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Modeling, Simulation and Control of a Laboratory Scale Continues Stirred Tank Heater

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ABSTRACT

This paper presents the modelling, controller formulation and simulation studies of a Continues Stirred Tank Heater (CSTH). The rig's analytical model is in a multi input multi output (MIMO) transfer function form considering the physical rig system parameters and limits. Trial and error method is used to design the Proportional Integral Derivative (PID) controller to improve the system's transient response for zero steady state error, as well as minimizing the rise time and overshoot. The system was simulated in MATLAB ® and system response with and without the controller was compared. The P controller improved the time response, response producing a zero steady state error and small steady state time compared to the system without controller. Increasing the temperature loop gain decreases the temperature output overshoot and transient time. While increasing the level loop gain affects the temperature loop negatively by increasing the overshoot and transient time. The P controller has given an unstable response by increasing the temperature output to the maximum limit producing a big steady state error. A PID controller has been offered as the solution to solve this problem

KEY WORDS: Continues Stirred Tank Heater, PID controller, Decoupling, Temperature.

1.INTRODUCTION

Interacting systems are used more commonly than noninteracting systems in the industry. These systems are utilized to have a constant temperature, perfect mixture and plain density. A laboratory size continuous stirred tank reactor (CSTR) in series with a Feeding Tank and a circulation pump can be defined as an interacting system for educational purposes. By using a system which consists of these three elements a wide variety of control problems and issues such as nonlinearity, linearization, coupled and decoupled loops, time delay and others, can be studied and solved. Hence interacting systems have high significance in process control systems for theoretical and practical studies and analysis[1-2]. A process or system dynamic behavior (time dependent or transient) can be elucidated by a set of equations which are defined as the process control model [3]. The system's model and its existence are vital in most of control strategies and control design. For system modeling, two main approaches can be used, system identification and analytical model [4]. One way is developing the mathematical model, which can be created from the dynamic and physical equations of a system [5].

PID controller, in comparison with the other control devices and algorithms, plays an important role in the industry and control purposes [6, 7, 8, 9]. It is known as the first and sometimes the best solution for the control problems and overcomes all other advanced controllers.

In spite of so many advantages such as the capability to be used in most processes control systems, straightforward and uncomplicated in use and simple implementation, sometimes the other controllers can be more useful than PID controllers [9].

In this paper, the dynamic equations of a laboratory interacting system have been derived. This set of equations includes the mass and energy balance equations of the thermal process control rig.

The system is presented as a multi-input multi-output transfer function block diagram. The mathematical model has been simulated. Two PID controllers have been designed, implemented and compared with other. The mathematical model and the system response have been evaluated.

2.THERMAL INTERACTING SYSTEM:

The thermal process control rig consists of two tanks, as shown in Fig. 1. The liquid circulates in a loop from process tank (the upper one) to the feeding tank (the lower one) by using the gravity, and its effects, and in the other side, from the feeding tank to the process tank with the help of a 300 W electrical pump.

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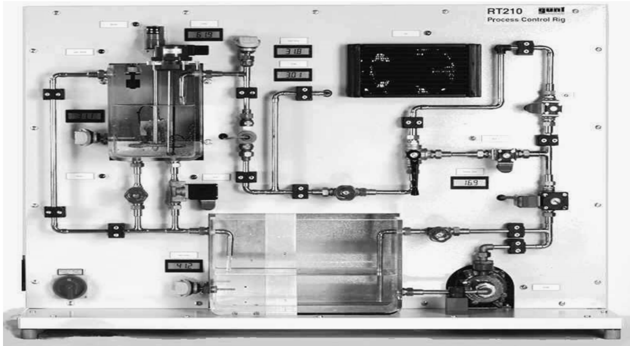


Figure1: Process Control Rig

The schematic diagram of the thermal rig is shown in Fig. 2. The volume of the liquid inside the process tank is controlled by valve 1 and valve 2. A proportional type valve is utilized as valve 1 to control the flows in while a regular open-close type valve is proffered as valve 2 to control the flows out. Process tank inlet feeds by a pipe with length of (150cm) from feeding tank. An electric heater inside the process tank controls the liquid temperature.

