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Investigation of Arise of Boundary Layer in a Boundary Value Problem for the First Order Partial Differential Equation Depending on Small Parameter

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ABSTRACT

In spite of the fact that investigation of rise(or lack of rise) of boundary layer in a boundary value problem for an ordinary linear equation whose coefficient of higher order derivative contains a small parameter under local conditions was conducted sufficiently we'll, but under non-local boundary conditions and mainly for partial equations such type problems were considered and investigated not enough.

In the paper we considered a boundary value problem for linear partial differential equation of first order under non-local boundary conditions when an elliptic type equation is transformed into hyperbolic one.

KEYWORDS: partial differential equation, perturbed equation, Cauchy-Riemann equation, fundamental solution, fundamental solution in direction, necessary conditions, Dirac's delta-function, Heaviside function, Dirac's complex argument function.

INTRODUCTION

It is known that in boundary value problem stated for ordinary linear differential equation whose coefficient of higher order derivative depends on a small parameter and if this parameter tends to zero the order of the equation decreases. Therefore may be limiting state of solution of the given boundary value problem doesn't satisfy boundary conditions [1],[2],[3]. So, if limiting state of the solution satisfies all the conditions of the problem, then a boundary layer arises in none of the boundaries. If even one of local conditions is not satisfied, then a boundary layer arises namely in the boundary where this condition is given.

Under non-local boundary conditions this problem , i.e. investigation of a boundary layer (on which boundary?) remains open [1]. We investigated these problems in [4], [5].

Here we'll consider a boundary value problem on a strip of plane for an elliptic type, linear partial equation of first order where the coefficient of the equation contains a small parameter. When this parameter tends to zero, this elliptic type equation becomes hyperbolic. Boundary conditions are not local.

$$\ell_\varepsilon u_\varepsilon \equiv \frac{\partial u_\varepsilon(x)}{\partial x_2} + i\varepsilon \frac{\partial u_\varepsilon(x)}{\partial x_1} = 0, \qquad x_1 \in R, \quad x_2 \in (0,\ 1),$$

(1)

here $\varepsilon \geq 0$ is a small parameter.

As first we construct an equation conjugated to equation (1). To this end we scalarly multiply the equation (1) by the function $\vartheta_\varepsilon(x)$. In the obtained expression we integrate by parts the internal integrals

$$0 = (\ell_\varepsilon u_\varepsilon, \vartheta_\varepsilon) \stackrel{def}{\equiv} \int_0^1 dx_2 \int_R \ell_\varepsilon u_\varepsilon \overline{\vartheta_\varepsilon}(x) dx_1 = \int_R dx_1 \int_0^1 \frac{\partial u_\varepsilon(x)}{\partial x_2} \overline{\vartheta_\varepsilon(x)} dx_2 + i\varepsilon \int_0^1 dx_2 \int_R \frac{\partial u_\varepsilon(x)}{\partial x_1} \overline{\vartheta_\varepsilon(x)} dx_1$$

. (*)

We take into account that the following natural conditions are fulfilled:

$$\lim_{x_1 \to \pm\infty} u_\varepsilon(x) = 0, \qquad (2)$$

Then, from (*) we get [6],[7]:

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$$\int_R u_\varepsilon(x_1,1)\overline{\vartheta_\varepsilon(x_1,1)}dx_1 - \int_R u_\varepsilon(x_1,0)\overline{\vartheta_\varepsilon(x_1,0)}dx_1 -$$

$$-\int_0^1 dx_2 \int_R dx_1 u_\varepsilon(x)\left[\frac{\partial \overline{\vartheta_\varepsilon(x)}}{\partial x_2} + i\varepsilon \frac{\partial \overline{\vartheta_\varepsilon(x)}}{\partial x_1}\right] = 0.$$

(3)

Thus, the relation conjugated to the equation (1) is obtained in the form:

$$\ell_\varepsilon^* \vartheta_\varepsilon \equiv \frac{\partial \vartheta_\varepsilon(x)}{\partial x_2} - i\varepsilon \frac{\partial \vartheta_\varepsilon(x)}{\partial x_1}. \tag{4}$$

Fundamental solution of the conjugated equation

In order to construct fundamental solution we consider the following inhomogeneous equation that corresponds to the equation (4):

$$\frac{\partial \vartheta_\varepsilon(x)}{\partial x_2} - i\varepsilon \frac{\partial \vartheta_\varepsilon(x)}{\partial x_1} = g(x), \tag{5}$$

here $g(x)$ is an arbitrary smooth function.

