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On Reliability Criterion Approach for Multiple Hypotheses Optimal Testing of Two Independent Markov Chains

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ABSTRACT

The problem of five hypotheses logarithmically asymptotically optimal (LAO) testing for a pair of simple homogeneous stationary Markov chains is studied. This problem is introduced by Ahlswede and Haroutunian on multiple hypotheses testing for many objects. We noticed Natarajan's theorem and its applications in hypotheses testing and show that this method of investigation, solvimg the problem is easier and gives identical results by procedure that was introduced by Haroutunian and Grigorian (2007).The problem of many hypotheses testing for one objects via large deviation techniques (LDT) for a model of simple homogeneous stationary Markov chains is solved by Yarmohammadi and Navaei (2008).

Keywords: Error probability , Reliability, Two independent objects, Markov chains.

1. INTRODUCTION

Ahlswede and Haroutunian in [1] formulated an ensemble of problems on multiple hypotheses testing for many objects and on identification of hypotheses under reliability requirement. The problem of many ($L > 2$) hypotheses testing on distributions of a finite state Markov chains is studied in [11] via large deviations techniques (LDT) also Navaei in [13] studied a model of hypotheses testing consisting of with two simple homogeneous stationary Markov chains with finite number of states such that having different distributions from four possible transmission probabilities. In this paper we solve the problem to multiple hypotheses testing for two independent objects and $M = 5$ distributions for the case of simple homogeneous stationary finite states of Markov chains. In section 2 we recall main definitions and results of [7] and [11] for many hypotheses testing and in section 3 present the problem of hypotheses testing for two independent objects via Markov chains.

2. On many hypotheses LAO testing for Markov chains

We remind the main definitions and results of paper [7] and [11] for the case of $M = 5$ distributions for further use.

Let $X = (x_0, x_1, \ldots, x_N)$, $x_n \in \chi = \{1,2,\ldots,I\}$, $X \in \chi^{N+1}$, $N = 0,1,2,\ldots,$ be vector of observations of a simple homogeneous irreducible stationary Markov chains with finite number I of states. The 5 hypotheses H_l concern the matrix of the transition probabilities

$$P_l = \{P_l(j \mid i), i = \overline{1,I}, j = \overline{1,I}\} , \ l = \overline{1,5} .$$

The stationary of the chain provides existence for each $l = \overline{1,5}$ of the unique stationary distributions $Q_l = \{Q_l(i), i = \overline{1,I}\}$, such that:

$$\sum_i Q_l(i)P_l(j \mid i) = Q_l(j), \sum_i Q_l(i) = 1, i = \overline{1,I} , \ j = \overline{1,I} .$$

$$Q_l \circ P_l = \{Q_l(i)P_l(j \mid i), i = \overline{1,I}, j = \overline{1,I}\} , \ l = \overline{1,5} .$$

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We denote by $D(Q \circ P \| Q_l \circ P_l)$ the Kullback-Leibler divergence

$$D(Q \circ P \| Q_l \circ P_l) = \sum_i Q(i) P_l(j \mid i)[\log Q(i) P(j \mid i) - \log Q_l(i) P_l(j \mid i)]$$
$$= D(Q \| Q_l) + D(Q \circ P \| Q \circ P_l),$$

Of a joint distribution

$$Q \circ P = \{Q(i)P(j \mid i), i = \overline{1,I}, j = \overline{1,I}\} ,$$

From joint to distribution $Q_l \circ P_l$, where the divergence for marginal distribution is

$$D(Q \| Q_l) = \sum_i Q(i)[\log Q(i) - \log Q_l(i)], l = \overline{1,5}$$

The probability of vector $X \in \chi^{N+1}$ of the Markov chains with transition probabilities P_l and stationary distribution Q_l, is the following

$$Q_l \circ P_l^N(X) \stackrel{\Delta}{=} Q_l(x_0) \prod_{n=1}^{N} P_l(x_n \mid x_{n-1}), l = \overline{1,I} ,$$

$$Q_l \circ P_l^N(A) \stackrel{\Delta}{=} \bigcup_{x \in A} Q_l \circ P_l^N(X) , A \in \chi^{N+1} .$$