This thermal rig is aimed to control the temperature and the level of the liquid (distilled water) inside the process tank, by controlling the inlet flow rate and the heating power.

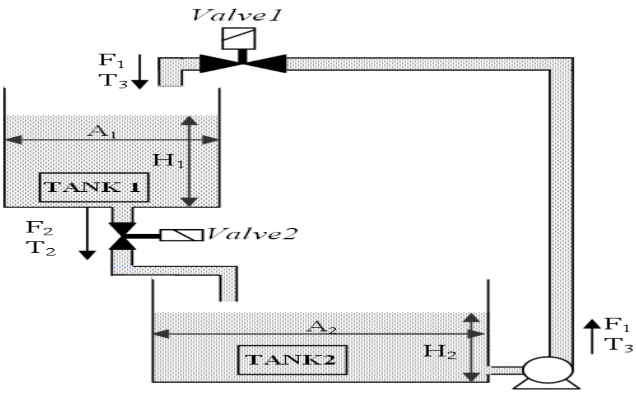


Figure2: Process Control Rig Diagram

The tank's liquid level and liquid temperature were controlled by manipulating of two main inputs which are the liquid inlet flow-rate and heating power. In order to obtain the system transfer functions, both tanks are analyzed separately and then the equations are combined together to form a complete system.

2.1FLOW RATE SYSTEM MODEL OF TANK 1:

Mass conservation equation of tank 1 can be written as [3, 10]:

$$m_1^o - m_2^o = \rho A dh_1 / dt \quad (1)$$

where m_1 is the flow-in mass flow rate, m_2 is the flow-out mass flow rate and h_1 is liquid mass inside the process tank A is the process tank base area.

$$m = \rho f \quad (2)$$

where ρ is the liquid density, f ,is the liquid flow rate.

$$f_2 = h / R \quad (3)$$

where$R$ is the valve resistance.
Substituting these equations gives:

$$f_1^o = A_1 dh_1^o / dt + h_1^o / R \quad (4)$$

(4) is the Steady State System equation. At unsteady state system the equation becomes:

$$\bar{f}_1 = A_1 d\bar{h}_1 / dt + \bar{h}_1 / R \quad (5)$$

By subtracting (4) from (1):

$$(\bar{f}_1 - f_1^o) = A_1 d(\bar{h}_1 - h_1^o) / dt + (\bar{h}_1 - h_1^o) \quad (6)$$

Laplace transformation of (6) yields:

$$\frac{H_1(s)}{F_1(s)} = \frac{R}{A_1 R s + 1} \quad (7)$$

2.2 TEMPERATURE SYSTEM MODEL OF TANK 1:

Heat conservation equation of tank 1 can be written as [3, 10]:

$$\mathrm{m_1 Cp T_1 + Q - m_2 Cp T_2 = Cp \frac{dM_1 T_2}{dt}} \quad (8)$$

where, Q is the heating power, t_1 is the temperature of flow-in liquid and t_2 is the temperature of flow-out and process tank's liquid at steady state; Cp is the specific heat capacity of liquid and Mt is liquid mass inside the process tank.

$$\mathrm{m_1} = \rho \mathrm{f_1}, \ \mathrm{m_2} = \rho \mathrm{f_2}$$
$$M_1 = \rho \cdot V_1 \quad and\ V_1 = A_1 \cdot h_1 \quad (9)$$

where V is the volume.

Substituting (8) into (9)

$$\mathrm{t_1^o f_1^o} + \frac{1}{\rho \mathrm{Cp}} \mathrm{q^o} = A_1 \frac{\mathrm{dh_1^o t_2^o}}{\mathrm{dt}} + \frac{1}{\mathrm{R}} \mathrm{h_1^o t_2^o} \quad (10)$$

Unsteady State Equation is:

$$\bar{\mathrm{t}}_1 \bar{\mathrm{f}}_1 + \frac{1}{\rho \mathrm{Cp}} \bar{\mathrm{q}} = A_1 \frac{\mathrm{d}\bar{\mathrm{h}}_1 \bar{\mathrm{t}}_2}{\mathrm{dt}} + \frac{1}{\mathrm{R}} \bar{\mathrm{h}}_1 \bar{\mathrm{t}}_2 \quad (11)$$

