

Suppliers Selected to Optimize the Supply Chain through the Combination of A, B, C :(Experimental Study)

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ABSTRACT

In the recent years , supply chain management has become one of the most important factor for gaining competitive advantages. As we know , supplier selection shall be viewed as complex-objective decision-making problem and multi objective problem. They involve both qualitative and quantitative factors such as price, quality , flexibility and being on time.

This paper uses linguistic variables that presented by experts to evaluate and determine the performance of each supplier to determine the weight of each criterion and criteria used. Linguistic variables sealed are expressed by triangular and trapezoidal fuzzy numbers ,and uses the method of multi criteria decision making fuzzy environment for selected suppliers and a method used to calculate weight and fuzzy (MCDM).

KEYWORDS: Supplier Selection, Fuzzy Numbers, Multi Criteria Decision Making, Fuzzy Topsis, Linguistic Variables.

1- INTRODUCTION

In the today competitive corporate environment ,supply chain management and supplier selection process have received maximum attention from professional managements.to improve performance of business operations at a reduced cost and delivery time.Levi et al.(2000) have mentioned that ,supply chain management is a set of approaches utilized to efficiently integrate suppliers , manufactures , ware houses , and stores ,so that merchandise is produced and distributed at right quantities ,to the right locations ,and to the right time , in order to minimize system-wide costs while satisfying service level requirements (S.Sinha &S.Sarmah,2008).

Recently ,relationship between supplier and consumption has been considered seriously. If there is a long-term relations between the two supply chain companies will be a major obstacle for competitors.

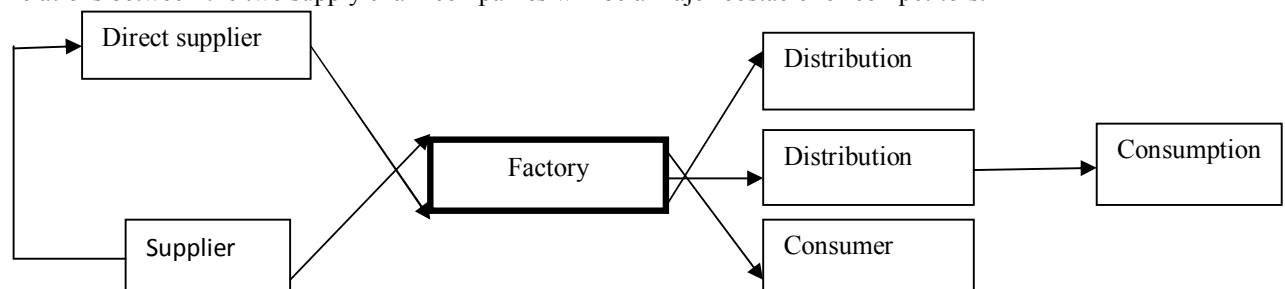


Figure 1. flow of goods and materials through the supply chain

Supply chain management involves the coordination of independently managed business organizations who seek to maximize their individual profits ,and one of the major issues of supply chain management is to develop suitable mechanisms to coordinate different activities that are controlled by different members of the chain (S.Sinha &S.Sarmah,2008).

2- LITERATURE REVIEW

One of the most important processes performed in the organizations today is the evaluation , selection and improvement of suppliers. Dickson (1996) identified twenty three criteria for supplier selection based on the extensive survey , the result shows the quality is the most important parameter followed by delivery and performance history. A number of quantitative techniques have been used to supplier , selection problem such as

weighing method, statistical methods, Analytic Hierarchy Process (AHP), Data Envelopment Analysis etc (Sreekumar & Mahapatra, 2009).

Muralidharan et al. (2002) compared the advantages and limitations of nine previously enveloped methods of supplier rating, and combined multiple criteria decision making and analytic hierarchy processes to construct multi-criteria group decision making model for supplier rating. The attributes of quality, delivery, price, technique capability, finance, attitude, facility, flexibility and service were used for supplier evaluation, and the attributes of knowledge, skill, attitude and experience were used for individual assessments. Sarkis and Talluri (2002) suggested that purchasing function has been attracting growing interest as a critical component of supply chain management, and multiple factors have been considered in supplier selection and evaluation, including strategic, operational, tangible and intangible measures within planning horizon, culture, technology, relationship, cost, quality, time and flexibility. (Wang, Chang, and Wang, 2007)

Manoj Kumar et al.(2004) formulated supplier selection problem as a fuzzy mixed integer goal programming problem .Satyanarayana raju et al.(2009) considered supplier selection problem as a multi-objective decision making problem and formulated through fuzzy goal programming approach . Kagnicioglu (2006) has used fuzzy multi-objective model with capacity , demand and budget constraint for supplier selection problem.

