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The Comparison of Global Optimization Methods for Design of Time Signal in Digital Telecommunication and Introduction of LSA Algorithm

Ali Mohammadi, Kaveh Alikhani, Hoseyn Sariri, Fariborz Parandin, Mehrshad Shamsi

Islamic Azad University, Islam-Abad-e-Qarb Branch, Islam-Abad-e-Qarb, Iran

ABSTRACT

Considering the influence of sending signals in detection error, in this paper it is attempted to reduce error probability by global optimization methods and increase correct decision making in the receiver. To do this by mathematical relations and considering receiving signal as a vector, designing signal is an optimization problem solution. To solve the related optimization problem, three famous global optimization methods as GA, SA and MLSL are used and the mentioned methods are used and simulation results are presented and compared, also LSA algorithm is introduced as revised SA algorithm.
KEYWORDS: Optimized algorithms; Simulated Annealing; Genetic Algorithm; Multi-level Single Linkage; Local after Simulated Annealing.

INTRODUCTION

In digital telecommunication to send symbols corresponding with each symbol, a time signal is sent and in receiver side, these signals being gathered by channel noise are received and demodulated. Demodulation is gaining components of time signals on base vectors. According to the achieved components, the receiver detects the sent symbol. The noise causes error in detection and in this way optimization equation reduces error probability. The way to define the sent signals in vector space has direct link with error probability and by good selection, we can minimize this error. If the probability of optimized receiver error is considered as objective function, the determination of signals components under the limitation on the sending power to reduce error in the receiver is an optimization issue. There are various methods to find extermum points (minimum or maximum) of a function. In local optimization methods, the main ideas is using the current point information (such as function gradient) to determine movement direction to extermum point. This rule causes that the final answer be dependent upon the beginning point of algorithm and if objective function has various extermum points, the algorithm will not be convergent to the optimized extermum point (e.g. the lowest minimum) (in other words, it will be trapped in local extermum point). To solve this problem, it is necessary to use global optimization methods. The simplest method is random search as some points in search space will be randomly selected and are used the starting point of a local algorithm. The more the number of starting pints, the more is possible to find optimized point in this method and instead bear more calculation load. Local optimization algorithms are used to design signals of digital telecommunication signals [1,2]. The researches being carried out in using global algorithms in the design of digital telecommunication signal are papers [3,4] that used MLSL algorithm for the design of optimized signal. Being inspired by paper [3], this paper's objective is the design of optimized signals for digital telecommunication that by using these signals instead of the existing standard modulations, the error of receiving signal is reduced. In comparison with paper [3] some innovations are done in this project as the followings:

1. Simplifying the conditions of optimization problem in comparison with reference [3]that increased the speed of optimization algorithms.
2. Getting objective function gradient as analytical for MLSL method considering the items in reference [3], gradient of objective function is not achieved.
3. The comparison of three important methods in global optimization (MLSL, SA, GA) in signals design
4. A new algorithm called LSA (Local after Simulated Annealing)is proposed that is the extended type of SA algorithm and improved considerably its performance in being convergent to optimized point
5. The presentation of vector display of two-dimensional 32 optimized signals that are not reported in other papers.

Optimization problem theory

To determine the optimized signals, at first a objective function being correspondent with receiver error should be gained. In digital telecommunication, the messages are send in the form of symbol and for each symbol,

***Corresponding Author:** Ali Mohammadi, Islamic Azad University, Islam-Abad-e-Qarb Branch, Islam-Abad-e-Qarb, Iran.
E-mail address: almohammadi1390@yahoo.co.uk

corresponding signal $s_m(t)$ $, m=1,...,M$ is sent and $S_m(t)$ signal is displayed according to base signals as the followings:

$S_m(t) = \sum_{k=1}^{K} a^{m,k}\varphi_k(t) \quad , m = 1, \ldots, M$ (1)

Where K is the number of base signals, $\varphi_k(t)$ base signal of kth and $a^{m,k}$ the image of $S_m(t)$ on kth base signal. The sent signal is received after interfering with additive noise of channel in receiver and is turned into the following equation after sampling:

$y[t] = S_m[t] + n[t] \quad t = 1,2,..T$ (2)

Here the noise is assumpted to be separated from signal and noise samples are separated from each other with equal distribution of p_N. H_m shows sending signal $S_m(t)$ is expressed as the followings:

$H_m: y[t] = S_m[t] + n[t] \;\; t = 1,2,..M$ (3)

By displaying the received signal as vector $y \triangleq (y[1], \ldots \ldots y[T])$ and by assuming that the receiver works based on criteria MAP, decision making in the receiver is done as:

$\hat{m} = \arg\max \{P(H_m|y)|m = 1,..,M\}$ (4)

In the receiver by assuming that signal $s_m(t)$ is sent when the error is occur that :

$P(H_{m'}|y) > P(H_m|y), \exists\, m \neq m'$ (5)

So the probability of the fact that hypothesis H_m is supported is:

$P(c|H_m) = P(\{(H_m|y) > P(H_{m'}|y), \forall m' = m\}|H_m) = P(\left\{\mathrm{Ln}\frac{P(H_m|y)}{P(H_{m'}|y)} > 0, \forall m' \neq m\right\}|H_m)$ (6)

According to Bayes 'theorem $P(H_m|\mathbf{y}) = \frac{p(\mathbf{y}|H_m)P(H_m)}{p(\mathbf{y})}$ and the above equation we have:

$Ln\frac{P(H_m|y)}{P(H_{m'}|y)} = Ln\frac{P(y|H_m)P(H_m)}{P(y|H_{m'})P(H_{m'})}$ (7)

As the aim is to obtain the set of optimized signals for each source, so a reasonable assumption is that the previous probabilities are equal.

$P(H_m) = P(H_{m'}), \; \forall\, m = m'$ (8)

Considering this assumption and replacing equation (2-7) by (2-6) we can say that:

$P(C|H_m) = P\left(\left\{\ln\frac{P(y|H_m)}{P(y|H_{m'})} > 0, \forall\, m \neq m'\right\}\middle|H_m\right)$ (9)

Minimizing error probability is equal to maximizing correct decision-making probability. And:

$P(C) = \sum_{m=1}^{M} P(C|H_m)P(H_m)$ (10)

Considering the this assumption that the previous probabilities are equal, for maximizing P(c) we should maximize $\sum_{m=1}^{M} P(c|H_m)$ (11).