Linearising the nonlinear parts $\bar{\mathrm{t}}_1 \bar{\mathrm{f}}_1$ and $\bar{\mathrm{h}}_1 \bar{\mathrm{t}}_2$ yields [3, 10]:

$$\bar{t}_1 \bar{f}_1 = t_1^o f_1^o + t_1^o(\bar{f}_1 - f_1^o) + f_1^o(\bar{t}_1 - t_1^o)$$
$$\bar{t}_1 \bar{f}_1 = t_1^o \bar{f}_1 + \bar{t}_1 f_1^o - t_1^o f_1^o \quad (12)$$
$$\bar{h}_1 \bar{t}_2 = h_1^o t_2^o + h_1^o(\bar{t}_2 - t_2^o) + t_2^o(\bar{h}_1 - h_1^o)$$
$$\bar{h}_1 \bar{t}_2 = h_1^o \bar{t}_2 + \bar{h}_1 t_2^o - h_1^o t_2^o \quad (13)$$

Substituting (12) and (13) at (11) gives:

$$t_1^o \bar{f}_1 + \bar{t}_1 f_1^o - \bar{t}_1 f_1^o + \frac{1}{\rho \mathrm{Cp}} \bar{\mathrm{q}} =$$
$$A_1 \frac{\mathrm{d}}{\mathrm{dt}}(h_1^o \bar{t}_2 + \bar{h}_1 t_2^o - h_1^o t_2^o) + \frac{1}{\mathrm{R}}(h_1^o \bar{t}_2 + \bar{h}_1 t_2^o - h_1^o t_2^o) \quad (14)$$

Subtracting (10) from (14) yields:

$$\mathrm{t_1^o}(\bar{\mathrm{f}}_1 - \mathrm{f_1^o}) + \mathrm{f_1^o}(\bar{\mathrm{t}}_1 - \mathrm{t_1^o}) + \frac{1}{\rho \mathrm{Cp}}(\bar{\mathrm{q}} - \mathrm{q^o}) =$$
$$A_1 \frac{\mathrm{d}}{\mathrm{dt}}(h_1^o(\bar{t}_2 - t_2^o) + t_2^o(\bar{h}_1 - h_1^o)) + \frac{1}{\mathrm{R}}(h_1^o(\bar{t}_2 - t_2^o) + t_2^o(\bar{h}_1 - h_1^o)) \quad (15)$$

Substituting (6) in (15) and the Laplace transformation give:

$$T_2(s) = \left(\frac{\mathrm{R}/h_1^o}{A_1 R s + 1}\right)\left[(\mathrm{t_1^o} - t_2^o)\mathrm{F_1}(s) + f_1^o \mathrm{T_1}(s) + \frac{1}{\rho \mathrm{Cp}} \mathrm{Q}(s)\right] \quad (16)$$

From Fig. 2 it is clear that T1 the inlet temperature of process tank is the outlet temperature of feeding tank. T1 is a variable and has to be calculated. In order to produce the equation of T1, feeding tank level and temperature system must be analyzed.

2.3FLOW RATE SYSTEM MODEL OF TANK 2:

Mass conservation equation of tank 1 can be written as [3, 8]:

$$m_2^o - m_3^o = \rho A_2 dh_2 / dt \quad (17)$$

Using:

$$m = \rho f \ , \ f_3 = f_1 \ \text{ and } f_2 = h / R \quad (18)$$

Substituting (18) in (17) yields:

$$\frac{h_1^o}{R} - f_1^o = A_2 dh_2^o / dt \quad (19)$$

$$\frac{\bar{h}_1}{R} - \bar{f}_1 = A_2 d\bar{h}_2 / dt \quad (20)$$

By subtracting (19) from (20):

$$\frac{1}{R}\left(\bar{h}_1 - h_1^o\right) - \left(\bar{f}_1 - f_1^o\right) = A_2 d\left(\bar{h}_2 - dh_2^o\right)/ dt \quad (21)$$