Ibrahim and Vgur (2003) have used activity based costing (ABC) approach under the fuzzy variables by considering multi period of supplier-purchaser relationship for vendor selecting. Kumar et al. (2004) has used fuzzy goal programming for supplier selection. Kumar et al.(2006) used fuzzy multi-objective mathematical programming for supplier selection with three goals: cost minimization , quality maximization and on-time delivery maximization with constraints as demand , capacity , and quota flexibility .(Sreekumar & Mahapatra ,2009).

Amid et al.(2006) developed a fuzzy multi-objective linear model for a supplier selection problem ,to overcome the vanguardness of information involved in the selection process.Yuan chen et al.(2006)adopted fuzzy multi objective programming approach for vendor selection in iron & steel enterprise .Chengtung chen et al.(2006)presented fuzzy approach for supplier evaluation and selection in supply chain management.

Choi and Hartley (1996) evaluated supplier-performance based on consistency, reliability, relationship, flexibility,price, service, technological capability and finances, and also addressed 26 supplier-selection criteria. Verma and Pullman (1998) ranked the importance of the supplier attributes of quality, on-time delivery, cost, lead-time and flexibility. Vonderembse and Tracey (1999) discussed the supplier and manufacturing performances could be determined by supplier selection criteria and supplier involvement. Furthermore, they concluded that the supplier selection criteria could be evaluated by quality, availability, reliability and performance, while supplier involvement could be evaluated by product R&D and improvement, and supplier performance could be evaluated by stoppage, delivery, damage and quality. Additionally, manufacturing performance could be evaluated by cost, quality, inventory and delivery (Wang, Chang, and Wang,2007).

Pearson and Ellram (1995) examine the supplier selection and evaluation criterion in small and large electronic firms. The results confirm the importance of the quality criteria in the supplier selection and evaluation , the other criteria found to be relatively important are speed to market , design capability and technology. The result shows that the nature of industry and its competitive environment may have a greater influence on selection criteria in comparison to the size of the firm .Gnanasekaran et al. (2006) has applied Analytical Hierarchy Process (AHP) for effective supplier selection in a leading automobile component manufacturing company. The study shows that application of AHP enhances the decision making process and reduces the time taken to select the supplier. The paper uses Additive Normalization Method and Eigen vector Method to find priority vector. (Sreekumar & Mahapatra ,2009).

3- Fuzzy numbers and linguistic variables:

In this section ,we discuss some major definition of fuzzy numbers and linguistic variables.

3-1- Fuzzy numbers:

The fuzzy set theory proposed by Zadeh (1976) and Zadeh (1965) is suitable for dealing with the issue of uncertainly in systems modeling.Fuzzy theory set allows mathematical operators to apply to the fuzzy domain.

Generally, the fuzzy sets are defined by membership functions .The fuzzy sets represent the grade of any element X of X that have partial membership to A.The degree to which an element belongs to a set is defined by the value between 0 and 1.If an element X really belongs to A ,

$\mu_A(x)=1$ and clearly not, $\mu_A(x)=0$.Higher is the membership value , $\mu_A(x)$, greater is the belongingness of an element x to a set A.(S. Öntü et al.2008).For example ,a triangular fuzzy number is defined as (l, m, u), where $l \leq m \leq u$. The parameters l, m, u respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event.

$$\mu_A(x) = \begin{cases} \frac{x-l}{m-l} & l \leq x \leq m, \\ \frac{x-u}{m-u} & m \leq x \leq u, \\ 0, & \text{otherwise} \end{cases}$$

According to Zadeh theory (1965) ,the fuzzy addition ,the fuzzy multiplication ,fuzzy division and the fuzzy subtraction of triangle fuzzy numbers are also triangular fuzzy numbers.