It is known that the Fourier transformation [6] is one of the principal methods for constructing fundamental solution. Assume

$$\vartheta_\varepsilon(x) = \frac{1}{2\pi}\int_{R^2} e^{i(\alpha,x)}\tilde{\vartheta}_\varepsilon(\alpha)d\alpha, \tag{6}$$

$$g(x) = \frac{1}{2\pi}\int_{R^2} e^{i(\alpha,x)}\tilde{g}(\alpha)d\alpha, \tag{7}$$

$$\tilde{g}(\alpha) = \frac{1}{2\pi}\int_{R^2} e^{-i(\alpha,\xi)}g(\xi)d\xi, \tag{8}$$

here

$$(\alpha, x) = \alpha_1 x_1 + \alpha_2 x_2. \tag{9}$$

Now, the values of (6) and (7) write in (5):

$$\frac{1}{2\pi}\int_{R^2} e^{i(\alpha,x)} i\alpha_2\tilde{\vartheta}(\alpha)d\alpha - i\varepsilon\frac{1}{2\pi}\int_{R^2} e^{i(\alpha,x)} i\alpha_1\tilde{\vartheta}(\alpha)d\alpha =$$

$$= \frac{1}{2\pi}\int_{R^2} e^{i(\alpha,x)}\tilde{g}(\alpha)d\alpha,$$

$$\frac{1}{2\pi}\int_{R^2} e^{i(\alpha,x)}\left[(i\alpha_2 + \varepsilon\alpha_1)\tilde{\vartheta}(\alpha) - \tilde{g}(\alpha)\right]d\alpha = 0.$$

Hence, for $\vartheta(\alpha)$ we get the following equation:

$$(i\alpha_2 + \varepsilon\alpha_1)\tilde{\vartheta}(\alpha) = \tilde{g}(\alpha),$$

$$\tilde{\vartheta}(\alpha) = \frac{\tilde{g}(\alpha)}{i\alpha_2 + \varepsilon\alpha_1} \tag{10}$$

Taking into account (8) and substituting (10) into (6) we get:

$$\vartheta_\varepsilon(x) = \frac{1}{2\pi}\int_{R^2} e^{i(\alpha,x)}d\alpha \frac{1}{i\alpha_2 + \varepsilon\alpha_1}\cdot\frac{1}{2\pi}\int_{R^2} e^{-i(\alpha,\xi)}g(\xi)d\xi.$$

Thus, for fundamental solution we get the relation:

$$V_\varepsilon(x-\xi) = \frac{1}{4\pi^2}\int_{R^2}\frac{e^{i(\alpha,x-\xi)}}{i\alpha_2 + \varepsilon\alpha_1}d\alpha, \tag{11}$$

Considering that this relation is differentiable and having written instead of $\vartheta_\varepsilon(x)$ its value in equation (5) we get the following expression:

$$\frac{\partial V_\varepsilon(x-\xi)}{\partial x_2}-i\varepsilon\frac{\partial V_\varepsilon(x-\xi)}{\partial x_1}=\frac{1}{4\pi^2}\int_{R^2}\frac{i\alpha_2-i\varepsilon\cdot i\alpha_1}{i\alpha_2+\varepsilon\alpha_1}e^{i(\alpha,x-\xi)}d\alpha=$$

$$=\frac{1}{2\pi}\int_R e^{i\alpha_1(x_1-\xi_1)}d\alpha_1\cdot\frac{1}{2\pi}\int_R e^{i\alpha_2(x_2-\xi_2)}d\alpha_2=\delta(x_1-\xi_1)\delta(x_2-\xi_2)=\delta(x-\xi).$$

(12)

Remark 1. If we'll calculate the integrals contained in (5) we get the expression:

$$V_\varepsilon(x-\xi)=\frac{1}{2\pi}\cdot\frac{1}{x_2-\xi_2-i\varepsilon(x_1-\xi_1)},\tag{13}$$

If we write the value of this expression in (5) this doesn't show that we obtain $\delta(x-\xi)$.
Fundamental property of (13) is understood in a generalized sense.