Laplace transformation of (21) yields

$$H_2(s) = \frac{1}{A_2 R s} H_1(s) - \frac{1}{A_2 s} F_1(s) \quad (22)$$

Substitute H1(s) value from (7):

$$H_{2(s)} = -\frac{A_1 R}{A_2 \left(A_1 R s + 1\right)} F_{1(s)} \quad (23)$$

2.4 TEMPERATURE SYSTEM MODEL OF TANK 2:

Heat conservation equation of tank 2 can be written as [3, 8]:

$$\mathrm{m_2 Cp T_2 - m_3 Cp T_3 = Cp \frac{dM_2 T_3}{dt}} \quad (24)$$

Using:

$$\mathrm{m_2} = \rho \mathrm{f_2} = \rho \mathrm{h_1/R}, \ \mathrm{m_3} = \rho \mathrm{f_3} = \rho \mathrm{f_1},$$
$$M_2 = \rho \cdot V_2 \quad and\, V_2 = A_2 \cdot h_2 \quad (25)$$

Produces:

$$\mathrm{\frac{1}{R} t_2^o h_1^o - t_3^o f_1^o = A_2 \frac{dt_3^o h_2^o}{dt}} \quad (26)$$

The unsteady state system becomes:

$$\mathrm{\frac{1}{R} \bar{t}_2 \bar{h}_1 - \bar{t}_3 \bar{f}_1 = A_2 \frac{d\bar{t}_3 \bar{h}_2}{dt}} \quad (27)$$

Linearizing the nonlinear parts yields:

$$\frac{1}{\mathrm{R}}\left[\mathrm{h}_1^o(\bar{t}_2 - t_2^o) + t_2^o(\bar{h}_1 - \mathrm{h}_1^o)\right] - \left[t_3^o(\bar{f}_1 - f_1^o) + f_1^o(\bar{t}_3 - t_3^o)\right]$$
$$= A_2 \frac{d\left[\mathrm{h}_2^o(\bar{t}_3 - t_3^o) + t_3^o(\bar{h}_2 - \mathrm{h}_2^o)\right]}{\mathrm{dt}} \quad (28)$$

Laplace transformation of (28) produces

$$\frac{\mathrm{h}_1^o}{R} T_2(s) + \frac{\mathrm{t}_2^o}{R} H_1(s) - \mathrm{t}_3^o F_1(s) - A_2 \mathrm{t}_3^o s H_2(s)$$
$$= \left(A_2 \mathrm{h}_2^o s + \mathrm{f}_1^o\right) T_3(s) \quad (29)$$

Rearranging and substituting (7) and (23):

$$T_3(s) = \frac{\mathrm{h}_1^o / R}{\left(A_2 \mathrm{h}_2^o s + \mathrm{f}_1^o\right)} T_2(s) + \frac{\mathrm{t}_2^o - \mathrm{t}_3^o}{\left(A_2 \mathrm{h}_2^o s + \mathrm{f}_1^o\right)\left(A_1 R s + 1\right)} F_1(s) \quad (30)$$

It is clear from the Fig. 2 that T1 and T3 are the same ,assuming isothermal system, with a time delay due to the liquid flow through the pipe, so;

$$T_1 = e^{-\theta s} T_3 \quad (31)$$

The system block diagram can be performed as in Fig. 3 which matches to the general control figure of multi-input multi-output systems.

