a PI controller of a first order plus time-delay system. In this algorithm the weighting matrices Q and R of the cost function are adjusted by the natural frequency and damping ratio of the closed–loop system. In order to prove the optimality of this algorithm, it is applied to a servo-hydraulic actuator with nonlinear dynamics. In this case, in order to apply the algorithm to the system and obtain the PI tuning gains, the nonlinear governing system equations should be replaced by simplified alternative equations which are realistic as well. In the other word, high-order equations should be simplified. Then the simulation results of the nonlinear plant are compared to those of the related simplified model.
KEYWORDS: Optimal control - LQR - PI control - Time delay systems.

1. INTRODUCTION

Linear Quadratic Regulator (LQR) design technique is well known in modern optimal control theory and it has been widely investigated in literature. [1] In this method the system have to be linear and time invariant and all state variables are supposed to be measurable and the system is supposed to be observable and linear. Therefore, nonlinear equations of motion have to be linearized in order to be obtained in this approach. If the system satisfies the mentioned requirements, the optimal control law will be obtained by solving the algebraic Riccati equation [2]. This theory assumes that the settings of a regulating controller which is governing a process are found by using a mathematical algorithm that minimizes a particular cost function with weighting factors dependant to the problem definition. The "cost function" is often defined as a summation of the deviations of key measurements from their desired values [3]. Usually, the magnitude of the control action itself is included in the summation to keep the energy expended by the control action limited.

Proportional-Integral-Derivative (PID) controllers[4] is still the most widely used controller in industrial process and control during the last fifty years. The popularity of this method, it is mostly because of its simplicity in structure and its ease of implementation. In early 1940s, Ziegler and Nichols [5] proposed the first PID tuning method and surprisingly it is still widely used in industry. As industrial systems evolve and become more and more complex, their performance would become more challenging, so the Ziegler-Nichols method is insufficient in such applications. Therefore, during the last 70 years many methods have been proposed to determine parameters of PID controllers, including time response tuning [6], time domain optimization [7] frequency domain determination [8] the gain-phase margin method [9, 10] evolutionary algorithms [11, 12] robust and adaptive PID tuning [13, 14].
Time delay exists in almost all industrial processes especially mechanical types and its impact on the response of the system is dependent directly to the nature of the system [15]; therefore, it is crucial to propose different approaches to control it so there are plenty of methods investigated in literature [16].

The goal of this paper is to employ an optimal PI/PID tuning approach [17] to control a servo-hydraulic actuator. As the first step, in section 2, we will give a brief introduction of LQR method. In section 3, we will describe the employed PI/PID approach based on a low-order plus time delay model. Then, in accordance with the desired closed-loop natural frequency and damping ration we select the weighting matrices Q and R. As the next step, we model a servo-hydraulic actuator. On the other hand, our system is nonlinear, so it should be linearized in order to let us use the optimal control approach. As simplified time-delay model is determined, the time-delay term will be modeled in two manners and the results of system based on two manners will be concluded.

An optimal PI tuning algorithm for a first order plus time delay model is derived via the LQR approach in section 3. Various simulations are given in section 4. Section 5 proposes the optimal PID tuning algorithm and section 6 concludes the paper.

2. LQR solution, based on time-delay systems

Consider a linear process with time delay described by

$$\dot{x} = Ax(t) + Bu(t-L), \qquad (1)$$

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where A, B are given matrices derived from the plant transfer function and the control performance specification in terms of

$$J=\frac{1}{2}\int_0^{\infty}[x^T(t)Qx(t)+u^T(t)Ru(t)]dt, \qquad (2)$$

where Q and R are given weighting matrices with proper dimensions. In the presented set of equations, $0\le Q$, $R>0$ and $u(t)=0$, when $t<0$. The value of Q will be determined by natural frequency and damping ratio of the closed-loop system and the value of R in designing the controller has no sense[17].

The LQR problem goal is to find the optimal control *u(t)* such that *J* as the cost function in Eq. (2) is minimized. In order to achieve this goal, we decompose the dynamic process into two stages:

(I) When $0\le t<L$, $u(t-L)=0$. In this stage there is no input signal to process (1) therefore:

$$\dot{x}=Ax(t), \qquad 0\le t<L. \qquad (3)$$

(II) When $L\le t$, the process has a possible non-zero input signal. In this stage, let $\hat{u}(t)=u(t-L)$, $L\le t$ then we have:

$$\dot{x}=Ax(t)+B\hat{u}(t), \qquad L\le t. \qquad (4)$$

Through this transformation, Eqs. (3) and (4) are now both delay-free and the LQR result for delay-free process can then be applied. It is well known that the LQR solution to process (4) is [2]

$$\hat{u}\ (t)=-R^{-1}B^TPx(t), \qquad L\le t, \qquad (5)$$

where P is the positive definite solution of the Riccati equation:

$$A^TP+PA+PBR^{-1}B^TP+Q=0. \qquad (6)$$

Converting $\hat{u}\ (t)$ in Eq. (5) back to $u\ (t)$, we obtain the LQR solution to the original process (1) with the index (2) as