The second order type of Markov vector X is [see[6]] the square matrix of I^2 relative frequencies $\{N(i,j)N^{-1}, i = \overline{1,I}, j = \overline{1,I}\}$ of the simultaneous appearance in X of the states i and j on the pairs of neighbor places. It is clear that $\sum_{i,j} N(i,j) = N$. Denote by $\Gamma^N_{Q \circ P}$ the set of vectors X from χ^{N+1} which have the second order type such that for some joint PD $Q \circ P$

$$N(i,j) = NQ(i)P(j \mid i) , i = \overline{1,I}, j = \overline{1,I}.$$

The set of joint PD $Q \circ P$ on I^2 is denoted by $Q \circ P$. Non-randomized test $\phi_N(X)$ accepts one of the hypotheses $H_l, l = \overline{1,5}$ on the basis of the trajectory $X = (x_0, x_1, ..., x_N)$ of the $N+1$ observations. We denote $\alpha^{(N)}_{l|m}(\phi_N)$ the probability to accept the hypotheses H_l under the condition that $H_m, m \neq l$, is true. For $l = m$ we denote $\alpha^{(N)}_{m|m}(\phi_N)$ the probability to accept the hypotheses H_m. It is clear that

$$\alpha^{(N)}_{m|m}(\phi_N) = \sum_{l \neq m} \alpha^{(N)}_{l|m}(\phi_N), m = \overline{1,5}. \qquad (1)$$

To every trajectory X the test ϕ_N puts in correspondence one from 5 hypotheses. The space χ^{N+1} will be divided into 5 parts,

$$Q_l^N = \{X, \phi(N)(X) = l\}, l = \overline{1,5}, \text{ and } \alpha^{(N)}_{l|m}(Q_N) = Q_m \circ P_m(Q_l^N), m, l = \overline{1,5}.$$

We consider the matrix of "reliabilities",

$$E = \{E_{l|m}(\phi) = \lim_{N \to \infty} -\frac{1}{N} \log \alpha^{(N)}_{l|m}(\phi_N), m, l = \overline{1,5}\}. \qquad (2)$$

It follows from definitions (1) and (2) that:

$$E_{m|m} = \min_{l \neq m} E_{l|m} . \qquad (3)$$

Let P be a matrix of transition probabilities of some stationary Markov chains, and Q be the corresponding stationary PD. For given family of positive numbers $E_{1|1}, E_{2|2}, ..., E_{4|4}$, we consider the decision rule ϕ^* by the sets of distributions

$$R_l \stackrel{\Delta}{=} \{Q \circ P : D(Q \circ P \,||\, Q \circ P_l) \leq E_{l|l}\}\ ,\ l = \overline{1,4},$$

$$R_l \stackrel{\Delta}{=} \{Q \circ P : D(Q \circ P \,||\, Q \circ P_l) > E_{l|l}\}\ ,\ l = \overline{1,4}, \qquad (4)$$

And the functions:

$$E^*_{l|l}(E_{l|l}) \stackrel{\Delta}{=} E_{l|l}\ ,\ l = \overline{1,4},$$

$$E^*_{l|m}(E_{l|l}) = \inf_{Q \circ P \in R_1} D(Q \circ P \,||\, Q \circ P_m)\ , m = \overline{1,5}, l \neq m,\ l = \overline{1,4}, \qquad (5)$$

$$E^*_{5|m}(E_{1|1}, ..., E_{4|4}) \stackrel{\Delta}{=} \inf_{Q \circ P \in R5} D(Q \circ P \,||\, Q \circ P_m)\ , m = \overline{1,4},$$

$$E^*_{5|5}(E_{1|1}, ..., E_{4|4}) \stackrel{\Delta}{=} \min_{l = \overline{1,4}} E^*_{l|5}.$$

The main result of paper [7],[11] is :

Theorem 1: Let $\chi = \{1,2,...,I\}$ be a finite set of the states of the stationary Markov chains possessing an irreducible transition matrix P and A be a nonempty and open subset or convex subset of joint distributions $Q \circ P$ and Q_m is stationary distribution for p_m, them for the type $Q \circ P(X)$ of a vector X from $Q_m \circ P_m$ on χ :

$$\underset{N \to \infty}{Lim} -\frac{1}{N} \log Q_m \circ P_m^N \{X : Q \circ P(X) \in A\} = \inf_{Q \circ P \in A} D(Q \circ P \,||\, Q \circ P_m)\ .$$