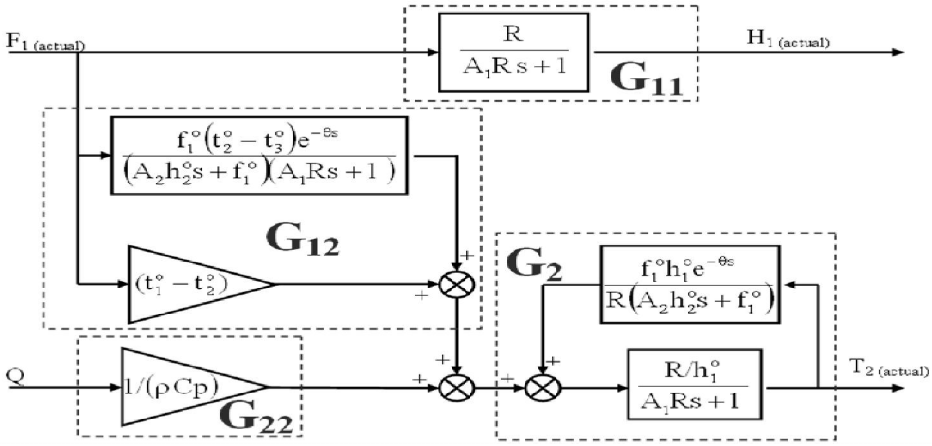


Figure 3: System block diagram

The system constants were considered as using distilled water, and initial conditions were assumed as followed in table 1.

Table 1: Constant and initial values

Constants	Initial Values
$A_1 = 0.185 \times 0.145\, m^2$	$h_1^o = 0.07m$
$A_2 = 0.185 \times 0.335\, m^2$	$h_2^o = 0.18m$
$\rho = 100\, kg / m^3$	$t_1^o = 32\ ^oC$
$Cp = 4186\, J / kg / C$	$t_2^o = 30\ ^oC$
$A_{Pipe} = 0.034159\, m^2$	$t_3^o = 27\ ^oC$
$L_{Pipe} = 1.67\, m$	$f_1^o = 0.000131 m^3 / \sec$

By substituting the values in table 1, the system becomes as shown in Fig. 4.

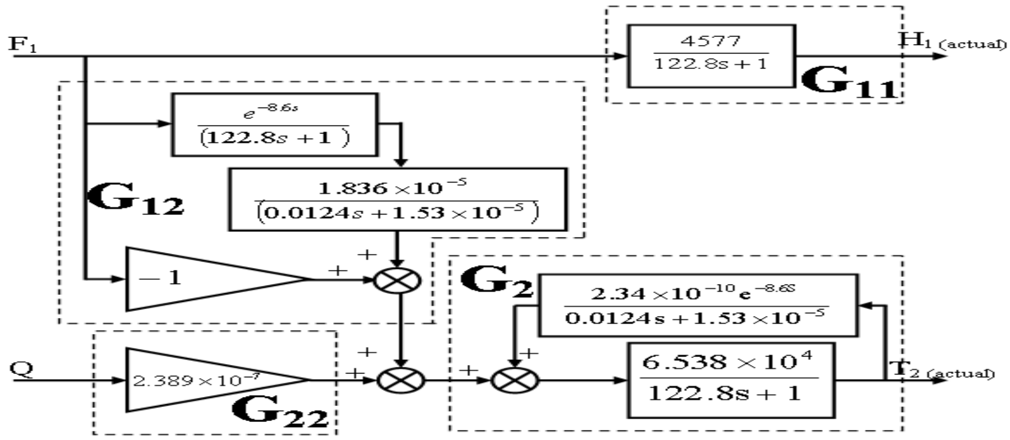


Figure 4: System block diagram

2.5 CALCULATING VALVE RESISTANCE (R):

Since

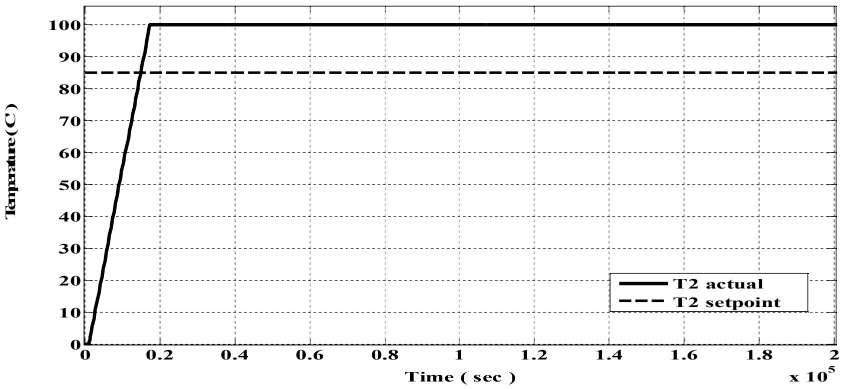
$$df_2 = dh_1 / R \text{ then } R = dh_1 / df_2 \qquad (32)$$

