J. Basic. Appl. Sci. Res., 2(3)2815-2818, 2012
© 2012, TextRoad Publication

ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

Approximate Explicit Solution of non-Newtonian Boundary Layer Over a Flat Plate by Homotopy Perturbation Method

N.Moallemi, I.Shafieenejad, J.Alaee, R.Bari Shahvari

Department of Mechanical Engineering, Islamic Azad University, Jask Branch, Jask, Iran

ABSTRACT

In this paper, an approximate analytical solution for the steady-state laminar non-Newtonian fluid flow past a flat plate that obeys power law is studied. With respect to nonlinearity of this problem a homotopy perturbation is proposed to solve non-linear differential equation. Then a comparative study between the numerical solution and the homotopy perturbation method is investigated. The results reveal that the proposed method is very effective, simple and it is proper method for engineer problem like this.
KEYWORDS: non-Newtonian, boundary layer, homotopy perturbation method, nonlinear equations.

INTRODUCTION

During the past few decades non-Newtonian fluids have become more appropriate and important than Newtonian fluids. This is due to their several industrial and technological applications. Extensive efforts on this topic are available in the literatures [1].

In 1998, JH.He proposed such a technique which is a coupling of the traditional perturbation method and homotopy in topology. In this paper, by mean`s of He`s homotopy perturbation method (HPM) an approximate solution of boundary layer equation for two-dimensional laminar viscous flow over flat plate is obtained. In [2] JH.He investigates a simple perturbation approach to Blasius equation. Most recently Ganji [3] discussed application of He's homotopy perturbation method to boundary layer equation flow and convection heat transferover a flat plate.

Fundamental of the homotopy perturbation method

To illustrate the homotopy perturbation method (HPM) for solving nonlinear differential equations, He considered the following nonlinear differential equation:

$$A(U)=f(r), \qquad r\in\Omega, \tag{1}$$

subject to the boundary condition

$$B\left(u,\frac{\partial u}{\partial n}\right)=0, \qquad r\in\Gamma \tag{2}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal vector drawn outwards from Ω. The operator A can generally be divided into two parts M and *N*. Therefore, (1) can be rewritten:

$$M(u)+N(u)=f(r), \qquad r\in\Omega \tag{3}$$

He [4] constructed a homotopy $v(r,q):\Omega\times[0,1]\to\Re$ Which satisfies

$$H(v,q)=(1-q)\left[M(v)-M(u_0)\right]+q\left[A(v)-f(r)\right]=0 \tag{4}$$

Which is equivalent to

$$H(v,q)=M(v)-M(u_0)+qM(u_0)+q\left[A(v)-f(r)\right]=0 \tag{5}$$

where $q\in[0,1]$ is an embedding parameter, and u_0 is the first approximation that satisfies the boundary condition. Obviously, we have

$$H(v,0)=M(v)-M(u_0)=0 \tag{6}$$

$$H(v,1)=A(v)-f(r)=0 \tag{7}$$

The changing process of q from zero to unity is just that of $H(v,q)$ from $M(v)-M(u_0)$ to $A(v)-f(r)$. In topology, this is called deformation and $M(v)-M(u_0)$ and $A(v)-f(r)$ are called homotopic. According to the homotopy perturbation method, the parameter q is used as a small parameter, and the solution of Eq. (4) can be expressed as a series in q in the form

$$v=v_0+qv_1+q^2v_2+q^3v_3+\ldots \tag{8}$$

When $q\to 1$, Eq. (4) corresponds to the original one, Eqs. (3) and (8) becomes the approximate solution of Eq. (3), i.e.

***Corresponding Author:** Nima Moallemi, Department of Mechanical Engineering, Islamic Azad University, Jask Branch, Jask, Iran.
E-mail:nima.moallemi@yahoo.com

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \ldots \quad (9)$$

The convergence of the series in Eq. (9) is discussed by He in [4].

Formulation of the problem

Solutions are possible under restricted conditions for some non_Newtonian fluid cases. The case of a power law fluid flowing over a flat plate is considered.

The change that must be made is in the relationship of the wall shear to the assumed velocity profile.

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}^{p} \quad (10)$$

Similar solutions are also possible under restricted conditions for some non_Newtonian fluid cases. For the case of power law fluid flowing over a flat plate is as follows:

$$\eta = y \left[\frac{U_e^{2-p} \rho}{\mu \, x} \right]^{\frac{1}{p+1}}, \quad \frac{u}{U_e} = f'(\eta) \quad (11)$$

And final differential equation is derived,

$$P(P+1) f''' + (f'')^{2-p} f = 0 \quad (12)$$

$$.C: \begin{cases} f = 0 & at \quad \eta = 0 \\ \frac{d}{d\eta}(f) = 0 & at \quad \eta = 0 \\ \frac{d}{d\eta}(f) = 1 & at \quad \eta = \infty \end{cases} \quad (13)$$

Note the close relationship to the Blasius, Newtonian case *p=1* is obtained.