One way to maximize the amount of above equation is maximizing each $P(c|H_m)$, It should be considered that some of $P(c|H_m)$ are dependent to each other and it is possible that increase in one of them causes reduction in another as their sum is decreased. Thus, maximization trend of each $P(c|H_m$ should be done by applying some limitations being dictated by other terms in (11). The mentioned limitations depend upon the arrangement of sent signals in vector space. As in each repeat, maximizing trend of limitations is changed. The result is that if applying such limitation is not possible, the speed (calculation output) of the trend is considerably reduced. A solution to simplify and increase the speed of maximization algorithm, is maximizing the least $P(c|H_m$ in equation (11). Although it is possible that such algorithm is not lead into the optimized answer for equation (11), but provide under optimization answer. Thus, here maximization of the following value is considered:

$\min_{m\in\{1,\ldots,M\}} P(C|H_m)$ (12)

for $p(\mathbf{y}|H_m)$ considering the independence of noise samples from each other we have:

$P(y|H_m) = \prod_{t=1}^{T} P(y[t]|H_m) = \prod_{t=1}^{T} P_N(y[t] - S_m[t])$ (13)

$\mathrm{Ln}\frac{P(y|H_m)}{P(y|H_{m'})} = \sum_{t=1}^{T} \mathrm{Ln}\frac{P(y[t]|H_m)}{P(y[t]|H_{m'})}$ (14)

Correctness of H_m (sending signal $S_m(t)$ means $y[t] = S_m[t] + n[t]$, thus:

$P(y[t]|H_m) = P_N(n[t])$ (15)

$P(y[t]|H_m) = P_N\left(n[t] + (S_m[t] - S_{m'}[t])\right) \;\; , \forall\, m \neq m'$ (16)

By combining the above equations we have:

$$\ln \frac{P(y|H_m)}{P(y|H_{m'})} = \sum_{t=1}^{T} \ln \frac{P_N(n[t])}{P_N\left(n[t]+\left(S_{m'}[t]-S_m[t]\right)\right)} = Q_{mm'} \qquad (17)$$

$Q_{mm'}$ is a random variable being dependent upon T independent random variable $n(T),\ldots,n(2),n(1)$. By replacing equation (17) by equation (9) we have:

$$\mathrm{P}(\mathrm{C}|\mathrm{H_m}) = \mathrm{P}(\mathrm{Q_{m1}} > 0, \mathrm{Q_{m2}} > 0, \ldots, \mathrm{Q_{m(m-1)}} > 0, \ldots, \mathrm{Q_{mM}} > 0) \qquad (20)$$

$Q_{mm'}$s are dependent to each other in the above equation and by assuming their fixed variance $\mathrm{P(c|H_m)}$ is maximized when we maximize $\min_{m\neq m'} E|Q_{mm'}|$ thus, optimization problem is turned into maximizing the following expression:

$$\min_{m\in\{1,..,M\}} \min_{m\neq m'} E \sum_{t=1}^{T} \ln \frac{P_N(n[t])}{P_N\left(n[t]+\left(S_{m'}[t]-S_m[t]\right)\right)} \qquad (21)$$

The above equation is objective function of optimization problem that can be expressed by function $K_N: R \to R$:

$$K_N(\delta) \triangleq \int_R \ln\left(\frac{P_N(\tau)}{P_N(\tau+\delta)}\right) P_N(\tau)d\tau \qquad (22)$$

Where K_N is [*]KL distance between probability density function of noise and its shift as $-\delta$. If probability density function of noise is symmetrical (it is not a limiting assumption), K_N function is a pair function. Table (1) shows density function of probability and KN shows some common noises being studied in this paper.

Table 1- Probability density function and multi-noise K_N (0) function

Noise		
Gaussian		
Laplacian		
Hyperbolic Secant	$\frac{1}{2\sigma}\mathrm{Sech}\left(\frac{\pi\tau}{2\sigma}\right)$	$-2\mathrm{Ln}\left(\mathbf{Sech}\left(\frac{\pi\delta}{4\sigma}\right)\right)$
Generalized Gaussian		
Cauchy		

In the above table, Γ denotes Gama function and it can be said that:

$$E \sum_{t=1}^{T} \ln \frac{P_N(n[t])}{P_N\left(n[t]+\left(S_{m'}[t]-S_m[t]\right)\right)} = \sum_{t=1}^{T} K_N\left(S_{m'}[t] - S_m[t]\right) \qquad (23)$$

And considering the value of $s_m(t)$s:

$$\mathrm{S_m[t]} = \sum_{k=1}^{K} a^{m,k}\varphi_k[t] \qquad (24)$$

The dependency of equation to images (components) of signals is clear. These components can be expressed as the form of the following vector:

$$a \triangleq (a^{1,1}, \ldots, a^{1,k}, .., a^{m,1}, .a^{m,k})$$

as $K_N(.)$ function is a pair function, the values are equal for $m > m'$ and $m < m'$, so to solve, considering one of these two parts is enough. Considering the above equation, solving the design of optimized signals equals:

$$\max_{a\in R^{MK}} \min \left\{\sum_{t=1}^{T} K_N\left(\sum_{k=1}^{K}\left(a^{m',k} - a^{m,k}\right)\varphi_k[t]\right) | m, m' \in \{1, \ldots, M\}, m' > m\right\} \qquad (25)$$

On the other hand, there is energy limitation for sent signals and this limitation is expressed by two limitations in average energy of signal and the limitation in moment energy of signal. In this paper, the limitation of moment energy of signal is considered and it should be $|\mathrm{S_m(t)}| \leq \mathrm{C}, \mathrm{m} = 1, .., \mathrm{M}$, where C is a definite value greater than zero. Considering this limitation, objective function for optimization problem is as the followings:

$$\max_{a\in R^{MK}} \left\{-\sum_{t=1}^{T} K_N\left(\sum_{k=1}^{K}\left(a^{m',k} - a^{m,k}\right)\varphi_k[t]\right) | m, m' \in \{1, \ldots, M\}, m' > m\right\} \qquad (26)$$

That they should be minimized under the above condition by changing the images of sent signals of $a \in R^{MK}$.