An experiment was conducted stating the relation between f_2 and h_1. The experiment was accomplished by changing process tank input flow and observing the liquid's level after it goes to steady state. The relation between the two variables as a first order equation was found to be $h_1 = 4576.9 f_2 - 0.1719$ producing [3, 8]:

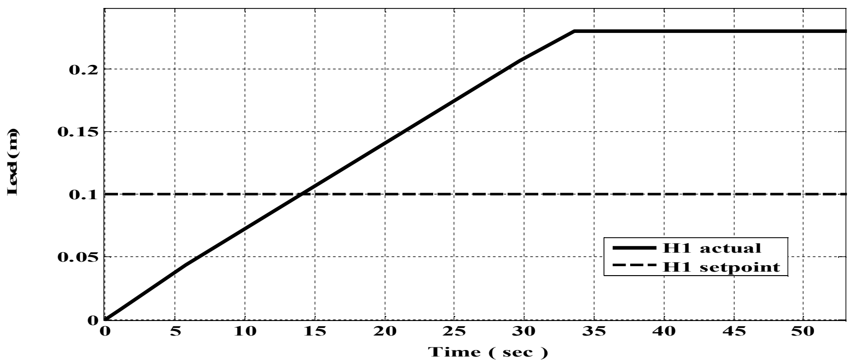
$$R = \frac{df_2}{dh_1} = 4576.9 \quad (33)$$

2.6 SYSTEM'S CONSTRAINTS:

In a process of control design the systems limitations should be considered for more accuracy and reliability. Limitations could be due to actuators physical structure, safety or process requires. In this process, the limitations considered were the inputs and the outputs minimum and maximum values. They were defined as:
Input flow rate: 0- 2.1e-4 m3/sec, heating power: 440 W, output temperature: 0 – 100 °C, liquid level= 0– 23 cm.



(a) Temperature System



(b) Level System

Figure 5: Systems Response with limitations

3.PID CONTROLLER:

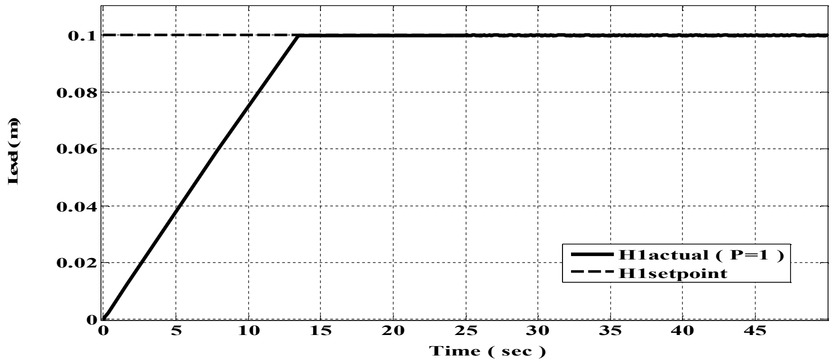
As mentioned in the first section of this paper, a PID controller is considered to control of the CSTH. The structure of the PID controllers which is defined in (34) is formed by three parameters.