Construction of fundamental solution in direction

Having applied the Fourier transformation to the equation (5) with respect to the variable x_1 we get :

$$\vartheta_\varepsilon(x)=\frac{1}{\sqrt{2\pi}}\int_R e^{i\alpha_1x_1}\widetilde{\vartheta}_\varepsilon(\alpha_1,x_2)d\alpha_1,\tag{14}$$

$$g(x)=\frac{1}{\sqrt{2\pi}}\int_R e^{i\alpha_1x_1}\widetilde{g}(\alpha_1,x_2)d\alpha_1,\tag{15}$$

$$\widetilde{g}(\alpha_1,x_2)=\frac{1}{\sqrt{2\pi}}\int_R e^{-i\alpha_1\xi_1}g(\xi_1,x_2)d\xi_1.\tag{16}$$

Substitute the values of (14) and (15) into (5):

$$\frac{1}{\sqrt{2\pi}}\int_R e^{i\alpha_1x_1}\frac{d\widetilde{\vartheta}_\varepsilon(\alpha_1,x_2)}{dx_2}d\alpha_1-i\varepsilon\frac{1}{\sqrt{2\pi}}\int_R e^{i\alpha_1x_1}i\alpha_1\widetilde{\vartheta}_\varepsilon(\alpha_1,x_2)d\alpha_1=$$

$$=\frac{1}{\sqrt{2\pi}}\int_R e^{i\alpha_1x_1}\widetilde{g}(\alpha_1,x_2)d\alpha_1,$$

$$\frac{1}{\sqrt{2\pi}}\int_R e^{i\alpha_1x_1}\left[\frac{d\widetilde{\vartheta}_\varepsilon(\alpha_1,x_2)}{dx_2}+\alpha_1\varepsilon\,\widetilde{\vartheta}_\varepsilon(\alpha_1,x_2)-\widetilde{g}(\alpha_1,x_2)\right]d\alpha_1=0.$$

Hence, for $\widetilde{\vartheta}_\varepsilon(\alpha_1,x_2)$ we get the following ordinary linear differential equation of first order:

$$\frac{d\widetilde{\vartheta}_\varepsilon(\alpha_1,x_2)}{dx_2}+\varepsilon\alpha_1\widetilde{\vartheta}_\varepsilon(\alpha_1,x_2)=\widetilde{g}(\alpha_1,x_2).\tag{17}$$

General solution of this equation is:

$$\widetilde{v}_\varepsilon(\alpha_1,x_2)=Ce^{-\varepsilon\alpha_1x_2}+\int_{x_2}^{x_2}e^{-\varepsilon\alpha_1(x_2-\xi_2)}\widetilde{g}(\alpha_1,\xi_2)d\xi_2,\tag{18}$$

here C and x_2 are arbitrary numbers. If $C=0$, $\alpha_1<0$, then $x_2=\infty$, if $\alpha_1>0$ then $x_2=-\infty$. Then

$$\tilde{\vartheta}_\varepsilon(\alpha_1,x_2)=\begin{cases}\int\limits_{\infty}^{x_2} e^{-\varepsilon\alpha_1(x_2-\xi_2)}\tilde{g}(\alpha_1,\xi_2)d\xi_2, & \alpha_1<0,\\ \int\limits_{-\infty}^{x_2} e^{-\varepsilon\alpha_1(x_2-\xi_2)}\tilde{g}(\alpha_1,\xi_2)d\xi_2, & \alpha_1>0.\end{cases} \tag{19}$$

Now, considering (16) we substitute the value of (19) into (14)

$$\vartheta_\varepsilon(x)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{0}e^{i\alpha_1x_1}d\alpha_1\int\limits_{\infty}^{x_2}e^{-\varepsilon\alpha_1(x_2-\xi_2)}d\xi_2\cdot\frac{1}{\sqrt{2\pi}}\int\limits_R e^{-i\alpha_1\xi_1}g(\xi)d\xi_1+$$

$$+\frac{1}{\sqrt{2\pi}}\int\limits_{0}^{\infty}e^{i\alpha_1x_1}d\alpha_1\int\limits_{-\infty}^{x_2}e^{-\varepsilon\alpha_1(x_2-\xi_2)}d\xi_2\cdot\frac{1}{\sqrt{2\pi}}\int\limits_R e^{-i\alpha_1\xi_1}g(\xi)d\xi_1=$$