Theorem 2: Let χ be a fixed finite set, for a family of distinct distributions $P_1, ..., P_5$ the following two statements hold. If the positive finite numbers $E_{1|1}, E_{2|2}, ..., E_{4|4}$, satisfy conditions:

$$0 < E_{1|1} < \min[D(Q_m \circ P_m \,||\, Q_m \circ P_1), m = \overline{2,5}], \qquad (6)$$

$$0 < E_{1|1} < \min[E^*_{l|m}(E_{m|m}), m = \overline{1, l-1}, D(Q_m \circ P_m \,||\, Q_m \circ P_l), m = \overline{l+1,5}], l = \overline{2,4}$$

Then:

a) there exists a LAO sequence of test ϕ^*, the reliability matrix of which $\{E^*_{l|m}(\phi^*)\}$ is defined in(5) , and all elements of it are positive,
b) even if one of conditions (6) is violated, then the reliability matrix of an arbitrary test necessarily has an element equal to zero,(the corresponding error probability dose not tend exponentially to zero).

Remark(1) : From the definition (5),(9) and (3) it follows that : $E^*_{4|5} > E_{m|m}\ , m\overline{1,4}$ and also,

$$E^*_{m|m} = E^*_{m|M}\ ,\ m = \overline{1,4}\ .$$

3. Problem of identification of distribution for two independent Markov chains with five distributions and formulation of results

In this section we expand the concept of section 2 for two independent homogenies stationary finite Markov chains. Let X_1 and X_2 be independent RV taking values in the same finite state of

Markov chains of set χ with one of 5 PDs, they are characteristics of corresponding independent objects, the random vector (X_1, X_2) assume values $(x^1, x^2) \in \chi \times \chi$.

Let $(X_1, X_2) = ((x_0^1, x_0^2),...,(x_n^1, x_n^2),...,(x_N^1, x_N^2)), x^i \in \chi,\ i = 1,2,...,n = \overline{1,N}$, be a sequence of results of $N+1$ independent observations of a simple homogeneses stationary Markov chains with finite number I of states. The statistication must define unknown PDs of the objects on the base of observed data. The selection for each objects and denoted it by Φ_N. The objects independence test Φ_N may be considered as the pair of the tests φ_N^1 and φ_N^2 for the respective separate objects. We will show the whole compound test sequence by Φ. The test φ_N^i is defined by a partition of the space χ^{N+1} on the 5 sets and to every trajectory X the test ϕ_N puts in correspondence one from 5 hypotheses. So the space χ^{N+1} will be divided into 5 parts,

$$g_{l,i}^N = \{X_i, \phi_N(X_i) = l\}, l = \overline{1,5}, i = 1,2 .$$

We define

$$\alpha_{l_1,l_2|m_1,m_2}(\Phi_N) = Q_{m1} \circ P_{m1}(g_{l_1}^N) Q_{m_2} \circ P_{m2}(g_{l_{2,2}}^N)$$

Be the probability of the erroneous acceptance by the test Φ_N of the hypotheses pair (H_{l_1}, H_{l_2}) provided that (H_{m_1}, H_{m_2}) is true, where $(m_1, m_2) \neq (l_1, l_2), m_i, l_i = \overline{1,5}$, $i = 1,2$. The probability to reject a true pair of hypotheses (H_{m_1}, H_{m_2}) by analogy with (1) is the following:

$$\alpha_{m_1,m_2|m_1,m_2}^N(\Phi_N) \stackrel{\Delta}{=} \sum_{(l_1,l_2)\neq(m_1,m_2)} \alpha_{l_1,l_2|m_1,m_2}^N(\Phi_N) \qquad (7)$$

We also study corresponding limits $E_{l_1,l_2|m_1,m_2}(\Phi_N)$ of error probability exponents of the sequence of test Φ, called reliabilities:

$$E_{l_1,l_2|m_1,m_2}(\Phi_N) \stackrel{\Delta}{=} \overline{\lim_{N\to\infty}} - \frac{1}{N} \log \alpha_{l_1,l_2|m_1,m_2}^N(\Phi_N), m_i, l_i = \overline{1,L}, i = 1,2 \qquad (8)$$

We denote by $E(\varphi^i)$ the reliability matrices of sequences of tests $\varphi^i, i = 1,2,$ for each of the objects. With using (7) and (8) it follows that :

$$E_{m_1,m_2|m_1,m_2}(\Phi_N) = \min_{(l_1,l)\neq(m_1,m_2)} E_{l_1,l|m_1,m_2}(\Phi) \qquad (9)$$

In this section we use the following lemma.