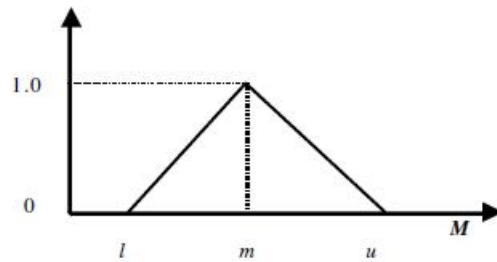


Figure2 . Triangular fuzzy number \tilde{M}

Table 1.Triangular fuzzy scale

Linguistic scale	Triangular fuzzy number	Triangular fuzzy reciprocal number
Equally important	(1, 1, 1)	(1, 1, 1)
Weakly more important	(2/3, 1, 3/2)	(2/3, 1, 3/2)
Strongly more important	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strongly more important	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)
Absolutely more important	(7/2, 4, 9/2)	(2/9, 1/4, 2/7)

3-1-1- Fuzzy analytic hierarchy process:

The AHP was developed in 1980s by Satty. It is a systematic decision making method which includes both qualitative and quantitative techniques. For the first time Buckley used fuzzy theory in AHP technique and called it fuzzy analytic hierarchy process in 1985. Calculation consistent ratio usually is done whitin the matrix fuzzy.

Definitions of the new fuzzy comparison matrices: The comparison matrix defined by Saaty employs 1-9 scales. The 1-9 scales are illustrated with the following comparison matrix and table 2.

$$(1) \quad A = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \dots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \dots & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \dots & \frac{w_3}{w_n} \\ \vdots & \vdots & \dots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \dots & \frac{w_n}{w_n} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Table2.Saatys scale for pairwise comparison

Saaty's scale	The relative importance of the two sub-elements
1	Equally important
3	Moderately important with one over another
5	Strongly important
7	Very strongly important
9	Extremely important
2, 4, 6, 8	Intermediate values

Our new fuzzy comparison matrix differs with Saaty's in that we use membership scales, instead of the 1-9 scales, as the values of the elements.

$$(2) \quad A = \begin{bmatrix} \frac{w_1}{w_1+w_1} & \frac{w_1}{w_1+w_2} & \dots & \frac{w_1}{w_1+w_n} \\ \frac{w_2}{w_2+w_1} & \frac{w_2}{w_2+w_2} & \dots & \frac{w_2}{w_2+w_n} \\ \vdots & \vdots & \dots & \vdots \\ \frac{w_n}{w_n+w_1} & \frac{w_n}{w_n+w_2} & \dots & \frac{w_n}{w_n+w_n} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \vdots & \vdots & \dots & \vdots \\ r_{n1} & r_{n2} & \dots & r_{nn} \end{bmatrix}$$

If this comparison matrix is consistent, it should satisfy:

$$(3) \quad r_{ii} = 0.5, r_{ij} + r_{ji} = 1, \frac{1}{r_{ij}} - 1 = \left(\frac{1}{r_{ik}} - 1\right) \times \left(\frac{1}{r_{ki}} - 1\right).$$

This method compares weights in pairs and is more straightforward and easier to use for the decision-makers. The meanings of our membership scales can also be expressed in the same way as Saaty's scale see table 3.

Table3. Scale for fuzzy pair-wise comparison.

Scale values	The relative importance of the two sub-elements
0.5	Equally important
0.55(or0.5 0.6)	Slightly important
0.65(or0.6 0.7)	Important
0.75(or0.7 0.8)	Strongly important
0.85(or0.8 0.9)	Very strongly important
0.95(or0.9 1.0)	Extremely important

Theoretically, the membership scales put forward in this paper and Saaty's scales should satisfy the following:

$$(4) \quad r_{ij} = \frac{a_{ij}}{a_{ij} + 1}.$$

The difference of our membership scales with Saaty's lies in that the values of membership scales falls within the range of [0,1].

Calculation of the priority weights. Let

$$(W = w_1, w_2, \dots, w_n),$$

$$(5) \quad w_i = \frac{b_i}{\sum_{i=1}^n b_i}.$$

where, $b_i = \frac{1}{\left[\sum_{j=1}^n \frac{1}{r_{ij}}\right] - n}$.

Consistency test of the comparison matrix. We can use the following equation to calculate the consistency index:

$$(6) \quad CI = \frac{\left[\sum_{i=1}^n \frac{(AW)_i}{nw_i} \right]}{n-1},$$

where the values of the elements in matrix A could be derived by applying equation (3) to matrix R.