$$\text{PID Controller} = K_p + \frac{K_I}{S} + K_D S \quad (34)$$

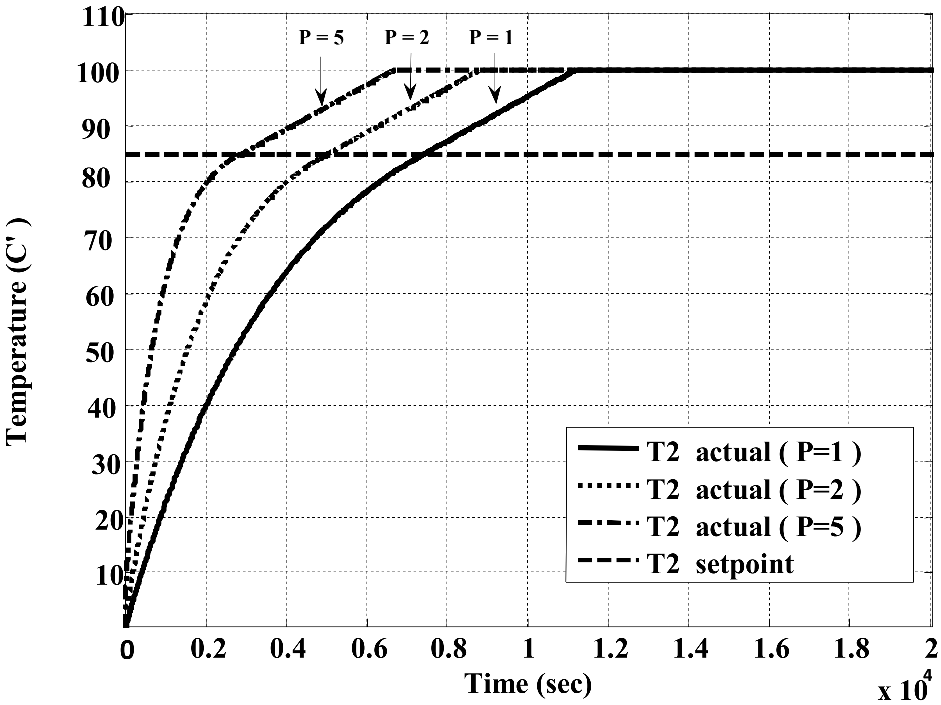
The PID controller should be designed carefully, in the presence the limitations and the interaction between the two loops. A trial and error approach was exercised to design the controller by observing the response of the system due to different control values.

As the level loop is a first order system, a P controller is enough to control the system and also a PI or a PID controller might be required for the heat loop because it is higher in order.

4.Results and Discussion:



(a) Level System



(b) Temperature System

Figure6: Response of the system with PID controller

Fig. 6 shows the system response to the PID controller with different values. It shows that the temperature system reaches the maximum of the output. If there were no limitations on the system output, the output would have increased continuously. This happened because of the limitation and the interaction between the two systems. To control the temperature and level of the process tank properly with PID control, the coupling between the two loops should be solved. The interaction between the two loops affects the efficiency of the PID controller. To solve this problem a decoupling block is used [11].

The decoupling method is easily understood, designed and implemented. As shown in Fig. 7, only one decoupling block is needed to cancel the effect of the input F1 on the output T2 [11].

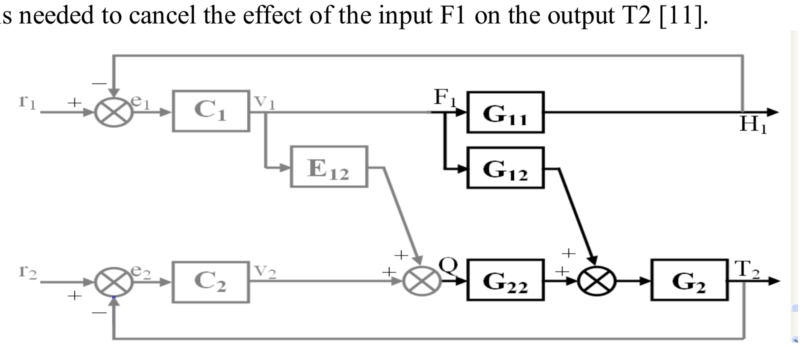


Figure 7: Decoupled system block diagram

From Fig. 7 we can see that [11]:

$$T_2(s) = G_{12}(s)\,G_2(s)\,v_1(s) + G_{22}(s)\,G_2(s)\,v_2(s) \\ \ldots + E_{12}(s)\,G_{22}(s)\,G_2(s)\,v_1(s) \tag{35}$$

The last part is added to eliminate the interaction of F1 (v1). So the addition of the two parts should be zero.