$$u(t)=\hat{u}\ (t+L)=-R^{-1}B^TPx(t+L), \qquad 0\le t. \qquad (7)$$

One sees from Eq. (7) that though the control law $\hat{u}\ (t)$ given in Eq. (5) is in time horizon of $L\le t$, the recovered *u(t)* actually gives the control signal for process (1) in the whole time horizon of $0\le t$. *x(t+L)* is not directly available at time *t*. By Eqs.(3)-(5), however, it can be expressed by the transmission of *x(t)* as

$$x(t+L)=\mathrm{e}^{(A-BR^{-1}B^TP)t}x(L)=\mathrm{e}^{(A-BR^{-1}B^TP)t}\mathrm{e}^{A(L-t)}x(t), \qquad (8)$$

when $0\le t<L$ and

$$x(t+L)=\mathrm{e}^{(A-BR^{-1}B^TP)t}x(L)=\mathrm{e}^{(A-BR^{-1}B^TP)L}x(t), \qquad (9)$$

when $L\le t$, If we factorize the matrix Q as $Q=H^TH$, the LQR solution to Eqs.(1) and (2) can thus be summarized in the following theorem.

Theorem 1. *For the linear process (1) with time delay, if (A, B) is controllable and (H, A) is observable, then the optimal control minimizing the criterion function (2) is given by*

$$u(t)=-R^{-1}B^TP\mathrm{e}^{(A-BP^{-1}B^TP)t}\mathrm{e}^{A(L-t)}x(t) \qquad 0\le t<L \qquad (10)$$

and

$$u(t)=-R^{-1}B^TP\mathrm{e}^{(A-BP^{-1}B^TP)L}x(t) \qquad L\le t \qquad (11)$$

where P is the positive definite solution to Eq. (6). The resultant system is also stable.

It might be concluded from Eq. (7) that the current control *u(t)* is actually a feedback of the future state at time of *(t+L)*. It implies that the controller has the prediction capability and thus may improve the closed-loop performance compared with traditional LQR or PID design. It is also noticed that during the starting period of time $t < L$, the control law (10) is time varying and generates a relatively large gain required to speed up the response. When $t=L$, Eq. (10) coincides with Eq. (11) and thus the control law is continuous. After that the feedback gain becomes constant, as seen in Eq. (11).

3. PI tuning for first-order modeling

In industry, a large class of processes has monotonic Input-output transients whose transfer functions can be approximated by a first-order plus time-delay (FOPTD) one:

$$G(s)=\frac{b}{s+a}\mathrm{e}^{-Ls} \qquad (12)$$

It should be noted that Eq. (12) is not the process itself but a model of it and is used only for the purpose of controller design. The controller, once designed, should be applied to the process but not the model. A PI controller [3]

$$u(t)=K_p\left(e(t)+\frac{1}{T_i}\int e(t)\,dt\right)=K_p\,e(t)+K_i\int e(t)\,dt \qquad (13)$$

is adequate for such a kind of processes. In this section, we will derive an optimal PI tuning algorithm via the LQR approach of the last section and the closed formulas for selecting Q and R in terms of the closed-loop specifications.

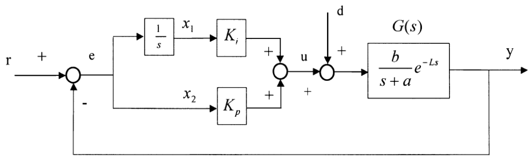

Fig. 1 Feedback control system

Consider a unity output feedback system shown in Fig.1. In the case of feedback design, the external set point does not affect the result and we put $r = 0$. It then follows from Fig. 1 that $(s+a)e=-b\mathrm{e}^{-Ls}u$, which is equivalent to the time-domain equation

$$\dot{e}=-ae-bu(t-L). \qquad (14a)$$

We have the identity

$$\frac{d}{dx}\int_0^t e(t)\mathrm{d}t=e \qquad (14b)$$

Let $x_1=\int_0^t e(t)\mathrm{d}t$ and $x_2=e$ such that $x=[x_1\ \ x_2]^{\mathrm{T}}$. Then Eq. (14a) and (14b) can be written in the following equivalent form:

$$\dot{x}=\begin{bmatrix}0 & 1\\ 0 & -a\end{bmatrix}x+\begin{bmatrix}0\\ -b\end{bmatrix}u(t-L) \qquad (15)$$

It should be emphasized that both the variables are available (see Fig. 1) and the state feedback of Kx is simply ($K_i\int_0^t e\mathrm{d}t + K_p e$), i.e., PI control. As a result, the state feedback gain to be derived by LQR will give us the required PI parameters.

In order to find the explicit expressions for K_i and K_p for ease of use, comparing Eq. (15) with Eq. (1) yields $A=\begin{bmatrix}0 & 1\\ 0 & -a\end{bmatrix}$ and $B=\begin{bmatrix}0\\ -b\end{bmatrix}$.