The comparison matrix will be considered to be consistent if there exists $CR = CI$
 $RI < 0.1$. The various values of RI are shown in table 4 (FENG KONG AND HONGYAN LIU,2005)

Table4.Values of RI

Size of matrix	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

3-2-Linguistic variable:

A linguistic variable is a variable whose values are expressed in linguistic terms.This paper uses linguistic variable $S = \{s_0, s_2, \dots, s_8\}$,which is defined by the linguistic term set (LTS) (Herrera et al. (2000)) The semantic element (SE) is defined in the unit interval [0, 1] of the linear triangular membership function using fuzzy set (xL, xm, xR) , as shown in Fig. 1, where xL and xR represent the left and right limits of the corresponding SE by the membership function, and xm indicates the value at which the membership grade equals 1.Applications can also use the trapezoid membership function for defining the SEs within LTS.(Wang, Chang, and Wang,2007)

Code	SE	(xL, xm, xR)
S_0	None	(0,0,0.12)
S_1	Very Low	(0,0.12,0.25)
S_2	Low	(0.12,0.25,0.37)
S_3	Almost Low	(0.25,0.37,0.5)
S_4	Medium	(0.37,0.5,0.62)
S_5	Almost High	(0.5,0.62,0.75)
S_6	High	(0.62,0.75,0.87)
S_7	Very High	(0.75,0.87,1)
S_8	Perfect	(0.87,1,1)

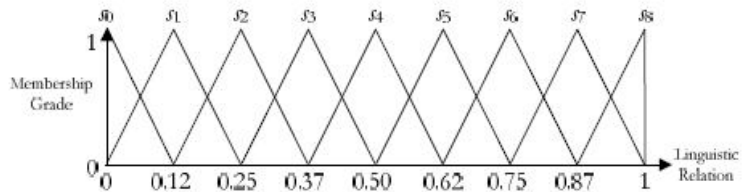


Figure 3.Difinition of linguistic variable.S. .(Wang, Chang, and Wang,2007)

The linguistic variables considered in this study are finite and totally ordered LTS, which requires the following properties (Herrera et al. (1995)):

- The set is ordered: $si \geq sj$ if $i \geq j$
- The negative operator is defined: $Neg(si) = sj$ such that $j = 8 - i$
- Maximization operator: $\max(si, sj) = si$ if $si \geq sj$
- Minimization operator: $\min(si, sj) = si$ if $si \leq sj$

Consequently, the results of negatively directed behaviors shall apply a negative operator to transform into a positive direction. .(Wang, Chang, and Wang,2007)

4- Fuzzy Topsis:

Topsis model has been proposed by Zimmermann (1991), Buckley (1985) , Zadeh (1965) ,Kaufmann and Gupta (1985) .The merit of using a fuzzy approach is to assign the relative importance of attributes using fuzzy numbers instead of precise numbers.(Önut, S. et al,2008)

- Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them, as Eq(7).

$$(7) \quad d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}$$

The problem can be described by following data:

- (I) a set of J possible candidates called $A = \{A_1, A_2, \dots, A_j\}$;
 - (II) a set of n criteria, $C = \{C_1, C_2, \dots, C_i\}$;
 - (III) a set of performance ratings of A_j ($j = 1, 2, 3, \dots, J$) with respect to criteria C_i ($i = 1, 2, 3, \dots, n$) called \tilde{x}_{ij} ($i = 1, 2, 3, \dots, n; j = 1, 2, 3, \dots, J$);
 - (III) a set of importance weights of each criterion w_i ($i = 1, 2, 3, \dots, n$).
- As stated above, problem matrix format can be expressed as follows:

$$(8) \quad \tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \tilde{x}_{j1} & \tilde{x}_{j2} & \dots & \tilde{x}_{jn} \end{bmatrix}$$

- Considering the different importance values of each criterion, the weighted normalized fuzzy decision matrix is constructed as:

$$(9) \quad \tilde{V} = [\tilde{v}_{ij}]_{n \times j} \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, J$$

where $\tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)w_i$

According to the briefly summarized fuzzy theory above, fuzzy TOPSIS steps can be outlined as follows:

Step 1: Choose the linguistic ratings ($\tilde{x}_{ij}, i = 1, 2, 3, \dots, n, j = 1, 2, 3, \dots, J$) for alternatives with respect to criteria. To obtain normalized decision matrix \tilde{r}_{ij} let $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$, $\tilde{x}_j^- = (a_j^-, b_j^-, c_j^-)$ and $\tilde{x}_j^+ = (a_j^+, b_j^+, c_j^+)$ we have

$$(10) \quad \tilde{r}_{ij} = \begin{cases} \tilde{x}_{ij} / \tilde{x}_j^+ = \left(\frac{a_{ij}}{a_j^+}, \frac{b_{ij}}{b_j^+}, \frac{c_{ij}}{c_j^+} \right) \\ \tilde{x}_j^- / \tilde{x}_{ij} = \left(\frac{a_j^-}{a_{ij}}, \frac{b_j^-}{b_{ij}}, \frac{c_j^-}{c_{ij}} \right) \end{cases}$$

Step 2: Calculate the weighted normalized fuzzy decision matrix. The weighted normalized value \tilde{v}_{ij} calculated by Eq. (6).

Step 3: Identify positive-ideal (A^*) and negative ideal (A_1^-) solutions. The fuzzy positive-ideal solution (FPIS, A^*) and the fuzzy negative-ideal solution (FNIS, A_1^-) are shown in Eqs.(11) and (12)

$$(11) \quad A^* = \{\tilde{v}_1^*, \dots, \tilde{v}_i^*\} = \left\{ \left(\max_j v_{ij} | i \in I \right) \right\}$$

$$i = 1, 2, \dots, n, \quad j = 1, 2, \dots, J$$

$$(12) \quad A_1^- = \{\tilde{v}_1^-, \dots, \tilde{v}_i^-\} = \left\{ \left(\min_j v_{ij} | i \in I \right) \right\}$$

$$i = 1, 2, \dots, n, \quad j = 1, 2, \dots, J$$

where I is criteria.

Step 4: Calculate the distance of each alternative from A* and A using Eqs. (13) and (14).

(13)

$$D_j^* = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_i^*) \quad j = 1, 2, \dots, J$$

(14)

$$D_j^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_i^-) \quad j = 1, 2, \dots, J$$

Step 5: Calculate similarities to ideal solution

(15)

$$CC_j = \frac{D_j^-}{D_j^* + D_j^-} \quad j = 1, 2, \dots, J$$

Step 6: Rank preference order. Choose an alternative with maximum CC j or rank alternatives according to CC j in descending order.

4-1- Extension TOPSIS methods for selecting suppliers:

\tilde{A} according to expanding primary, each fuzzy number like A can also be expressed by its intervals, namely

(16)

$$\tilde{A} = \bigcup_{\alpha} \alpha A_{\alpha}, \quad 0 < \alpha \leq 1$$

where

(17)

$$A_{\alpha} = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}$$

$$= [\min\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}, \max\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}].$$

A α is referred \tilde{A} as α -level sets or α -cuts of the fuzzy number/set. In order that the TOPSIS method can also be used to deal with fuzzy MCDM problems. The simplest extension is to change a fuzzy MCDM problem into a crisp one via defuzzification.

Another extension is to define the Euclidean distance between any two fuzzy numbers as a crisp value. Chen (2000) defines the Euclidean distance of two triangular fuzzy numbers

$$\tilde{m} = (m_1, m_2, m_3) \text{ and } \tilde{n} = (n_1, n_2, n_3) \text{ as}$$

(18)

$$d(\tilde{m}, \tilde{n}) = \sqrt{\frac{1}{3} [(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2]}.$$

Let $\tilde{X} = (\tilde{x}_{ij})_{n \times m}$ fuzzy decision matrix characterized by membership functions $\mu_{\tilde{x}_{ij}}(x)$ ($i = 1, \dots, n, j = 1, \dots, m$) and $\tilde{W} = (\tilde{w}_1, \dots, \tilde{w}_m)$ weights characterized by $\mu_{\tilde{w}_j}(x)$ ($j = 1, \dots, m$). If all the criteria/attributes, C1, ..., Cm, are assessed using the same set of fuzzy linguistic

variables, then the fuzzy decision \tilde{X} is of the same dimension and therefore needs no normalization. Otherwise, \tilde{X} has to be normalized.