$$=-\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{0}e^{i\alpha_1x_1}d\alpha_1\int\limits_{x_2}^{\infty}e^{-\varepsilon\alpha_1(x_2-\xi_2)}d\xi_2\cdot\frac{1}{\sqrt{2\pi}}\int\limits_R e^{-i\alpha_1\xi_1}g(\xi)d\xi_1+$$

$$+\frac{1}{\sqrt{2\pi}}\int\limits_{0}^{\infty}e^{i\alpha_1x_1}d\alpha_1\int\limits_{-\infty}^{x_2}e^{-\varepsilon\alpha_1(x_2-\xi_2)}d\xi_2\cdot\frac{1}{\sqrt{2\pi}}\int\limits_R e^{-i\alpha_1\xi_1}g(\xi)d\xi_1=$$

$$=-\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{0}e^{i\alpha_1x_1}d\alpha_1\int\limits_R e^{-\varepsilon\alpha_1(x_2-\xi_2)}\theta(\xi_2-x_2)d\xi_2\cdot\frac{1}{\sqrt{2\pi}}\int\limits_R e^{-i\alpha_1\xi_1}g(\xi)d\xi_1+$$

$$+\frac{1}{\sqrt{2\pi}}\int\limits_{0}^{\infty}e^{i\alpha_1x_1}d\alpha_1\int\limits_R e^{-\varepsilon\alpha_1(x_2-\xi_2)}\theta(\xi_2-x_2)d\xi_2\cdot\frac{1}{\sqrt{2\pi}}\int\limits_R e^{-i\alpha_1\xi_1}g(\xi)d\xi_1.$$

Hence, for fundamental solution we get:

$$V_\varepsilon(x-\xi)=-\frac{1}{2\pi}\int\limits_{-\infty}^{0}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\theta(\xi_2-x_2)d\alpha_1+$$
$$+\frac{1}{2\pi}\int\limits_{0}^{\infty}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\theta(x_2-\xi_2)d\alpha_1 \tag{20}$$

Verifying fundamental solution that we obtained above, after application of the Fourier transformation, similarly from (20) we get:

$$\frac{\partial V_\varepsilon(x-\xi)}{\partial x_2}=\frac{\delta(\xi_2-x_2)}{2\pi}\int\limits_{-\infty}^{0}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}d\alpha_1+$$

$$+\frac{\varepsilon}{2\pi}\int\limits_{-\infty}^{0}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1\theta(\xi_2-x_2)d\alpha_1+$$

$$+\frac{\delta(x_2-\xi_2)}{2\pi}\int\limits_{0}^{\infty}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}d\alpha_1-$$

$$-\frac{\varepsilon}{2\pi}\int\limits_{0}^{\infty}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1\theta(x_2-\xi_2)d\alpha_1=$$

$$=\frac{\delta(x_2-\xi_2)}{2\pi}\int\limits_{-\infty}^{0}e^{i\alpha_1(x_1-\xi_1)}d\alpha_1+\frac{\varepsilon\theta(\xi_2-x_2)}{2\pi}\int\limits_{-\infty}^{0}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1d\alpha_1+$$

$$+\frac{\delta(x_2-\xi_2)}{2\pi}\int\limits_{0}^{\infty}e^{i\alpha_1(x_1-\xi_1)}d\alpha_1-\frac{\varepsilon\theta(x_2-\xi_2)}{2\pi}\int\limits_{0}^{\infty}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1d\alpha_1=$$

$$=\frac{\delta(x_2-\xi_2)}{2\pi}\int\limits_R e^{i\alpha_1(x_1-\xi_1)}d\alpha_1+\frac{\varepsilon\theta(\xi_2-x_2)}{2\pi}\int\limits_{-\infty}^{0}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1d\alpha_1-$$

$$-\frac{\varepsilon\theta(x_2-\xi_2)}{2\pi}\int\limits_{0}^{\infty}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1d\alpha_1=\delta(x_1-\xi_1)\delta(x_2-\xi_2)+$$

$$+\frac{\varepsilon\theta(\xi_2-x_2)}{2\pi}\int\limits_{-\infty}^{0}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1d\alpha_1-\frac{\varepsilon\theta(x_2-\xi_2)}{2\pi}\int\limits_{0}^{\infty}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1d\alpha_1,$$

$$\frac{\partial V_\varepsilon(x-\xi)}{\partial x_1}=-\frac{\theta(\xi_2-x_2)}{2\pi}i\int_{-\infty}^{0}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1 d\alpha_1+$$
$$+\frac{\theta(x_2-\xi_2)}{2\pi}i\int_{0}^{\infty}e^{i\alpha_1(x_1-\xi_1)-\varepsilon\alpha_1(x_2-\xi_2)}\alpha_1 d\alpha_1.$$