Let $Q=\begin{bmatrix}q_1 & 0\\ 0 & q_2\end{bmatrix}$. Substituting $P=\begin{bmatrix}p_{11} & p_{12}\\ p_{12} & p_{22}\end{bmatrix}$ into Riccati equation (6) yields

$$\begin{bmatrix} 0 & 0 \\ 1 & -a \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}+\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix}- \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 \\ -b \end{bmatrix}R^{-1}\begin{bmatrix} 0 & -b \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}+ \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}=0 \quad (16)$$

Its positive definite analytical solution is

$$\begin{aligned} p_{12} &= \sqrt{q_1 R/b}, \\ p_{22} &= (-Ra+\sqrt{R^2a^2+Rb^2(2p_{12}+q_2)})/b^2, \\ p_{11} &= ap_{12}+R^{-1}b^2p_{12}p_{22}. \end{aligned} \quad (17)$$

Let

$$\begin{aligned} F = R^{-1}B^{\mathrm{T}}P = R^{-1}\begin{bmatrix} 0 & -b \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \\ = -R^{-1}b\begin{bmatrix} p_{12} & p_{22} \end{bmatrix} \end{aligned} \quad (18)$$

and

$$\begin{aligned} A_c = A-BF = \begin{bmatrix} 0 & 1 \\ 0 & -b \end{bmatrix}+\begin{bmatrix} 0 \\ -b \end{bmatrix}R^{-1}b\begin{bmatrix} p_{12} & p_{22} \end{bmatrix} \\ = \begin{bmatrix} 0 & 1 \\ -R^{-1}b^2p_{12} & -\sqrt{a^2+R^{-1}b^2(2p_{12}+q_2))} \end{bmatrix}. \end{aligned} \quad (19)$$

The optimal controller in Eq. (10) and (11) then reduces to

$$u(t)=\begin{cases} -Fe^{A_ct}e^{A(L-t)}x(t), & 0\le t<L \\ -Fe^{A_cL}x(t), & L\le t \end{cases}. \quad (20)$$

Remark 1. Note that given the process's A and B the optimal controller in Eq. (20) depends only on the gain F in Eq. (18), or only on p_{12} and p_{22} in the solution (17) of Riccati equation. Now, if the matrix Q in Eq. (16) is replaced by a general form of $Q=\begin{bmatrix} q_1 & q_{12} \\ q_{12} & q_2 \end{bmatrix}$ then its positive definite analytical solution is $p_{22}=(-Ra+\sqrt{R^2a^2+Rb^2(2p_{12}+q_2)})/b^2$, $p_{12}=\sqrt{q_1 R/b}$, and $p_{11}=ap_{12}+R^{-1}b^2p_{12}p_{22}$. Note that p_{12} and p_{22} remain the same as those in Eq. (17) though the Q matrix is non-diagonal. This shows that choosing Q with a diagonal form will not lose the generality in the PI optimal controller design via the LQR approach for the case in Fig. 1.

To obtain the feedback gains in Eq. (20) explicitly, one needs to calculate $\exp(A_ct)$ and $\exp(A(L-t))$. It follows from the Laplace inverse transformation that

$$\begin{aligned} \exp(A(L-t)) = l^{-1}(sI-A)^{-1}|_{(L-t)} \\ = \begin{bmatrix} 1 & (1-\exp(-a(L-t)))/a \\ 0 & \exp(-a(L-t)) \end{bmatrix} \end{aligned} \quad (21)$$

As for $\exp(A_ct)$, let $\hat{a}_1=\sqrt{a^2+R^{-1}b^2(2p_{12}+q_1)}$, $\hat{a}_2=R^{-1}b^2p_{12}$, α_1 and α_2 be the roots of the equations $s^2+\hat{a}_1s+\hat{a}_2=0$, therefore those roots are $\alpha_1=(-\hat{a}_1+\sqrt{\hat{a}_1^2-4\hat{a}_2})/2$ and $\alpha_2=(-\hat{a}_1-\sqrt{\hat{a}_1^2-4\hat{a}_2})/2$.

Then we have

$$\exp(A_ct)=l^{-1}(sI-A_c)^{-1}=\begin{bmatrix} f_{11}(t) & f_{12}(t) \\ f_{21}(t) & f_{22}(t) \end{bmatrix}, \quad (22)$$

where

$$f_{11} = \frac{1}{\alpha_1 - \alpha_2}\left[(\alpha_1 + \hat{a}_1)e^{\alpha_1 t} - (\alpha_2 + \hat{a}_1)e^{\alpha_2 t}\right],$$

$$f_{12} = \frac{1}{\alpha_1 - \alpha_2}\left[e^{\alpha_1 t} - e^{\alpha_2 t}\right],$$

$$f_{21} = \frac{-\hat{a}_2}{\alpha_1 - \alpha_2}\left[e^{\alpha_1 t} - e^{\alpha_2 t}\right] \text{ and}$$

$$f_{22} = \frac{1}{\alpha_1 - \alpha_2}\left[\alpha_1 e^{\alpha_1 t} - \alpha_2 e^{\alpha_2 t}\right].$$

Recall that $u = Kx = [K_i, K_p][\int_0^t e\mathrm{d}t, e]^{\mathrm{T}}$. Substituting Eqs. (18), (19), (21) and (22) into Eq. (20) gives us the explicit expressions for the PI parameters.