If $\tilde{x}_{ij} = (a_{ij}, b_{ij}, d_{ij})$ ($i = 1, \dots, n, j = 1, \dots, m$) are triangular fuzzy numbers, then normalization process can be conducted by

(19)

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*} \right), \quad i = 1, \dots, n; \quad j \in \Omega_b$$

(20)

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \right), \quad i = 1, \dots, n; \quad j \in \Omega_c$$

where

(21)

$$d_j^* = \max_i d_{ij}, \quad j \in \Omega_b$$

(22)

$$a_j^- = \min_i a_{ij}, \quad j \in \Omega_c$$

Normalized criteria/attribute values/ratings are between zero and one. So, the ideal solution can be defined as $A^*Z\{1, \dots, 1\}$. As such, the negative ideal solution can be defined as $A^-Z\{0, \dots, 0\}$. Note that if there is no need to normalize the fuzzy decision matrix $\tilde{X} = (\tilde{x}_{ij})_{n \times m}$,

then the ideal and the negative ideal solutions can be respectively defined as

$$(23) \quad A^* = \{x_1^*, \dots, x_m^*\} = \{(\max_j d_{ij} | j \in \Omega_b), (\min_j a_{ij} | j \in \Omega_c)\}$$

$$(24) \quad A^- = \{x_1^-, \dots, x_m^-\} = \{(\min_j a_{ij} | j \in \Omega_b), (\max_j d_{ij} | j \in \Omega_c)\}$$

Let $(r_{ij})_\alpha = [(r_{ij})_\alpha^L, (r_{ij})_\alpha^U]$ and $(w_j)_\alpha = [(w_j)_\alpha^L, (w_j)_\alpha^U]$ level sets of \tilde{r}_{ij} and \tilde{w}_j

$$(25) \quad RC_i = \frac{\sqrt{\sum_{j=1}^m (w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^m (w_j r_{ij})^2} + \sqrt{\sum_{j=1}^m (w_j (r_{ij} - 1))^2}},$$

$$(26) \quad i = 1, \dots, n,$$

(27) where

Obviously, RC_i is an interval number. It can be captured by the following pair of fractional numbers $(w_j)_\alpha^L \leq w_j \leq (w_j)_\alpha^U, j = 1, \dots, m,$

$$(28) \quad (RC_i)_\alpha^L = \text{Min} \frac{\sqrt{\sum_{j=1}^m (w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^m (w_j r_{ij})^2} + \sqrt{\sum_{j=1}^m (w_j (r_{ij} - 1))^2}}$$

s.t.

$$(w_j)_\alpha^L \leq w_j \leq (w_j)_\alpha^U, \quad j = 1, \dots, m$$

$$(r_{ij})_\alpha^L \leq r_{ij} \leq (r_{ij})_\alpha^U, \quad j = 1, \dots, m$$

$$(29) \quad (RC_i)_\alpha^U = \text{Max} \frac{\sqrt{\sum_{j=1}^m (w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^m (w_j r_{ij})^2} + \sqrt{\sum_{j=1}^m (w_j (r_{ij} - 1))^2}}$$

s.t.

$$(w_j)_\alpha^L \leq w_j \leq (w_j)_\alpha^U, \quad j = 1, \dots, m$$

$$(r_{ij})_\alpha^L \leq r_{ij} \leq (r_{ij})_\alpha^U, \quad j = 1, \dots, m$$

According to the fact

$$(30) \quad \frac{\partial RC_i}{\partial r_{ij}} = \frac{r_{ij} \sqrt{\frac{\sum_{j=1}^m (w_j(r_{ij}-1))^2}{\sum_{j=1}^m (w_j r_{ij})^2}} + w_j^2(1-r_{ij}) \sqrt{\frac{\sum_{j=1}^m (w_j r_{ij})^2}{\sum_{j=1}^m (w_j(r_{ij}-1))^2}}}{\left(\sqrt{\sum_{j=1}^m (w_j r_{ij})^2} + \sqrt{\sum_{j=1}^m (w_j(r_{ij}-1))^2} \right)^2} > 0,$$

$$\frac{\partial RC_i}{\partial r_{ij}} > 0 \quad j = 1, \dots, m,$$

Therefore RC_i is a monotonically increasing function of r_{ij} (j=1,..,m), which means RC_i reaches its maximum at r_{ij}=1 and arrives at its minimum when r_{ij}=0. Therefore, the above pair of fractional programming model (r_{ij})_α^U is simplified as (r_{ij})_α^L