The definitions of parameters of an optimization problem

Global optimization problem is finding a solution among a set of acceptable answer that objective function has the least amount for it. A global optimization problem for function f(x) is defined as the followings:

[*] Kullback_Leibler

Definition: assuming $f(x): R^D \to R$ and $S \subseteq R^D$ (R is the set of real numbers). By finding $x^{**} \in S$ as for all $x \in S$ we have, $f(x^{**}) \le f(x)$, then, $x^{**}$is as the global minimize of objective function and $f(x^{**})$ is global minimum. Searching zone of S is determined as $S = \{x \mid L_i \le x_i \le U_i, i = 1,...,D\}$ in which L_i, U_i are respectively, lower zone and upper zone of variable x_i. Any local minimum f(x*) is consisting of a tension zone that is defined as the followings: local minimum tension zone f(x*) is the set of points in search space that start of a local algorithm is convergent from those points to x* point.

Simulation design: Signals design is done for two states M=8,16 and T=50, $C = \sqrt{10}$ (equations (25), (26) for 5 noises whose distribution function (PN) and $K_N(\delta)$ is shown in table (1). In addition, base signals are considered as two types of sine-cosine and sine-sine and considering these two types of base signal, the condition of the problem is closely considered.

Base signals sine-cosine: Two signals in this base are written as the followings:

$$\varphi_1(t) = \sqrt{\frac{2}{T}}\sin 2\pi\omega\frac{t}{T} \ , \ \varphi_2(t) = \sqrt{\frac{2}{T}}\cos 2\pi\omega\frac{t}{T} \ , \omega = 10 \, , t = 0,1, \dots T-1 \qquad (27)$$

Where t is discrete, T is the number of sampling in the related time distance and ϖ is a parameter defining the number of periods in which signal is non-zero and signal s(t) is written as the followings:

$$S_m(t) = \alpha^{m,1}\varphi_1(t) + \alpha^{m,2}\varphi_2(t) = \alpha_m \sin x + \beta_m \cos x \qquad , \alpha_m = \alpha^{m,1}\sqrt{\frac{2}{T}} \ , \beta_m = \alpha^{m,2}\sqrt{\frac{2}{T}} \ , x = 2\pi\omega\frac{t}{T}$$

(28)

Considering the condition $S_m(t) < C$,when maximum signal $S_m(t)$ is smaller than C, the condition of the problem is established for all the times (t), to determined the related maximum, its derivative is equal to zero.

$$\frac{dS_m(t)}{dt} = \frac{2\pi\omega}{T}\alpha_m\cos(x) - \frac{2\pi\omega}{T}\beta_m\sin(x) = 0 \qquad (29)$$

As the result we will have:

$$\alpha_m\cos(x') = \beta_m\sin(x') \to x' = \frac{2\pi\omega t'}{T} = \tan^{-1}\left(\frac{\alpha_m}{\beta_m}\right) + 2n\pi \ , n = 0,1,2,.. \qquad (30)$$

$$t' = \frac{T}{2\pi\omega}\tan^{-1}\left(\frac{\alpha_m}{\beta_m}\right) \to \cos\left(2\pi\omega\frac{t'}{T}\right) = \frac{\beta_m}{\alpha_m}\sin\left(2\pi\omega\frac{t'}{T}\right) \quad , \ S(t') = \frac{\beta_m^2}{\alpha_m}\sin\left(2\pi\omega\frac{t'}{T}\right) + \alpha_m\sin\left(2\pi\omega\frac{t'}{T}\right) \quad (31)$$

And

$$\sqrt{((\alpha^{m,1})^2 + (\alpha^{m,2})^2)} < \sqrt{\frac{T}{2}}C \qquad (32)$$

$$\sin\left(2\pi\omega\frac{t'}{T}\right) = \frac{\tan\left(2\pi\omega\frac{t'}{T}\right)}{\sqrt{1+\tan^2\left(2\pi\omega\frac{t'}{T}\right)}} = \frac{\frac{\alpha_m}{\beta_m}}{\sqrt{1+\left(\frac{\alpha_m}{\beta_m}\right)^2}} = \frac{\alpha_m}{\sqrt{\alpha_m^2+\beta_m^2}} \to S(t') = \left(\frac{\beta_m^2}{\alpha_m} + \alpha_m\right)\left(\frac{\alpha_m}{\sqrt{\alpha_m^2+\beta_m^2}}\right)$$

$$= \sqrt{\alpha_m^2 + \beta_m^2} < C$$

So the condition of the problem expresses that $S_m(t)$ signals in two-dimensional vector space should be confined in a circle with the radius of $\sqrt{\frac{T}{2}}C$.

Base signals sine-sine: Frequency difference of two sinus signals make them orthogonal and thus the following two signals are achieved.

$$\varphi_1(t) = \sqrt{\frac{2}{T}}\sin 2\pi\omega_1\frac{t}{T} \ , \ \varphi_2(t) = \sqrt{\frac{2}{T}}\sin 2\pi\omega_2\frac{t}{T} \ , \omega_1 = 10, \omega_2 = 11 \, , t = 0,1, \dots T-1 \quad (33)$$

By considering the problem condition and signal $S_m(t) < C$:

$$S_m(t) = \alpha^{m,1}\varphi_1(t) + \alpha^{m,2}\varphi_2(t) = \alpha_m \sin 2\pi\omega_1\frac{t}{T} + \beta_m \sin 2\pi\omega_2\frac{t}{T} \qquad , \alpha_m = \alpha^{m,1}\sqrt{\frac{2}{T}}, \beta_m = \alpha^{m,2}\sqrt{\frac{2}{T}} \qquad (34)$$