Theorem 2. *The LQR optimal control for process (12) with state equation (15) is given in the form of a PI controller (13), where for* $0 \le t < L$, [1]

$$K_i(t) = R^{-1}b[p_{12}f_{11}(t) + p_{22}f_{21}(t)], \quad (23a)$$

$$K_p(t) = R^{-1}b\left\{\frac{1}{a}p_{12}f_{11}(t) + \frac{1}{a}p_{22}f_{21}(t) + \left[p_{12}f_{12}(t) - \frac{1}{a}p_{12}f_{11}(t) + p_{22}f_{21}(t) - \frac{1}{a}p_{22}f_{21}(t)\right]e^{-a(L-t)}\right\} \quad (23b)$$

and for $L \le t$,

$$K_i(t) = R^{-1}b[p_{12}f_{11}(L) + p_{22}f_{21}(L)], \quad (24a)$$

$$K_p(t) = R^{-1}b\left[p_{12}f_{12}(L) + p_{22}f_{22}(L)\right], \quad (24b)$$

where constants p_{12} *and* p_{22} *are given in Eq. (17),* $f_{ij}(t)$, $i = 1,2; j = 1,2,$ *are given in Eq. (22),* q_1, q_2 *and R are tuning parameters.*

In an ordinary LQR design, the selection of *Q* and *R* matrix is quite technical and affects the system performance a lot. In order to overcome this difficulty, we now derive a direct relationship between q_1 and q_2, and the damping ratio ξ and natural frequency ω_n of the closed-loop system.

Theorem 3. *When* $L \le t$, *the damping ratio* ξ *and natural frequency* ω_n *of the LQR optimal closed-loop system in Eqs. (13) and (15) is*

$$\omega_n = \sqrt{R^{-1}b\sqrt{q_1 R}},$$

$$\xi = \frac{\sqrt{a^2 + R^{-1}b(2\sqrt{q_1 R} + q_2 b)}}{2\sqrt{R^{-1}b\sqrt{q_1 R}}} \quad (25)$$

or equivalently, in order to have the desired ξ and ω_n, q_1 and q_2 should be chosen as

$$q_1 = \frac{\omega_n^4 R}{b^2},$$

$$q_1 = \frac{\left[(4\xi^2 - 2)\omega_n^2 - a^2\right]R}{b^2} \quad (26)$$

Proof. When $L \le t$, the closed-loop system becomes

$$\dot{x} = A_c x = \begin{bmatrix} 0 & 1 \\ -R^{-1}b\sqrt{q_1 r} & -\sqrt{a^2 + R^{-1}b(2\sqrt{q_1 r} + q_2 b)} \end{bmatrix},$$

whose characteristic equation

$$\Delta = s(s + \sqrt{a^2 + R^{-1}b(2\sqrt{q_1 r} + q_2 b)}) + R^{-1}b\sqrt{q_1 R}.$$

It thus has

$$\omega_n^2 = R^{-1}b\sqrt{q_1 R},$$
$$2\xi\omega_n = \sqrt{a^2 + R^{-1}b(2\sqrt{q_1 R} + q_2 b)}.$$

The theorem is then obvious.

Remark 2. For the system (15) with $Q = \mathrm{diag}\{q_1, q_2\}$ with q_1 and q_2 chosen according to Eq. (26), the performance index (2) becomes [1]

$$J = R\left[\int_0^\infty [\frac{\omega_n^4}{b^2}\left(\int_0^t e(t)\mathrm{d}t\right)^2 + \frac{(4\xi^2 - 2)\omega_n^2 - a^2}{b^2} e(t)^2 + u(t)^2]\mathrm{d}t\right],$$

i.e., J is proportional to R. This implies that R makes no sense in the design of controller gain F in Eq. (18) and thus we can always choose $R=1$ when Theorem 3 is applied.

In view of the above development, an optimal PI tuning algorithm for process (12) can be summarized as follows for ease of reference.

An optimal PI tuning algorithm
Initialization: Obtain a, b, L and set $R=1$.
Step 1. Choose the closed-loop ω_n and ξ.
Step 2. Calculate q_1 and q_1 from (26).
Step 3. Calculate p_{12} and p_{22} from (17), A_c from (19) and $\exp(A_c t)$ from (22).
Step 4. Calculate the PI parameters from (23) and (24).

Remark 3. In the proposed algorithm, ξ and ω_n are only user-specified parameters. To our experience, choosing $\xi \in [0.7,\ 0.9]$ and $\omega_n \in [10,\ 15]$ would give a satisfactory result. Normally, we can use defaults $\xi = 0.7$ and $\omega_n = 14$. For a better performance, a better tuning procedure may be employed.

4. Modeling studies [4]

The PI tuning algorithm proposed in the last section will be applied to a servo-hydraulic dynamical model.