Solution

Eqs. (12, 13) will be solved by HPM. If three terms of these approximations are considered with respect to accuracy of this work, we will obtain:

$$f(\eta) = f_0(\eta) + q f_1(\eta) + q^2 f_2(\eta) + \cdots \quad (14)$$

Where f_i, are functions yet to be determined for $i = 0,1,2,...$ after separating the linear and nonlinear parts of the equation, we apply homotopy perturbation to Eq. (12).

$$(1-q)\left(n(n+1)\frac{d^3}{d\eta^3} f(\eta) - n(n+1)\frac{d^3}{d\eta^3} U_0 \right) + q\left(n(n+1)\frac{d^3}{d\eta^3} f(\eta) + \left(\frac{d^2}{d\eta^2} f(\eta) \right)^{2-p} f(\eta) \right) = 0 \quad (15)$$

Substituting f from Eq. (14) into Eq. (15) and some simplification and rearranging based on powers of p -terms, we have three iterations solved separately.

The differential equation of the zero order with the boundary conditions is obtained as follows:

$$p(p+1)\left(\frac{d^3}{d\eta^3} f_0(\eta) - \frac{d^3}{d\eta^3}(U_0) \right) = 0 \quad B.C: \begin{cases} f = 0 & at \quad \eta = 0 \\ \frac{d}{d\eta}(f) = 0 & at \quad \eta = 0 \\ \frac{d}{d\eta}(f) = 1 & at \quad \eta = \delta \end{cases} \quad (16)$$

We note that U_0 is initial guess and with respect to our problem, it is considered parabola.

$$U_0 = \left(\frac{1}{2\delta} \right) \eta^2 \quad (17)$$

We conclude that $f_0 = U_0 = \left(\frac{1}{2\delta} \right) \eta^2$ (18)

The first order equation is

$$p\left(p+1\right)\left(\frac{d^3}{d\eta^3}f_1\left(\eta\right)\right)+f_0\left(\frac{d^2}{d\eta^2}f_0\right)^{2-p}+p\left(p+1\right)\frac{d^3}{d\eta^3}\left(U_0\right)=0 \quad (19)$$

Subject to boundary conditions

$$\begin{cases} f=0 & at \quad \eta=0 \\ \frac{d}{d\eta}\left(f\right)=0 & at \quad \eta=0 \\ \frac{d}{d\eta}\left(f\right)=0 & at \quad \eta=\delta \end{cases} \quad (20)$$

For first order solution, we substitute the zero-order solution f_0 in to Eq. (19) and some simplifications along boundary conditions are considered.

The second order equation is Subject to boundary conditions (20).

$$p\left(p+1\right)\left(\frac{d^3}{d\eta^3}f_2\left(\eta\right)\right)+f_0\left(2-p\right)\left(\frac{d^2}{d\eta^2}f_0\right)^{1-p}\left(\frac{d^2}{d\eta^2}f_1\right)+$$

$$f_1\left(\frac{d^2}{d\eta^2}f_0\right)^{2-p}=0 \quad (21)$$

Collecting these results, finally the velocity field is obtained by the homotopy perturbation method.

RESULTS AND DISCUSSION

Since, Eq. (12) cannot be easily solved by the analytical method; so it is solved by HPM. These results are in qualitative agreement with the numerical solution presented bellow. It is concluded; this method is very effective and has high accuracy for problem like this.

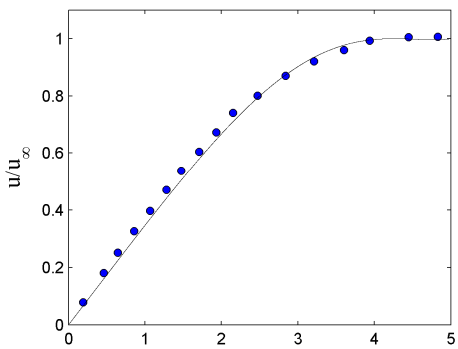

Figure 1: Velocity profile for $p=0.5$ and compared with numerical solution (points)

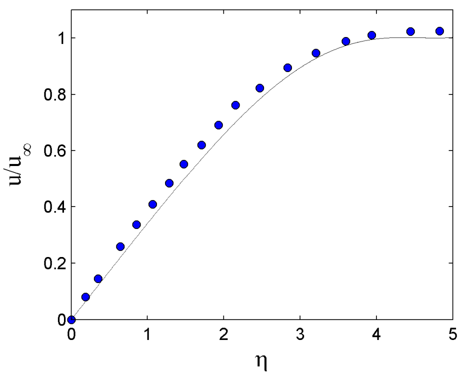

Figure 2: Velocity profile for $p=0.9$ and compared with numerical solution (points)

REFERENCES

[1]- Siddiqui AM, Ahmed M, Ghori QK. Thin film flow of non-Newtonian fluids on a moving belt. Chaos, Solitons & Fractals, Vol. 33, no. 3, pp. 1006–16, 2007.

[2]-He, J.H., "A Simple Perturbation Approach to Blasius Equation" Applied Mathematics and Computation 140 (2003) 217–222.

[3] -Esmaeil pour, M., Ganji, D., "Application of He's Homotopy Perturbation Method to Boundary Layer Flow and Convection Heat Transfer Over a Flat Plate"…

[4]-He, J.H., M.A. Abdou, "New Periodic Solutions for Nonlinear Evolution Equations Using Exp-function Method", Chaos, Solitons and Fractals (2004) 287–292.