Substituting these expressions into the left hand side of the equation (5) instead of $\vartheta_\varepsilon(x)$ we get relation (12).

Definition Fundamental solution (20) is a fundamental solution of equation (5) in the direction of x_2 [8].

By immediate verification we can easily show that the function:

$$V_\varepsilon(x-\xi)=\theta(x_2-\xi_2)\delta(x_1-\xi_1+i\varepsilon(x_2-\xi_2)),$$

(21)

is a fundamental solution of the equation (5). This is clear from the following relations:

$$\frac{\partial V_\varepsilon}{\partial x_2}=\delta(x_2-\xi_2)\delta(x_1-\xi_1+i\varepsilon(x_2-\xi_2))+i\varepsilon\theta(x_2-\xi_2)\delta'(x_1-\xi_1+i\varepsilon(x_2-\xi_2)),$$

$$\frac{\partial V_\varepsilon}{\partial x_1}=\theta(x_2-\xi_2)\delta'(x_1-\xi_1+i\varepsilon(x_2-\xi_2)),$$

$$\frac{\partial V_\varepsilon}{\partial x_2}-i\varepsilon\frac{\partial V_\varepsilon}{\partial x_1}=\delta(x-\xi)+i\varepsilon\theta(x_2-\xi_2)\delta'(x_1-\xi_1+i\varepsilon(x_2-\xi_2))-$$
$$-i\varepsilon\theta(x_2-\xi_2)\delta'(x_1-\xi_1+i\varepsilon(x_2-\xi_2))=\delta(x-\xi).$$

Necessary conditions

We scalarly multiply both hand sides of the equation (1) by the fundamental solution (21) and get:

$$0=(\ell_\varepsilon u_\varepsilon,V_\varepsilon)\equiv\int_R dx_1\int_0^1\frac{\partial u_\varepsilon(x)}{\partial x_2}\overline{V_\varepsilon(x-\xi)}dx_2+i\varepsilon\int_0^1 dx_2\int_R\frac{\partial u_\varepsilon(x)}{\partial x_1}\overline{V_\varepsilon(x-\xi)}dx_1=$$
$$=\int_R dx_1\left[u_\varepsilon(x)\overline{V_\varepsilon(x-\xi)}\Big|_{x_2=0}^{1}-\int_0^1 u_\varepsilon(x)\frac{\partial\overline{V_\varepsilon(x-\xi)}}{\partial x_2}dx_2\right]+$$
$$+i\varepsilon\int_0^1 dx_2\left[u_\varepsilon(x)\overline{V_\varepsilon(x-\xi)}\Big|_{x_1=-\infty}^{\infty}-\int_R u_\varepsilon(x)\frac{\partial\overline{V_\varepsilon(x-\xi)}}{\partial x_1}dx_1\right]=$$
$$=\int_R\left[u_\varepsilon(x_1,1)\overline{V_\varepsilon(x_1-\xi_1,1-\xi_2)}-u_\varepsilon(x_1,0)\overline{V_\varepsilon(x_1-\xi_1-\xi_2)}\right]dx_1-$$
$$-\int_R dx_1\int_0^1 dx_2 u_\varepsilon(x)\overline{\left[\frac{\partial V_\varepsilon(x-\xi)}{\partial x_2}-i\varepsilon\frac{\partial V_\varepsilon(x-\xi)}{\partial x_1}\right]},$$

hence

$$\int_R\left[u_\varepsilon(x_1,1)\overline{V_\varepsilon(x_1-\xi_1,1-\xi_2)}-u_\varepsilon(x_1,0)\overline{V_\varepsilon(x_1-\xi_1-\xi_2)}\right]dx_1=$$
$$=\begin{cases}u_\varepsilon(\xi), & \xi_1\in R,\ \xi_2\in(0,1),\\ \frac{1}{2}u_\varepsilon(\xi), & \xi_1\in R,\ \xi_2=0,\ \xi_2=1.\end{cases}\tag{22}$$