Today, hydraulic drive systems are widely used in industry due to their many advantages. High transmitted power to components weight ratio, lubrication and spontaneous heating transmission due to fluid properties, the ability to apply large torques, very quick response, high bandwidth and high torque to inertia ratio are among these benefits.

Servo-hydraulic systems include various components such as servo-valves, actuators, pumps which have very complex, nonlinear and time variant dynamics. For instance, by changing the operating temperature, Temperature-sensitive parameters such as density, viscosity and balk modulus change.

Servo-hydraulic systems have wide range of applications such as Production systems, Materials testing machines, Active suspension systems, Fatigue testing, Flight simulation, Paper machines, Ships, Robotic equipment. Also in Aircraft and Aerial industries in which power to weight ratio and controlling them is important, hydraulic systems are an ideal choice to mobilize flight control surfaces.

For hydraulic systems which work long time and have variable temperatures, changes of parameters are not negligible. Therefore, it is necessary to define different controllers to compensate such changes in order to ensure optimal system performance.

However, electric motors are used in many of these applications, but motion control systems that require much force or very wide bandwidth, use effectively electro-hydraulic components. Servo-hydraulic systems are usually used for applications with bandwidth greater than 20 Hz or control power greater than 15 kW.

Besides the ability to provide high forces and quick response; servo-hydraulic systems have other benefits than electric systems. For example, hydraulic systems are more rigid and for a specified power level frequency, they have higher resonance, higher control loop gain and improved dynamic performance. Also, they have the important feature of self cooling. Hydraulic fluids, as cooling medium, dissipate heat effectively from actuators and control components. Nevertheless, such systems have several nonlinear effects which can make the modeling and parameter determination complicated and subsequently system control will become difficult.

According to the mentioned points, providing an appropriate dynamic model which is not only simple but also realistic is very important. This model should be as simple as possible; because designing a controller for a complex model is one of the major problems itself. On the other hand, the model must be precise enough to describe the behavior of the real system in a desirable manner.

In this paper, firstly a nonlinear dynamical model of a rotary servo-hydraulic system is provided .The resulting model consists of servo valve dynamics (including torque motor, valve spool and the fluid flow through orifices) and hydro-motor. Then the model will be approximated by a time-delay transfer function and finally, the Pi tuning approach will be applied to the model to control it.

4.1 System Dynamics Modeling

A rotary Servo-hydraulic system consists of the following main parts: electric motor, hydraulic pump, filter, safety valve, accumulators, pressure gauge, pressure transmitter, servo-hydraulic valve, rotary hydraulic motor (hydro-motor), one-way valve, interface board, computer and encoder.
The components of a servo-hydraulic valve and their inter connection is shown in Figure.(2)

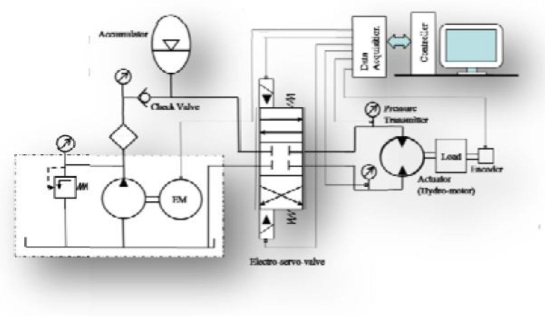

Fig. 2 Rotary servo-hydraulic system

4.2 Servo-valve modeling

A Flapper-Nozzle servo valve consists of three major parts: electric torque motor, hydraulic booster and spool valve. A diagram of the double flapper nozzle is shown in Figure. (3).

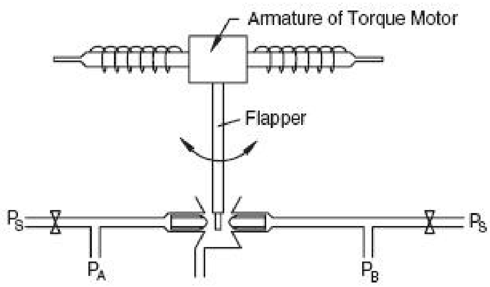

Fig. 3 A double flapper nozzle

Supply pressure (the setting of the relief valve or the setting of a pressure compensated pump) is supplied to the points identified with P_s (Figure. (3)). Fluid flows across the fixed orifices and enters the center manifold. Orifices are formed on each side between the flapper and the opposing nozzles. As long as the flapper is centered, the orifice is the same on both sides and the pressure drop to the return is the same. Pressure at A equals the pressure at B, and the spool is in force balance. Suppose the torque motor rotates the flapper clockwise. Now, the orifice on the left is smaller than the orifice on the right, and the pressure at A will be greater than the pressure at B. This pressure difference shifts the spool to the right. As the spool shifts, it deflects a feedback spring. The spool continues to move until the spring force produces a torque that equals the electromagnetic torque produced by the current flowing through the coil around the armature. At this point, the armature is moved back to the center position, the flapper is centered, the pressure becomes equal at A and B, and the spool stops. The spool stays in this position until the current through the coil changes. Because of the feedback spring, the spool has a unique position corresponding to each current through the coil ranging from 0 to rated current. At rated current, the spool is shifted to its full open position.