$$(31) \quad (RC_i)_\alpha^L = \text{Min} \frac{\sqrt{\sum_{j=1}^m (w_j(r_{ij})_\alpha^L)^2}}{\sqrt{\sum_{j=1}^m (w_j(r_{ij})_\alpha^L)^2} + \sqrt{\sum_{j=1}^m (w_j((r_{ij})_\alpha^L - 1))^2}}$$

$$\text{s.t. } (w_j)_\alpha^L \leq w_j \leq (w_j)_\alpha^U, \quad j = 1, \dots, m$$

$$(32) \quad (RC_i)_\alpha^U = \text{Max} \frac{\sqrt{\sum_{j=1}^m (w_j(r_{ij})_\alpha^U)^2}}{\sqrt{\sum_{j=1}^m (w_j(r_{ij})_\alpha^U)^2} + \sqrt{\sum_{j=1}^m (w_j((r_{ij})_\alpha^U - 1))^2}}$$

$$\text{s.t. } (w_j)_\alpha^L \leq w_j \leq (w_j)_\alpha^U, \quad j = 1, \dots, m$$

For n alternatives, we have n fuzzy relative closenesses, these models can be solved with SLOVER in Excel or LINGO software, in this paper has been used of Excel for modeling and solving. Let $\alpha_1, \dots, \alpha_N$ be different alpha levels satisfying $0 < \alpha_1 < \dots < \alpha_N = 1$ then the defuzzified values of \tilde{RC}_i can be defined by

$$(33) \quad (RC_i)_{ALC}^* = \frac{1}{N} \sum_{j=1}^N \left(\frac{(RC_i)_{\alpha_j}^L + (RC_i)_{\alpha_j}^U}{2} \right), \quad i = 1, \dots, n$$

5- Case study:

In this section, we examine three numerical items (B1, B2, B3), as a supplier, the company wants to choose one of them and ranking them according to five below items

- A1= Supplier profitability
- A2=Flexibility
- A3=Facilities and technological capabilities
- A4=Quality
- A5=Delivery time

Every important decision factor for the criteria has been estimated in Table5:

Table 5.Important factor of five decision criteria

	A1	A2	A3
B1	H	VH	MH
B2	VH	VH	VH
B3	VH	H	H
B4	VH	VH	VH
B5	M	MH	MH

Performance of three options to five indicators of decision-makers is in table 6:

Table 6.Performance indicators than the options of decision makers

Decision-makers			Options	Criteria
B3	B2	B1		
MG	G	MG	C1	A1
MG	G	G	C2	
F	G	VG	C3	
F	MG	G	C1	A2
G	MG	VG	C2	
MG	G	G	C3	
G	G	F	C1	A3
VG	VG	VG	C2	
VG	G	MG	C3	
VG	MG	G	C1	A4
G	VG	VG	C2	
VG	MG	G	C3	
F	F	F	C1	A5
VG	VG	VG	C2	
MG	VG	G	C3	

Table7.Data obtained from the combined judgments decision

0.833	0.633	0.433	1	1	0.9	1	0.933	0.767	1	1	0.9	0.967	0.867	0.7	Wj
A5			A4			A3			A2			A1			
7	5	3	10	9.667	8.333	9	7.667	5.667	8.667	7	5	9.333	4.667	5.667	C1
9.667	8.667	7	10	10	9	10	9.667	8.333	10	10	9	9.667	8.333	6.333	C2
9.667	8.333	6.333	9.667	8.667	7	9.667	8.667	7	9.667	8.667	7	9	8	6.333	C3

Table8.Decision matrix was normalized

0.833	0.633	0.433	1	1	0.9	1	0.933	0.767	1	1	0.9	0.967	0.867	0.7	Wj
A5			A4			A3			A2			A1			
0.724	0.517	0.31	1	0.967	0.833	0.9	0.767	0.567	0.867	0.7	0.5	0.966	0.793	0.586	C1
1	0.897	0.724	1	1	0.9	1	0.967	0.833	1	1	0.9	1	0.862	0.655	C2
1	0.862	0.655	0.967	0.867	0.7	0.967	0.867	0.7	0.967	0.867	0.7	0.931	0.828	0.655	C3