In the expression (22), the first expression determines internal value of $u_\varepsilon(\xi)$, by its boundary values. The second expression in (22), are necessary conditions since this expression determines boundary value of the unknown function by the same boundary values. Now, distinguish these conditions:

$$\frac{1}{2}u_\varepsilon(\xi_1,0)=\int_R\left[u_\varepsilon(x_1,1)\overline{V_\varepsilon(x_1-\xi_1,1)}-u_\varepsilon(x_1,0)\overline{V_\varepsilon(x_1-\xi_1,0)}\right]dx_1\,,$$

$$\frac{1}{2}u_\varepsilon(\xi_1,1)=\int_R\left[u_\varepsilon(x_1,1)\overline{V_\varepsilon(x_1-\xi_1,0)}-u_\varepsilon(x_1,0)\overline{V_\varepsilon(x_1-\xi_1,-1)}\right]dx_1\,.$$

In these expressions we write the values of fundamental solution having taken then from (21):

$$\frac{1}{2}u_\varepsilon(\xi_1,0)=\int_R u_\varepsilon(x_1,1)\theta(1)\delta(x_1-\xi_1+i\varepsilon)dx_1-\int_R u_\varepsilon(x_1,0)\theta(0)\delta(x_1-\xi_1)dx_1\,,$$

$$\frac{1}{2}u_\varepsilon(\xi_1,1)=\int_R u_\varepsilon(x_1,1)\theta(0)\delta(x_1-\xi_1)dx_1-\int_R u_\varepsilon(x_1,0)\theta(-1)\delta(x_1-\xi_1-i\varepsilon)dx_1\,.$$

Since the second expression from the above obtained one is an identity the first expression is represented in the form [9], [10]:

$$u_\varepsilon(\xi,0)=u_\varepsilon(\xi_1-i\varepsilon,1)\,. \qquad (23)$$

It is seen from this necessary condition that in the given strip a natural boundary condition for the equation (1) is given in the form of linear combination of expressions contained in (23). Thus, for the equation (1) we take a boundary condition in the form:

$$u_\varepsilon(\xi_1,0)+u_\varepsilon(\xi_1-i\varepsilon,1)=\varphi(\xi_1)\,,\quad \xi_1\in R \qquad (24)$$

Then, from (23) and (24) we get:

$$u_\varepsilon(\xi_1,0)=u_\varepsilon(\xi_1-i\varepsilon,1)=\frac{\varphi(\xi)}{2}\,. \qquad (25)$$

Now, we'll use the first part of the main relation

$$u_\varepsilon(\xi)=\int_R u_\varepsilon(x_1,1)\theta(1-\xi_2)\delta(x_1-\xi_1-i\varepsilon(1-\xi_2)dx_1-$$

$$-\int_R u_\varepsilon(x_1,0)\theta(-\xi_2)\delta(x_1-\xi_1+i\varepsilon\xi_2)dx_1=$$

$$=\int_R u_\varepsilon(x_1,1)\;\delta(x_1-\xi_1-i\varepsilon(1-\xi_2)dx_1=u_\varepsilon(\xi_1+i\varepsilon(1-\xi_2),1)$$

(26)

Thus, the solution of boundary value problem (1), (24) is obtained in the form (26). Hence

$$u_0(\xi)=\lim_{\varepsilon\to 0}u_\varepsilon(\xi)=u_0(\xi_1,1)\,, \qquad (27)$$

Finally, if we'll use (25), then for the limiting state of the solution we get the following:

$$u_0(\xi)=\frac{1}{2}\varphi(\xi_1)\,, \qquad (28)$$

It is seen from the obtained expression that the limiting state of the solution of boundary value problem (28) satisfies the given boundary condition (24), i.e. boundary layer doesn't arise.

Remark 2. Instead of boundary condition (24) of the given problem we can take the following condition:

$$\alpha_1(\xi_1)u_\varepsilon(\xi_1,0)+\alpha_2(\xi_1)u_\varepsilon(\xi_1-i\varepsilon,1)=\varphi(\xi_1)\,,\quad \xi_1\in R\,,$$

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