4.2.1. Torque motor dynamics

In order to simplify the electrical modeling of the torque motor of the servo valve, it is modeled as RL series circuits in which the back-emf effects produced by load are neglected.

The transfer function of a RL series circuit is [4]

$$\frac{I(s)}{V(s)} = \frac{1}{sL_c + R_c} \tag{27}$$

In which L_c is the self inductance of the motor coil and R_c is the total resistance of the motor coil and the servo amplifier resistant, which is provided by the manufacturer.

4.2.2. Spool valve dynamics

A servo valve is a very complex device which demonstrates a high order nonlinear response. Finding a proper mathematical model requires good understanding of its parameters. In fact, many parameters such as nozzle and orifice, springs constants, spool geometry etc which are set by the producers are not available for costumers.

In reality, all physical systems demonstrate some nonlinear behaviors which are originated from a physical motion restriction or some factor such as friction, hysteresis, mechanical abrasion, erosion, looseness etc. To model a complex servo valve, any inherent nonlinear effect should be neglected. Then by a small disturbance, the spool valve dynamic is approximated by a linear model. Such models are mostly based on first order or second order differential equations and their coefficients are set so that they fit the frequency response of the tables provided by the produce or manufacturer. A simple first order or second order is just an approximation of the real behavior. However, servo valve is selected among different types so that the natural frequency is at least 3 times greater than the natural frequency of the actuator. Therefore, it is very important to model the valve response precisely in rather low frequency ranges. Therefore the spool valve dynamics can be approximated with an acceptable second-order transfer function [4]:

$$\frac{A_v(s)}{I_v(s)} = \frac{K_v\omega_v^2}{s^2 + 2\zeta_v\omega_v s + \omega_v^2} \tag{28}$$

in which I_v is the input torque motor current and A_v is the resultant opening surface of the valve.

4.3. Actuator modeling consisting of servo-hydro motor

The ratio between the mass flow of the control valve and the internal pressure of the actuator lacuna is very important since viscosity of the fluid varies with temperature. Leakage effects should be considered in the actuator dynamics too. On the other hand, compressibility of the oil makes an elastic effect inside the cylinder lacuna which interacts with the piston mass. This factor should be considered in analysis of all hydraulic systems which mostly results in restriction of available bandwidth. This effect is modeled by the mass conservation equation. The compressibility equation is [4]

$$\frac{V_t}{2\beta}\dot{P}_L = C_d A_v \sqrt{\frac{P_s - P_L sign(A_v)}{\rho}} - D_m\dot{\theta} - C_L P_L \tag{29}$$

in which C_L is the leakage coefficient, β is the fluid balk module, θ is the position angle, V_t is the net fluid volume under pressure and D_m is the actuator volumetric displacement.

4.4. System equations:

The proposed actuator has a nonlinear nature which can be modeled by the following set of equations [4]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = w_1 x_3 - w_2 x_2 - w_3 \\ \dot{x}_3 = p_1 A_v \sqrt{P_s - x_3 sigma(A_v)} - p_2 x_3 - p_3 x_2 \end{cases} \tag{30}$$

in which

Table 1 - Parameters of system equation coefficients

$x_1 = \theta$	$x_2 = \omega = \dot{\theta}$	$x_3 = P_L$
$w_1 = \frac{D_m}{J_m}$	$w_2 = \frac{B_v}{J_m}$	$w_3 = \frac{T_L}{J_m}$
$p_1 = \frac{2\beta C_D}{V_t\sqrt{\rho}}$	$p_2 = \frac{2\beta C_L}{V_t}$	$p_3 = \frac{2\beta D_m}{V_t}$

In the mentioned set of equations, the parameter which should be controlled is $x_2 = \omega = \dot{\theta}$, therefore Eqs.30 are reduced to

$$\begin{cases} \dot{x}_2 = w_1 x_3 - w_2 x_2 - w_3 \\ \dot{x}_3 = p_1 A_v \sqrt{P_s - x_3 sigma(A_v)} - p_2 x_3 - p_3 x_2 \end{cases} \quad (31)$$

The above set of equations is a real model of servo-hydraulic valve but it is very complex even now and it is not capable to be transformed to a second-order transfer function, therefore it should be simplified. The following values are resulted from simplification, in which

$w_1 = 1$.

$w_2 = 0$, because the value of B_v is negligible in comparison with the value of J_m.

$w_3 = 0$, because the value of T_L is negligible in comparison with the value of J_m.

$P_1 = 0$, because the value of C_D is negligible in comparison with the values J_m and ρ.

$P_2 = 4.5$.

$P_3 = 0$, because the value of D_m is negligible in comparison with the value V_t.