Lemma[6] , [8]: If elements $E_{l|m}(\varphi^i), m,l = \overline{1,5}$, $i = 1,2,$ are strictly positive , then the following equalities hold for $\Phi = (\varphi^1, \varphi^2)$:

$$E_{m_1,m_2|m_1,m_2}(\Phi_N) = E_{l_1|m_1}(\varphi^1) + E_{l_2|m_{21}}(\varphi^2), \text{ if } m_1 \neq l_1 \text{ , } m_2 \neq l_2 \text{ ,} \qquad (10.a)$$

$$E_{m_1,m_2|m_1,m_2}(\Phi_N) = E_{l_i|m_i}(\varphi^i), \text{ if } m_{3-i} = l_{3-i} \text{ , } m_i \neq l_i \text{ ,} i = 1,2 \qquad (10.b)$$

Consider for a given positive elements $E_{m,m|m,5}$ and $E_{m,m|5,m}, m = \overline{1,4}$, the family of regions :

$$R_m^{(1)} \stackrel{\Delta}{=} \{Q \circ P : D(Q \circ P \,\|\, Q \circ P_m) \leq E_{m,m|5,m}\}, m = \overline{1,4} \text{ ,}$$

$$R_m^{(2)} \stackrel{\Delta}{=} \{Q \circ P : D(Q \circ P \,\|\, Q \circ P_m) \leq E_{m,m|m,5}\}, m = \overline{1,4} \text{ ,}$$

$$R_5^{(1)} \overset{\Delta}{=} \{Q \circ P : D(Q \circ P \| Q \circ P_m) > E_{m,m|5,m}\}, m = \overline{1,4} \,,$$

$$R_5^{(2)} \overset{\Delta}{=} \{Q \circ P : D(Q \circ P \| Q \circ P_m) > E_{m,m|m,5}\}, m = \overline{1,4} \,,$$

Definition (2): The test sequence $\Phi^* = (\varphi_1, \varphi_2, ..)$ we call LAO for the model with two objects if for given positive value of certain 4 elements of the reliability matrix $E(\Phi^*)$ the procedure provides maximal value for other element of it. Consider the following numbers:

$$E^*_{m,m|m,5} \overset{\Delta}{=} E_{m,m|m,5} \;,\; E^*_{m,m|5,m} \overset{\Delta}{=} E_{m,m|5,m} \;, m = \overline{1,4} \tag{11.a}$$

$$E^*_{m_1,m_2|l_1,l_2} \overset{\Delta}{=} \inf_{Q \circ P : Q \circ P \in R^i_{l_i}} D(Q \circ P \| Q \circ P_m) \;, l_i \neq m_i \;, m_3 - i = l_3 - i \;\;, i = 1,2 \tag{11.b}$$

$$E^*_{m_1,m_2|l_1,l_2} \overset{\Delta}{=} E^*_{m_1,m_2|m_1,l_2} + E^*_{m_1,m_2|l_1,m_2} \;\;,\;\; m_i \neq l_i \;\;, i = 1,2 \tag{11.c}$$

$$E^*_{m_1,m_2|m_1,m_2} \overset{\Delta}{=} \min_{(l_1,l) \neq (m_1,m_2)} E^*_{m_1,m_2|l_1,l_2} \tag{11.d}$$

Our aim is to find LAO test from the set of the compound tests $\Phi = (\varphi^1, \varphi^2)$ when strictly positive elements $E_{m,m|m,5}$ and $E_{m,m|5,m}$, $m = \overline{1,4}$, of the reliability matrix are given.

We must notice that for the elements $E_{m,m|m,5}$ and $E_{m,m|5,m}$, $m = \overline{1,4}$, of the test for two object can be positive only two subsets of tests $\Phi = (\varphi^1, \varphi^2)$:

$$A \overset{\Delta}{=} \{\Phi = (\varphi^1, \varphi^2) : E_{m|m}(\varphi^1) > 0, E_{m|m}(\varphi^2) > 0 \;, m = \overline{1,4}\},$$

$$\beta \overset{\Delta}{=} \{\Phi = (\varphi^1, \varphi^2) : \exists m' \in [1,4] : E_{m'|m'}(\varphi^1) = 0, E_{m'|m'}(\varphi^2) = 0 \quad \text{and for other}$$
$$m < 5, E_{m|m}(\varphi^1) > 0, E_{m|m}(\varphi^2) > 0\}$$