Considering the previous trend:

$$\frac{dS_m(t)}{dt} = \frac{2\pi\omega_1}{T}\alpha_m\cos\left(2\pi\omega_1\frac{t}{T}\right) + \frac{2\pi\omega_2}{T}\beta_m\cos\left(2\pi\omega_2\frac{t}{T}\right) = 0 \qquad (35)$$

$$\frac{\cos 2\pi\omega_1\frac{t'}{T}}{\cos 2\pi\omega\frac{t'}{T}} = \frac{\beta_m\omega_2}{\alpha_m\omega_1} \to \sin 2\pi\omega_2\frac{t'}{T} = \sqrt{\left(1-\cos^2 2\pi\omega_2\frac{t'}{T}\right)} = \sqrt{\left(1-\left(\frac{\alpha_m\omega_1}{\beta_m\omega_2}\right)^2\cos^2 2\pi\omega_1\frac{t'}{T}\right)} \quad (36)$$

$$S_m(t') = \alpha_m \sin\left(2\pi\omega_1 \frac{t'}{T}\right) + \beta_m\sqrt{1-\left(\frac{\alpha_m\omega_1}{\beta_m\omega_2}\right)^2 + \left(\frac{\alpha_m\omega_1}{\beta_m\omega_2}\right)^2 \sin^2 2\pi\omega_1 \frac{t'}{T}} < |\alpha_m| + |\beta_m| \quad (37)$$

Here as the above equation doesn't have answer as analytical and it should be solved as numerical, $S_m(t') \approx |\alpha_m| + |\beta_m|$. Thus,

$$|\alpha_m| + |\beta_m| < C \rightarrow |a^{m,1}| + |a^{m,2}| < \sqrt{\frac{T}{2}}C \quad (38)$$

The resulting zone is a diamond. Considering inequalities (32), (38), searching zone of optimization problem is determined easily and is easily applied in the related algorithms. Also, direct application of condition $S_m(t) < C$ causes that searching zone has little difference with the zones of equation (32), (38) and according to figure (27)the space signals getting at base signal of sine-cosine is greater than the space signals can have at base signal sine-sine. And it means as more energy for base signals sine-cosine. From this space, it is resulted that designing with base signal sine-cosine creates less error in the receivers. In other words, correct decision making is increased in the receiver.

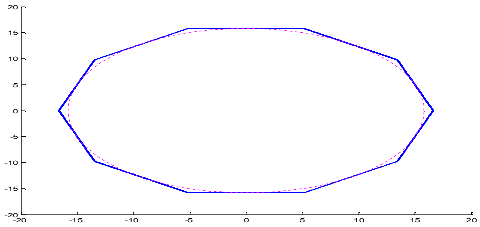

Figure 1a: sine-cosine

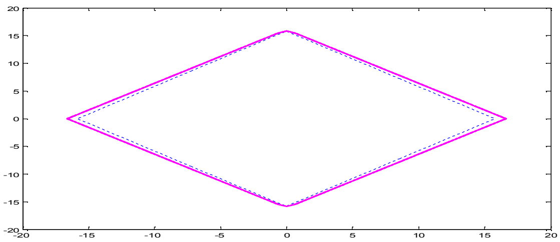

Figure 1b; sine-sine

Simulation

Genetic algorithm: The genetic algorithm being used in this paper is SGA. SGA is the simplest genetic algorithm consisting of 3 main operators of all genetic algorithms. These three operators are including selection, cross over and mutation. To do the algorithm, at first an initial population (here p=40) is selected randomly then objective function values are calculated for all members and the members participate in the production of the next generation depending upon its value. 24 superior combination of each generation are used for construction of the new generation (probability coefficient of cross over 0.6), to escape from being trapped in local extermum point, mutation parameter is used and probability coefficient of this parameter is selected as 0.05. then values of objective function is calculated for new generation and here the condition of ending algorithm is investigated and in case of fulfilling the condition, the algorithm task is finished, otherwise, this task continues. Spatial arrangement of signal is displayed for some states being achieved by GA algorithm in figures (2) to (5):

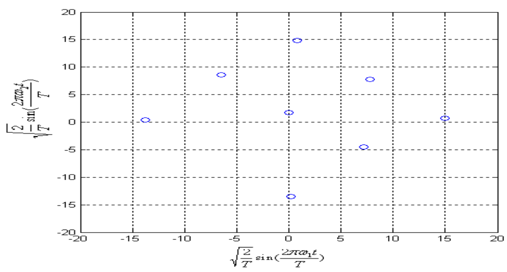

Figure 2- Spatial arrangement of 8-signal for Cauchy noise (base signal sine-sine)

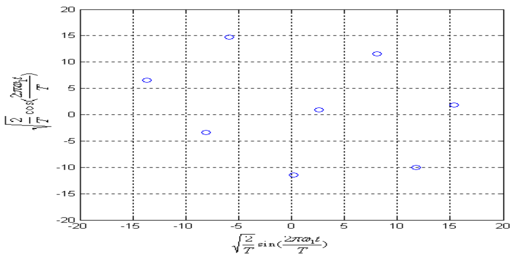

Figure 3- Spatial arrangement of 8-signal for Gaussian noise (base signal sine-cosine)

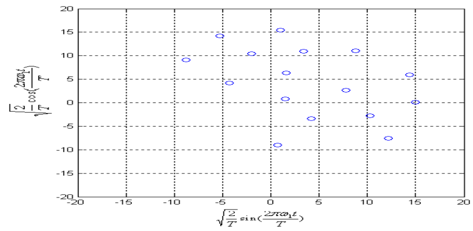

Figure 4- Spatial arrangement of 16-signal for Laplacian noise (sine-cosine)

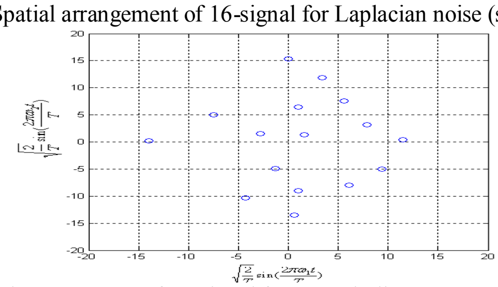

Figure 5- Spatial arrangement of 16-signal for Hyperbolic Secant noise (sine-cosine)

The results showed that in genetic algorithm, there is no guarantee to reach a good point and for repetitious execution of algorithm, we reach different answers and this is due to algorithm trapping in local minimums. In addition, if it is allowed that algorithm is executed for a long time, it cannot be sure that algorithm reaches a good point. But it is possible that after many times execution, we achieve a good answer that is not economical.