Therefore the results of simplification can be presented as:

Table 2 - Value of parameters after applying simplifications

$w_1 = 1$	$w_2 = 0$	$w_3 = 0$
$P_1 = 0$	$P_2 = 4.5$	$P_3 = 0$

On the other hand, a second-order transfer function can be introduced as [3]

$$G(s)_{\text{model}} = \frac{a_0 \omega_o^2}{s^2 + 2\omega_o \zeta_o s + \omega_o^2} \quad (32)$$

in which $\zeta_o = 1$ and $\omega_o = 10$ are the damping ratio and natural frequency of the open-loop system.

Transfer function in Eq. (32) can be reformulated to the following transfer function which is more familiar to us due to its variables:

$$G(s) = \frac{\frac{b}{L}}{(s+a)(\frac{1}{L}+s)} \quad (33)$$

Even now, the presented Eq. (33) is not similar to Eq. (12) and that is because of the presence of one of the time-delay approximations in the equation $(1/L + s)$. Therefore this term should be replaced by the time-delay term:

$$e^{-Ls} = \frac{1}{1+Ls} \quad (34)$$

By putting Eq. (34) into Eq. (33) and reformulating the whole equation, Eq. (12) will be resulted in which

Table 3 - Value of parameters for open-loop system

$L(sec)$	$a(Rad/sec)$	$b(Rad/sec)$
0.07	4.5	9

Considering the simplification process and using the results of equation (32) and (33), an optimal control for the system is obtained in which the values of ω_n, the natural frequency and ζ_n, the damping ratio of the closed-loop system are consequently 14 and 0.7.

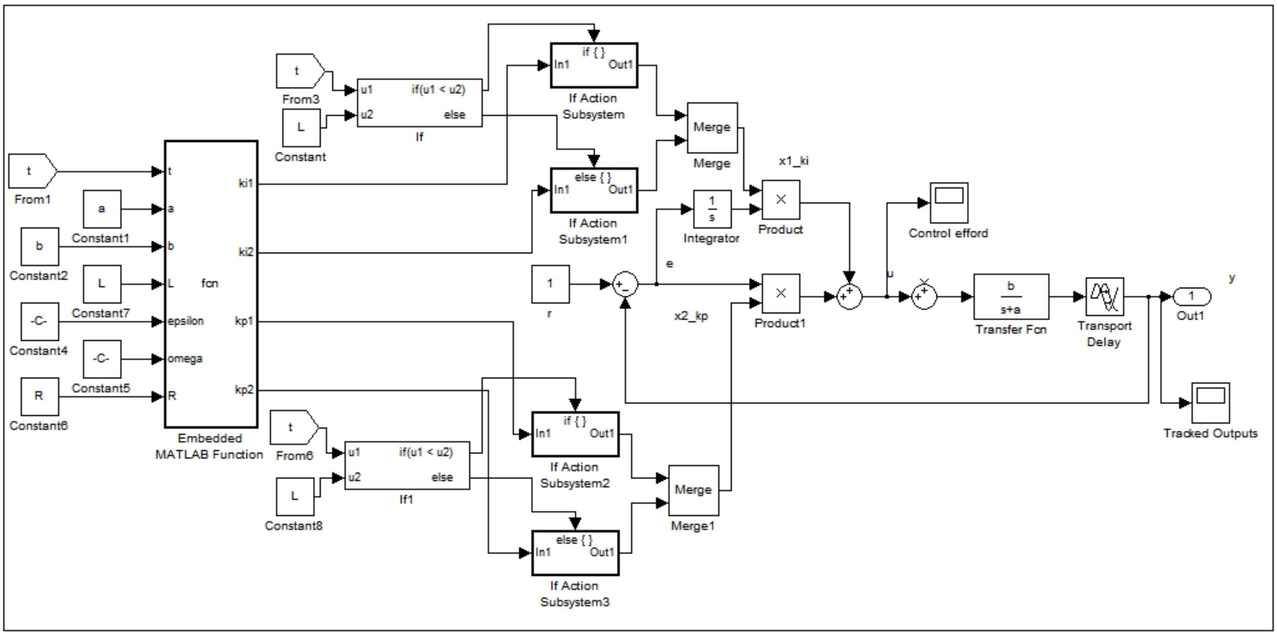

Fig. 4 The Simulation program in Simulink

5. Simulation studies

An optimal PI tuning via LQR method is presented in section 3 and 4 and the modeling of a servo-hydraulic actuator is presented in section 4.

By applying the PI optimal algorithm of sections 4 and 5 to the approximated model of section 5, PI optimal gains will be resulted which are used to control the model behavior in an optimal manner.

The resulted transfer function from the model is

$$G(s)=\frac{9}{s+4.5}\mathrm{e}^{-0.07s} \qquad (35)$$

By applying the model parameters of Table. (3) to the PI tuning method, the following coefficients are obtained:

$$K_p=[(4.839\cos(9.997t)-2.565\sin(9.997t)-2.307\mathrm{e}^{\left(\frac{9t}{2}\right)}\cos(9.997t)+2.261\mathrm{e}^{\left(\frac{9t}{2}\right)}\sin(9.997t)]\left(\mathrm{e}^{\left(\frac{-49t}{5}\right)}\right) \qquad 0\le t<L$$

$$K_i=[(21.777\cos(9.997t)-11.544\sin(9.997t)]\left(\mathrm{e}^{\left(\frac{-49t}{5}\right)}\right) \qquad 0\le t<L$$

$$K_p=0.819 \qquad L\le t$$

$$K_i=4.644 \qquad L\le t$$

which are time-variant for $0\le t<L$ and constant for $L\le t$. The trends of PI gains are shown in Figures. (7) and (8).