Theorem 2: Let all distributions P_m , $m = \overline{1,5}$, are different, that is $D(P_l \| P_m) > 0, l \neq m, \; m = \overline{1,5}$, then the following three statements are valid :

a) when given elements $E_{m,m|m,5}$ and $E_{m,m|5,m}$, $m = \overline{1,4}$, meet the following conditions:

$$0 < E_{1,1|5,1} < \min_{l=\overline{2,5}} D(Q_l \circ P_l \| Q_l \circ P_1), \tag{12}$$

$$0 < E_{1,1|1,5} < \min_{l=\overline{2,5}} D(Q_l \circ P_l \| Q_l \circ P_1), \tag{13}$$

$$0 < E_{m,m|5,m} < \min[\min_{l=\overline{1,m-1}} E^*_{m,m|l,m} \,, \min_{l=\overline{m+1,5}} D(Q_l \circ P_l \| Q_l \circ P_m)], m = \overline{2,4} \,, \tag{14}$$

$$0 < E_{m,m|m,5} < \min[\min_{l=\overline{1,m-1}} E^*_{m,m|m,l} \,, \min_{l=\overline{m+1,5}} D(Q_l \circ P_l \| Q_l \circ P_m)], m = \overline{2,4} \,, \tag{15}$$

Then there exists a LAO test sequence $\Phi^* \in A$, the reliability matrix of which $E(\Phi^*) = \{E_{m_1,m_2|l_1,l_2}(\Phi^*)\}$ is defined in (16) and all elements of it are positive,

b) even if one of conditions (12)-(15) is violated , then there exists at least one element of the matrix $E(\Phi^*)$ equal to 0,

c) For given positive numbers $E_{m,m|m,5}$ and $E_{m,m|5,m}$, $m = \overline{1,4}$, the reliability matrix $E(\Phi)$ of the tests $\Phi \in \beta$ necessarily contains elements equal to zero.

Proof: a) Inequalities (11) imply that inequalities (6) hold simultaneously for the both objects.
With, using remark (1) we can rewrite inequality (6) for both objects as follows:

$$0 < E_{1|5}(\varphi^1) < \min_{l=\overline{2,5}} D(Q_l \circ P_l \,\|\, Q_l \circ P_1), \tag{16}$$

$$0 < E_{1|5}(\varphi^2) < \min_{l=\overline{2,5}} D(Q_l \circ P_l \,\|\, Q_l \circ P_1), \tag{17}$$

$$0 < E_{m|5}(\varphi^1) < \min[\min_{l=\overline{1,m-1}} E^*_{m|l}(\varphi^1), \min_{l=\overline{m+1,5}} D(Q_l \circ P_l \,\|\, Q_l \circ P_m)],\ m = \overline{2,4}\ , \tag{18}$$

$$0 < E_{m|5}(\varphi^2) < \min[\min_{l=\overline{1,m-1}} E^*_{m|l}(\varphi^2), \min_{l=\overline{m+1,5}} D(Q_l \circ P_l \,\|\, Q_l \circ P_m)],\ m = \overline{2,4}\ , \tag{19}$$

We shall prove , for example the inequality (18) , which are the consequence of the inequality (14). Consider the tests $\Phi \in A$ such that $E_{m,m|m,5}(\Phi) = E_{m,m|m,5}$ and $E_{m,m|m.l}(\Phi) = E^*_{m,m|m,5}$, $l = \overline{1,m-1}, m = \overline{1,4}$. The corresponding error probabilities $\alpha_{m,m|m,M}(\Phi_N)$ and $\alpha_{m,m|m,l}(\Phi_N)$ are given as products defined by (10.b). Because $\Phi \in A$, then we can write:

$$E_{m|l}(\varphi) \overset{\Delta}{=} \overline{\lim_{N\to\infty}} -\frac{1}{N}\log(1-\alpha_{m|l}(\varphi^1_N))\ ,\ m = \overline{2,4}\ , \tag{20}$$

$$E_{m|l}(\varphi) \overset{\Delta}{=} \overline{\lim_{N\to\infty}} -\frac{1}{N}\log(1-\alpha_{m|l}(\varphi^2_N))\ ,\ m = \overline{2,4}\ , \tag{21}$$