SA algorithm: In the presented SA algorithm, the initial temperature is selected as T_0=30 that in initial stages, all combinations are accepted. Temperature change is based on $T_k = \alpha T_{k-1}$ where $\alpha = 0.97$ is selected till temperature reduction is done slowly. In the algorithm at first a combination is selected randomly and objective function is calculated. Then, the combination is changed in one dimension (That the amount of this change is dependent upon the amount of its temperature) and objective function is reduced and is calculated for new combination. But the reduced objective function for new combination is less than the previous combination. The new combination is accepted and objective function amount is calculated for this combination. Otherwise, by one probability (that is dependent upon the temperature of that stage), the new combination is accepted. This is done for all parameters. To reach the better solution, it is necessary to do some repetition in each temperature. This number in the applied algorithm is 50 and then the temperature is reduced according to temperature function. Algorithm is finished when whether the temperature is achieved final temperature (in applied algorithm T_f =1) or an optimized point is repeated for definite numbers that N is the number of repetitions in each temperature and in this paper this amount is N=50. Spatial arrangement of signal is displayed for some states attained by SA algorithm in figures (6) to (9).

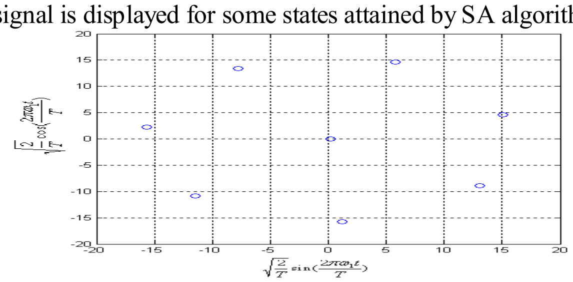

Figure 6- Spatial arrangement of 8-signal for Gaussian noise (sine-cosine)

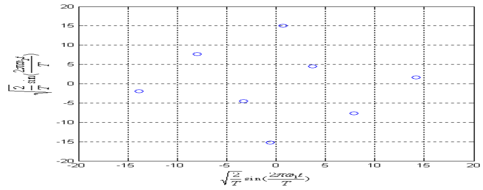

Figure 7- Spatial arrangement of 8-signal for Cauchy noise (sine-sine)

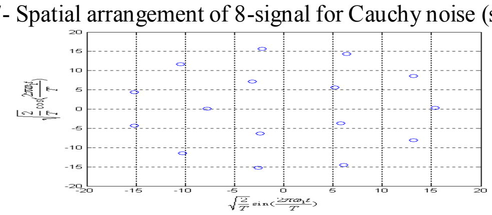

Figure 8- Spatial arrangement of 16-signal for Laplacian noise (sine-cosine)

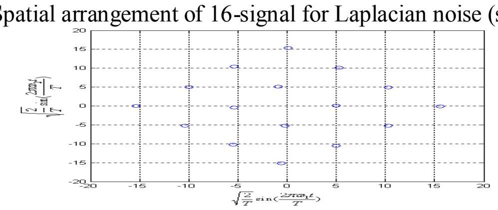

Figure 9- Spatial arrangement of 16-signal for Hyperbolic Secant noise (sine-sine)

The results show the followings: SA algorithm acts rapidly or it doesn't reach absolute minimum but always it reaches a good point. Thus, it has high reliability coefficient in comparison with GA algorithm.

MLSL algorithm: To simulate MLSL algorithm, a local algorithm should be used. In this paper, Newton method is used. The parameters of MLSL algorithm in simulations are considered as Np=1000, ξ=0.05, $\rho = 4$. Also algorithm task is finished when ending condition is fulfilled. Ending condition is considered as two forms, at first equations (35), (36) should be satisfied and second objective function should be repeated for a definite amount for definite number. The results show that in most cases, algorithm reaches a point (minimum) that these items show high reliability coefficient of algorithm. Spatial arrangement of signal is displayed for some states being achieved by MLSL algorithm in figures (10) to (13):

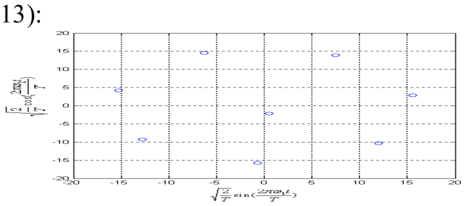

Figure 10- Spatial arrangement of 8-signal for Gaussian noise (sine-cosine)

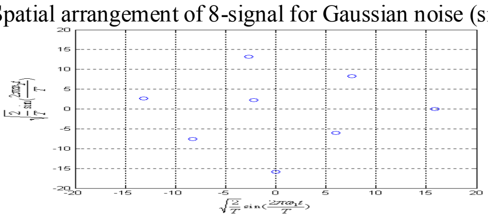

Figure 11- Spatial arrangement of 8-signal for Cauchy noise (sine-sine)

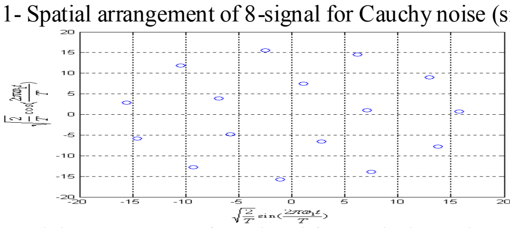

Figure 12- Spatial arrangement of 16-signal for Laplacian noise (sine-cosine)

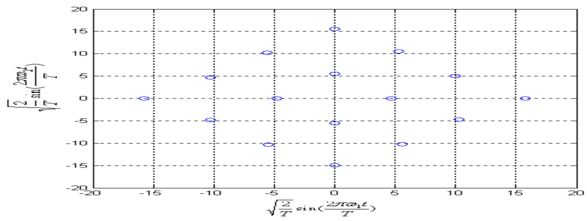

Figure 13- Spatial arrangement of 16-signal for Hyperbolic Secant noise (sine-sine)

Extending SA method: As we know if SA algorithm is executed for a long time, it is reached to absolute minimum and if we limit the execution of algorithm, in fact it will be convergent to a point in tension zone of absolute minimum pint. Thus, by SA algorithm, we can find a good point to use in local algorithm. Thus, the combinations achieved by execution of SA algorithm, are applied as starting point of a local algorithm that in this paper, it is called LSA method. The results of using LSA are shown in figures (14) to (17) that show considerable improvement in comparison with normal SA.