All of the parameters provided up to now are simulated in the Simulink environment which is shown in Figure. (4).

To see what causes different shapes of response, PI parameters for both models are shown below

As shown in Figure. (5), the settling times for both lines are similar and both models demonstrate a similar behavior against a step input. On the other hand the step input for an actuator with 0.07 sec delay time, is damped in0.7 sec. Such similarities between the responses of the simplified model (Eq. (33)) and its approximated model (Eq. (12)), demonstrate that the delay-time approximation applied to the simplified transfer function is acceptable and its response is reliable.

The presented trends in Figures. (5) and (6) demonstrate that the approximation applied to the model to adapt Eq. (32) to the first-order transfer function in Eq. (12) is acceptable and

Conclusion

Time delay is a very common phenomenon in industry. In this paper, an optimal PI tuning algorithm based on LQR method has been used to provide an aerospace servo-hydraulic actuator controller. Therefore the gain

The provided actuator has 0.07 sec delay-time and its behavior is settled in 0.7 sec which is acceptable for such actuators.

Figures and Drawings

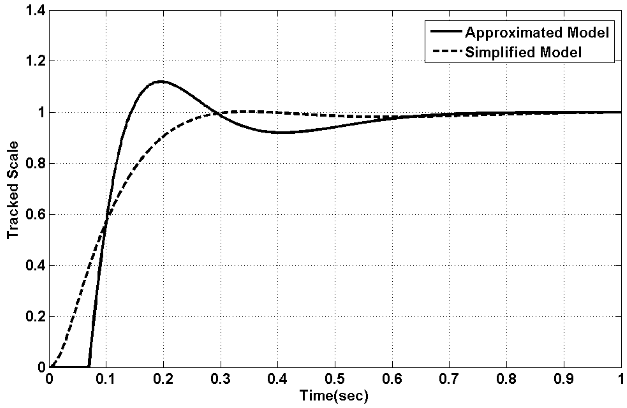

Fig. 5 Tracked outputs vs. Simulation time

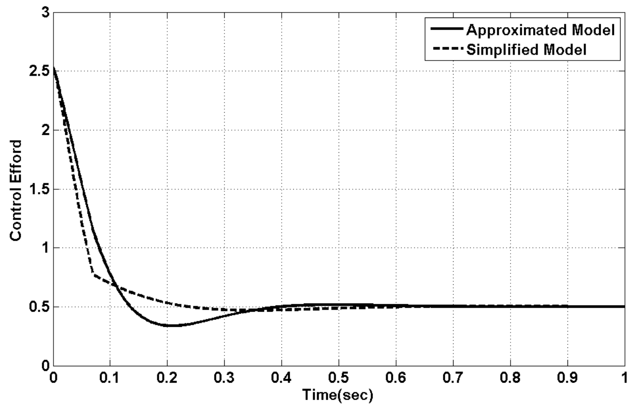

Fig. 6 Control effort vs. Time

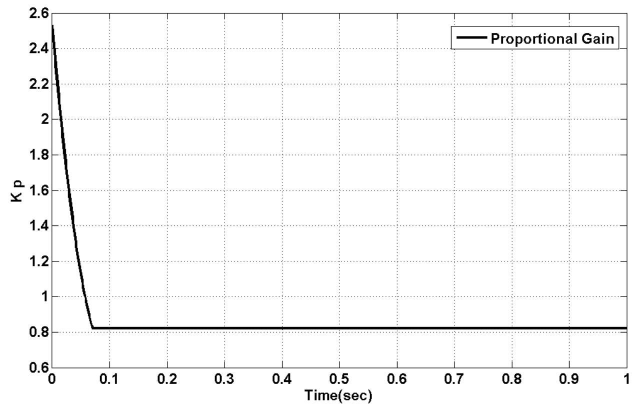

Fig. 7 K_p vs. Time

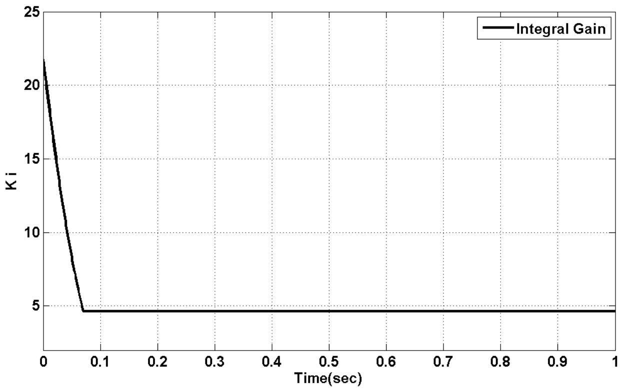

Fig. 8 K_i vs. Time

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