According to (12),(16),(17)and (20),(21) we obtain that :

$$E^*_{m,m|5,m}(\Phi) = E^*_{m|5}(\varphi^1),\ m = \overline{2,4}\ , \tag{22}$$

$$E^*_{m,m|l,m}(\Phi) = E^*_{m|l}(\varphi^1),\ m = \overline{2,4}\ , \tag{23}$$

There for (18) is consequence of (14).
It follows from remark (1) and (16)-(19) that conditions (6) of the theorem (1) take place for both objects.
According to Theorem (1) there exist LAO sequences of tests $\varphi^{*,1}$ and $\varphi^{*,2}$ for the first and the second objects such that the elements of the matrices $E(\varphi^{*,1})$ and $E(\varphi^{*,2})$ are determined according to (5). We consider the sequence of tests Φ^*, which is composed of the pair of sequences of tests $\varphi^{*,1}$ and $\varphi^{*,2}$ and also we will show that Φ^* is LAO and other elements of the matrix $E(\Phi^*)$ are determined according to (11).

From (16)-(19),(6),and remark (1) it follows that the requirements of lemma are fulfilled. With using lemma we can deduce that the reliability matrix $E(\Phi^*)$ can be obtained from matrices $E(\varphi^{*,1})$ and $E(\varphi^{*,2})$ as in (10.a) , (10.b) .
When conditions (12)-(15) take place we obtain according to (10.b) , (5) , (22) , (23) and remark (1) , that the elements $\{E_{m_1,m_2|l_1,l_2}(\Phi^*), m_i \neq l_i, m_{3-i} = l_{3-i}$, $i = 1,2$ of the matrix $E(\Phi^*)$ are determined by relation (11.b). From (10.a) and (10.b) we obtain (11.c). The equality in (11.d) is a particular case of (6). From (11.b) it be follows that all elements of $E(\Phi^*)$ are positive.
Now we show that the compound test Φ^* for two objects is LAO, that is it is optimal suppose that for given $E_{m,m|m,5}, E_{m,m|5,m} = \overline{1,4}$, there exist a test $\Phi' \in A$ with matrix $E(\Phi')$, such that it has at least one element exceeding the respective element of the matrix $E(\Phi^*)$. This contradicts to the fact, that LAO tests have been used for the objects X_1 and X_2.

b)When one of the inequalities (12)-(15) is violated, then from (24) we see, some of elements in the matrix $E(\Phi^*)$ must be equal to zero.

c) When $\Phi \in \beta$, then from (15.a) and remark (1) it follows that the elements $E_{m',m'|5,5} = 0.$

Remark (2): $\forall\, \Phi \in \beta,$ from independence of two objects, the relation (11) and remark (1) we can write:

$$E_{m',m'|5,m'}(\Phi) = \overline{\lim_{N\to\infty}} -\frac{1}{N}\log(1-\alpha_{m'|m'}(\varphi_N^2)) > 0, \qquad (24)$$

$$E_{m',m'|m',5}(\Phi) = \overline{\lim_{N\to\infty}} -\frac{1}{N}\log(1-\alpha_{m'|m'}(\varphi_N^1)) > 0, \qquad (25)$$

And for $\Phi \in \beta$, we obtain :

$$E_{m_1,m_2|l_1,l_2}(\Phi) = E_{m_1|l_1}(\varphi^1) + E_{m_2|l_2}(\varphi^2), \ \ if \quad m_1 \neq l_1 \ , \ m_2 \neq l_2 \ , \qquad (26)$$

$$E_{m_1,m|l_1,m}(\Phi) = E_{m_1|l_1}(\varphi^1), \ E_{m_2,m|m,l_2}(\Phi) = E_{m_2|l_2}(\varphi^2), \ \ if \quad m_i \neq l_i \ , \ m_2 \neq l_2 \ , \qquad (27)$$

With using (26) and (27) we have:

$$E_{m',m_2|m',l_2}(\Phi) = E_{m_2|l_2}(\varphi^2) + E_{m',m'|m',5}(\Phi), \ \ if \quad m_2 \neq l_2 \ , \qquad (28)$$

$$E_{m_1,m'|l_1,m'}(\Phi) = E_{m_1|l_1}(\varphi^1) + E_{m',m'|5,m'}(\Phi), \ \ if \quad m_1 \neq l_1 \ , \qquad (29)$$

We must notice that in this case elements $E_{m',m'|5,5}(\Phi) = 0.$

Remark (3): The similar result may be recived if we take alternatively: $E_{m,1|m,5}$ $m = \overline{2,5}$ instead of $E_{1,1|1,5}$

$E_{1,m|5,m}$ $m = \overline{2,5}$ Instead of $E_{1,1|5,1}$ and other elements also the same.

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