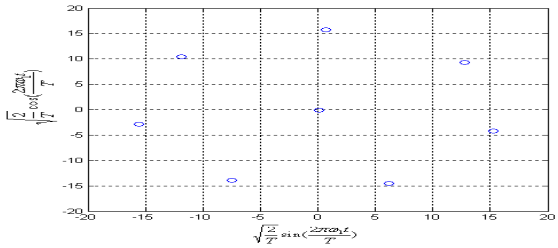

Figure 14- Spatial arrangement of 8-signal for Gaussian noise (sine-cosine)

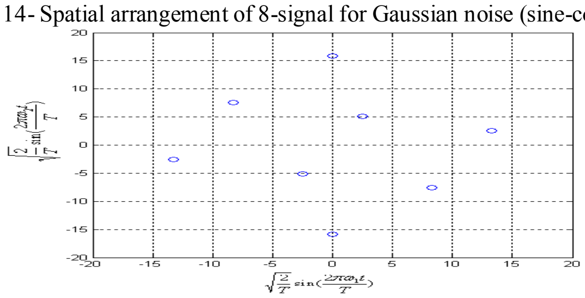

Figure 15- Spatial arrangement of 8-signal for Cauchy noise (sine-sine)

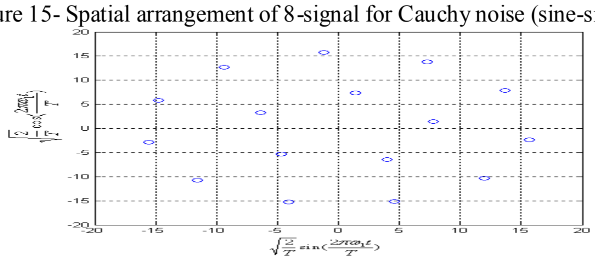

Figure 16- Spatial arrangement of 16-signal for Laplacian noise (sine-cosine)

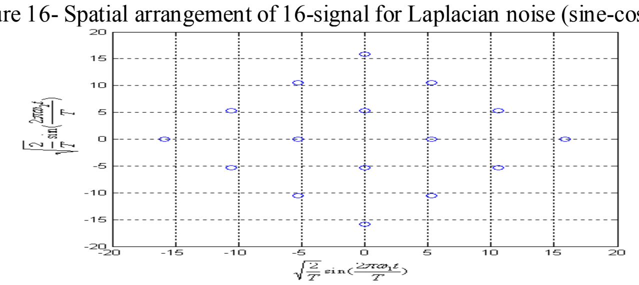

Figure 17- Spatial arrangement of 16-signal for Hyperbolic Secant noise (sine-sine)

Comparing methods

For comparing methods, each algorithm is done 10 times for each noise and base signal and the amount of final objective function (F(a)), the number times that objective function was called by algorithm (N) and the average of these two values are obtained for 10 executions. The comparison of the results of these tables shows that GA algorithm has low reliability coefficient (similar answers in different executions) in finding less minimum and it is trapped in local minimums. While, the number of calculation of objective function (calculation load) GA is a lot. As it was said, SA algorithm has rather high reliability coefficient. If it is allowed that algorithm, continue its work, almost all answers led into one point that is near to absolute minimum. This case requires taking too much time. But in this project, due to the comparison of the methods, it is attempted to create equal conditions for algorithms and due to this, the results of SA algorithm don't have high reliability coefficient. LSA algorithm presents better results in comparison with SA and its reliability coefficient is improved in it. MLSL algorithm has high reliability coefficient and rapidly reaches an optimized point. Optimization gradient of objective function in this algorithm is very steep and due to this, algorithms reach very rapidly to optimized point. GA algorithm due to low reliability coefficient and being trapped in local points are set aside. SA algorithm is put aside due to weak answers and the fact that LSA algorithm is extended type of SA algorithm and the main comparison is done between LSA algorithm and MLSL algorithm. To compare two important parameters in algorithms are evaluated as:

1. The amount of objective function (minimum)
2. The number of times objective function is calculated in algorithm

To do this, charts (1) to (4) are plotted to compare the methods easily. In the following charts x axis shows different kinds of noises and y axis shows the amount of objective function and the number of times objective function is calculated, respectively. Considering the fact that objective function optimized amount is different for each noise, to compare MLSL and LSA algorithms in different noises, we normalize objective function amount. Thus, the amount of each y in each noise is divided by maximum absolute value of ys for two methods. This normalization trend is done for the number of calculation of objective function. In the charts, MLSL algorithm is displayed by dart color bar and LSA algorithm is shown by light color bar. In this figures, the bar in the lowest place shows the best performance, both in terms of the amount of function (F(a)) and the number of times in which objective function is calculated (NUMFUN) abbreviations being used in the charts are including:

G: Gaussian, L: Laplacian, H: Hyperbolic Secant, GG: Generalized Gaussian, C: Cauchy