Table9. Weighted decision matrix was normalized

A5	A4			A3			A2			A1					
0.60	0.33	0.13	1.00	0.97	0.75	0.90	0.72	0.43	0.87	0.70	0.45	0.93	0.69	0.41	C1
0.83	0.57	0.31	1.00	1.00	0.81	1.00	0.90	0.64	1.00	1.00	0.81	0.97	0.75	0.46	C2
0.83	0.55	0.28	0.97	0.87	0.63	0.97	0.81	0.54	0.97	0.87	0.63	0.90	0.72	0.46	C3

The positive ideal solution and the negative ideal solutions are:

$$A^*=[(1,1,1),(1,1,1), (1,1,1), (1,1,1), (1,1,1)]$$

$$\bar{A}=[(0,0,0), (0,0,0), (0,0,0), (0,0,0), (0,0,0)]$$

Finally,the following results are obtained according to α different levels

Table 10. High and low level indicator and a close relative priorities of suppliers of per different levels a.

C3		C2		C1		α
$(CC_{C3})^u_\alpha$	$(CC_{C3})^l_\alpha$	$(CC_{C2})^u_\alpha$	$(CC_{C2})^l_\alpha$	$(CC_{C1})^u_\alpha$	$(CC_{C1})^l_\alpha$	
0.963	0.6806	1	0.781	0.903	0.555	0
0.954	0.6987	0.994	0.797	0.889	0.575	0.1
0.944	0.7168	0.988	0.813	0.875	0.595	0.2
0.933	0.7347	0.982	0.829	0.86	0.616	0.3
0.923	0.7526	0.975	0.845	0.636	0.845	0.4
0.912	0.7704	0.968	0.861	0.831	0.656	0.5
0.901	0.7882	0.961	0.876	0.816	0.676	0.6
0.89	0.8058	0.954	0.891	0.801	0.696	0.7
0.88	0.8234	0.947	0.905	0.786	0.716	0.8
0.869	0.8408	0.94	0.919	0.771	0.736	0.9
0.858	0.8582	0.932	0.932	0.755	0.755	1
<u>0.841</u>	$(CC_{C3})^*_ALC$	<u>0.913</u>	$(CC_{C2})^*_ALC$	<u>0.743</u>	$(CC_{C1})^*_ALC$	
2		1		3		Rank

Distance between (0,1) is divided to 5 distances and used linguistic variable for every distance in table 11

Table 11. Situation assessment suppliers

Relative proximity index	Situation assessment
(CC_i)	
$CC_i \in [0, 0.2)$	Refusal supplier
$CC_i \in [0.2, 0.4)$	Acceptance supplier with high-risk
$CC_i \in [0.4, 0.6)$	Acceptance supplier with low-risk
$CC_i \in [0.6, 0.8)$	Approved supplier
$CC_i \in [0.8, 1]$	Approved supplier with a high priority

Based on values obtained for the relative proximity index:

Table 12. Situation assessment suppliers in case study

Options	Relative proximity index	Situation assessment
	(CC_i)	
C1	0.743 ∈ [0.6, 0.8)	Approved supplier
C2	0.913 ∈ [0.8, 1]	Approved supplier with a high priority
C3	0.841 ∈ [0.8, 1]	Approved supplier with a high priority

6- Conclusions

Fuzzy MCDM has found wide applications in the solution of real world decision making problems. Most of the researchers proposed advantage of supply chain management. Many companies design and implement an appropriate supply chain management are considered as an important tool for advantage competitive. One of the key success factors in creating supply chain is close relationship between supplier and buyer. Therefore choosing suppliers is the most important issue in implementing successful supply chain. Generally, choosing supplier is faced with inherently vague data therefore using of Fuzzy set is suitable. In other words when numerical variables can't be used as performance indicators, linguistic variables can be used.

Topsis method is suitable and flexible for selecting suppliers, and combines it for crisp MCDM with the fuzzy extension principle and performs defuzzification at the very end of decision analysis process. Compared with the other fuzzy versions of the TOPSIS method, the proposed fuzzy TOPSIS method produces an exact fuzzy

estimate rather than a crisp point estimate or an exaggerated fuzzy estimate for the relative closeness of each decision alternative.(Y.M. Wang , T.M.S. Elhag,2006)

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