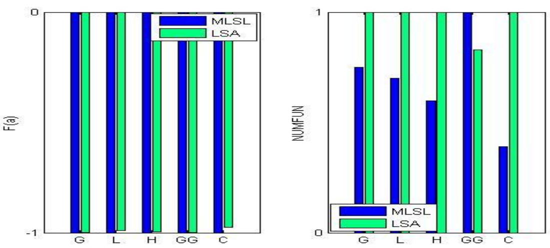

Chart 1- The comparison of two methods for 8-state with base signal sine-cosine

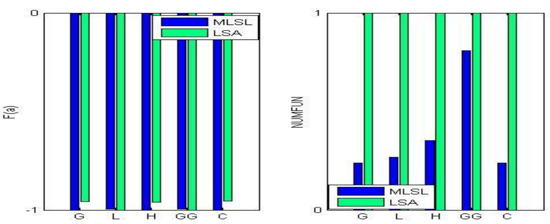

Chart 2- The comparison of two methods for 8-state with base signal sine-sine

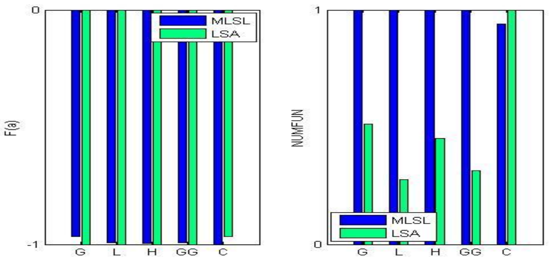

Chart 3- The comparison of two methods for 16-state with base signal sine-cosine

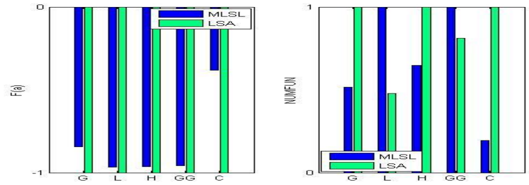

Chart 4- The comparison of two methods for 16-state with base signal sine-sine

As it is shown in the charts, most of the time, MLSL algorithm do better and in some cases (16-singal), LSA algorithm do better (lower minimum). Objective functions are compared for all combinations achieved by execution of algorithms and the best combinations are determined for different states. Spatial arrangements of these optimized signals are shown in figures (18) to (27) that can be used in digital telecommunication systems.

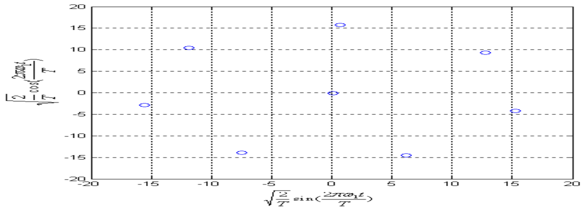

Figure 18- Arrangement of optimized 8-signal, Gaussian, Generalized Gaussian (sine-cosine)

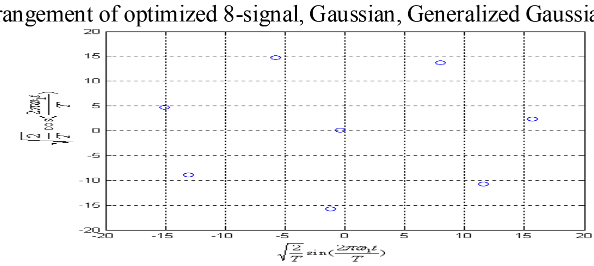

Figure 19- Arrangement of optimized 8-signal, Laplacian (sine-cosine)

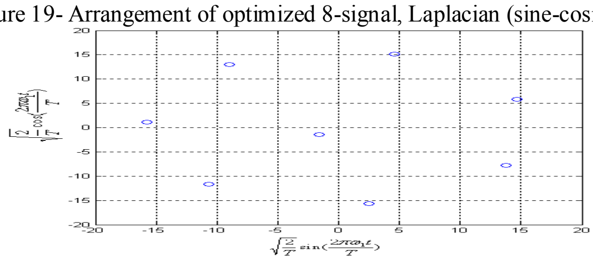

Figure 20- Arrangement of optimized 8-signal, Hyperbolic Secant (sine-cosine)

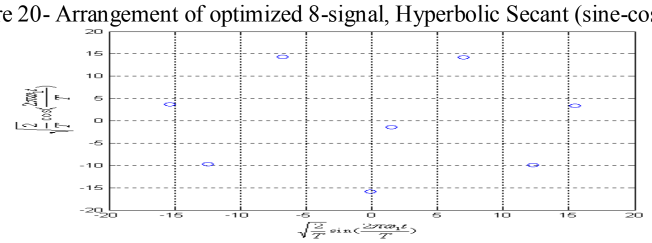

Figure 21- Arrangement of optimized 8-signal, Cauchy (sine-cosine)

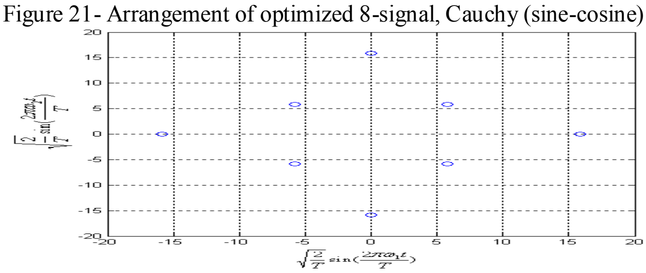

Figure 22- Arrangement of optimized 8-signal, Gaussian (sine-sine)

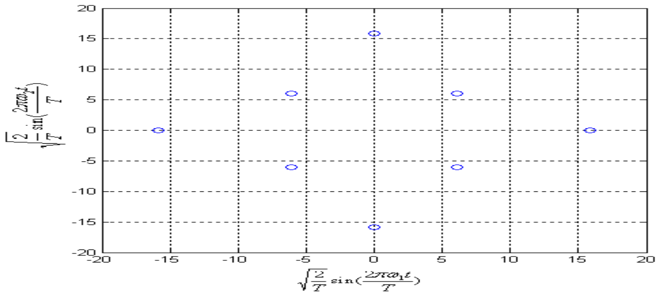

Figure 23- Arrangement of optimized 8-signal, Generalized Gaussian (sine-sine)

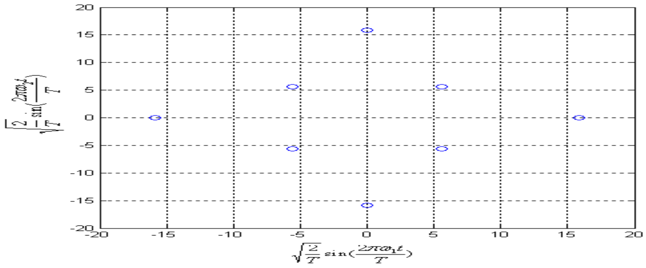

Figure 24- Arrangement of optimized 8-signal, Laplacian , Hyperbolic Secant and Cauchy (sine-sine)

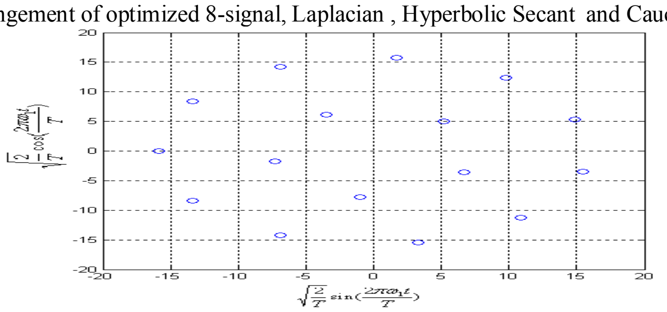

Figure 25- Arrangement of optimized 16-signal, Laplacian ‹Gaussian‹ Generalized Gaussian‹ Hyperbolic Secant and Cauchy (sine-cosine)

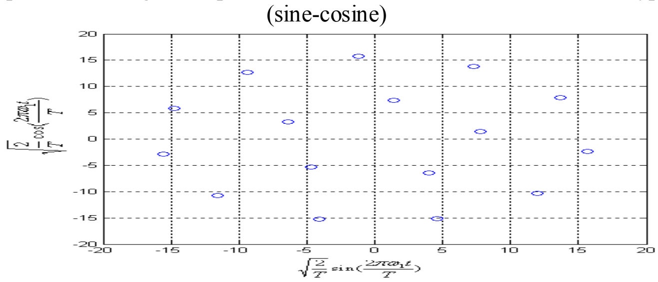

Figure 26- Arrangement of optimized 16-signal, Laplacian (sine-cosine)

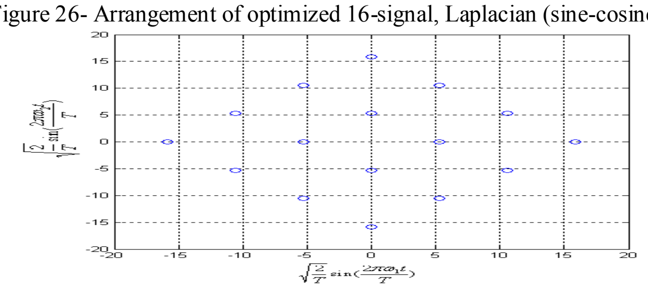

Figure 27- Arrangement of optimized 16-signal, Laplacian ‹Gaussian‹ Generalized Gaussian‹ Hyperbolic Secant and Cauchy (sine-sine)

The design of optimized 32-point signal: finally by MLSL algorithm, 32-point optimized signals are designed and its results are shown with the shapes of signal spatial arrangement. The studies show that there is not reference showing the results of 32-point optimized signals,

Table 2- The amount of objective function for 32-point with base signal sine-cosine

Noise	F(a)	Noise	F(a)
Gaussian	-10.2283	Gaussian	-16.5708
Laplacian	-13.9723	Laplacian	-22.0469
Hyperbolic Secant	-11.6071	Hyperbolic Secant	-18.6667
Generalized Gaussian	-16.1595	Generalized Gaussian	-26.4802
Cauchy	-4.6169	Cauchy	-7.4119

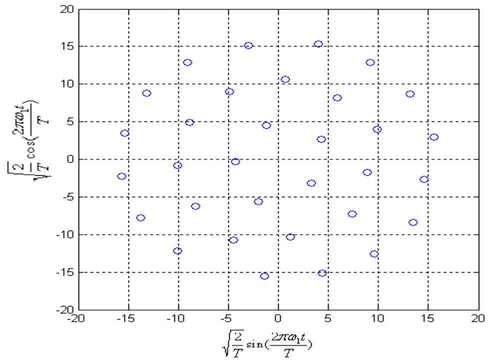

Figure 28- Spatial arrangement of optimized 32-signal for all noises (sine-cosine)

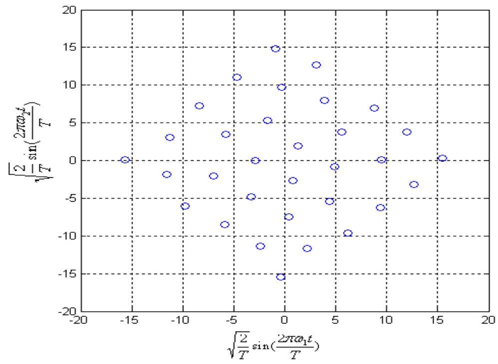

Figure 29- Spatial arrangement of optimized 32-signal for all noises (sine-sine)

As it is show in the above figures, optimized 32-signal are the same for all the noises and this is due to the less distance between the points. In reference [2], this point is referred. Considering this reference when the distance between the points are less, KL leveled surfaces are in the form of circle and when the distance is increased, these surfaces are in different forms for different noises. As the value of C is fixed (signal components are selected in a fixed range), it was predicted that for more signals, the leveled surfaces are equal for different noises and the same vector arrangement is achieved for different noises.

Conclusion

The design of two-dimensional optimized signals in digital telecommunication was analyzed and simulated by three global optimization methods. According to the results of simulation, MLSL algorithm do better and had access to better results. About SA algorithm we should say that by this algorithm we can find a good starting point for starting the work of a local algorithm and in this regard LSA algorithm is recommended.

GA algorithm didn't get good results and showed that it doesn't have the required reliability (The answers in some executions, had little difference with each other) to be applied in the design of optimized signals. This algorithm is used in some cases in which there is not extra information (e.g. gradient) in the problem. In this paper, optimized two-dimensional signals of 8, 16, 32 are achieved for different noises and if in digital telecommunication instead of standard modulations, the signals in the paper are used, the probability of error is reduced in detection. The important point is that in 16-signal at base signal sine-sine; the same modulation spatial arrangement QAM is the optimized state.

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[2] D. H. Johnson and G. C. Orsak, "Relation of signal set choice to the performance of optimal non-Gaussian detectors", IEEE Trans. Comm., vol. 41(9), pp. 1319-1328, 1993.
[3] A. J. Kearsley , "Global and Local Optimization Algorithms for Optimal Signal Set Design", Journal of Research of the National Institute of Standards and Technology, vol 106, pp 441-454 , 2001.
[4] M. Nakhkash, “The application of ground penetrating radar to the characterization of multi-layered media,” Ph.D. Thesis, University of Liverpool, UK, 1999.