$$G_{12}(s)\,G_2(s)\,v_1(s) + E_{12}(s)\,G_{22}(s)\,G_2(s)\,v_1(s) = 0 \\ E_{12}(s) = -\frac{G_{12}(s)}{G_{22}(s)} = \frac{1.523\,s^2 + 0.01428\,s - 3.06\,e^{-6}}{1.523\,s^2 + 0.01428\,s + 1.53\,e^{-5}} \tag{35}$$

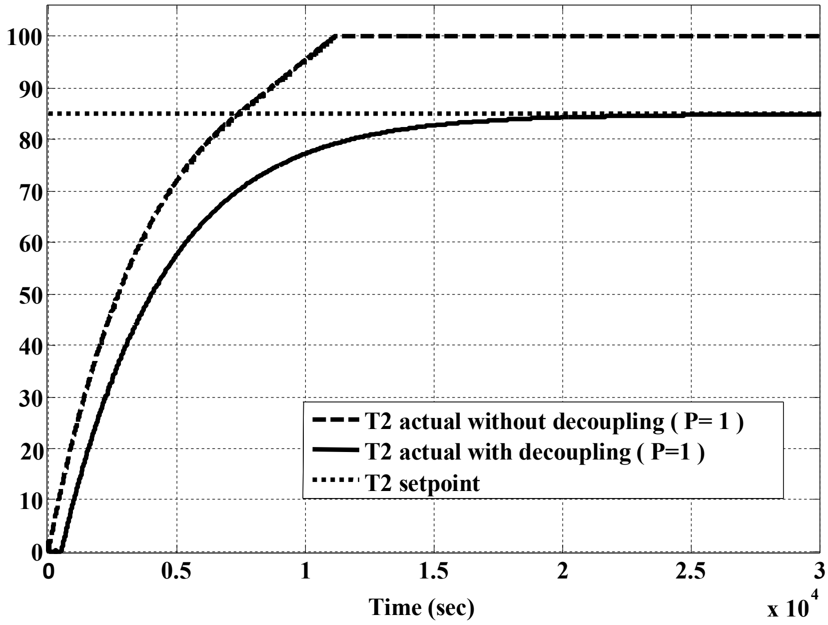
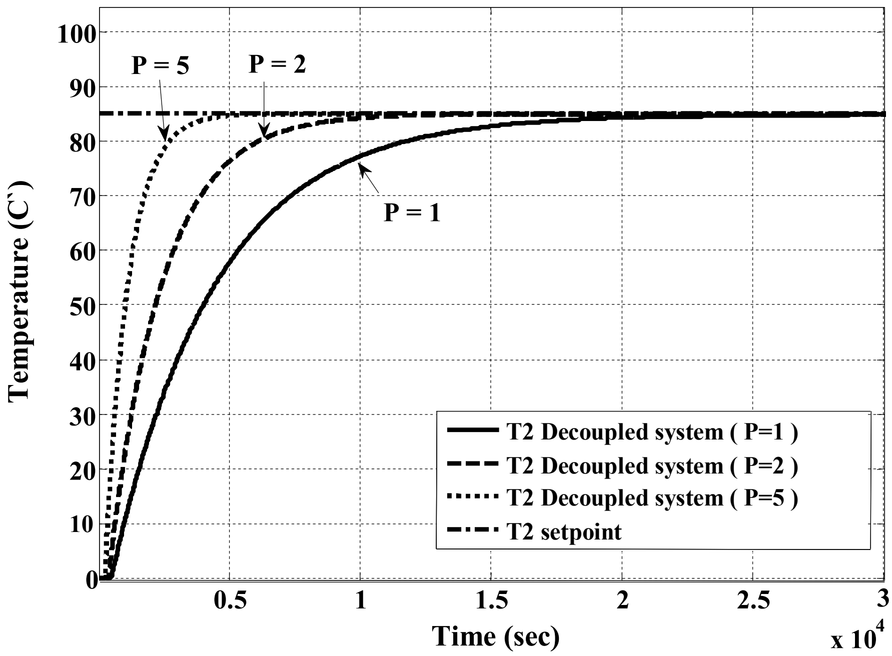
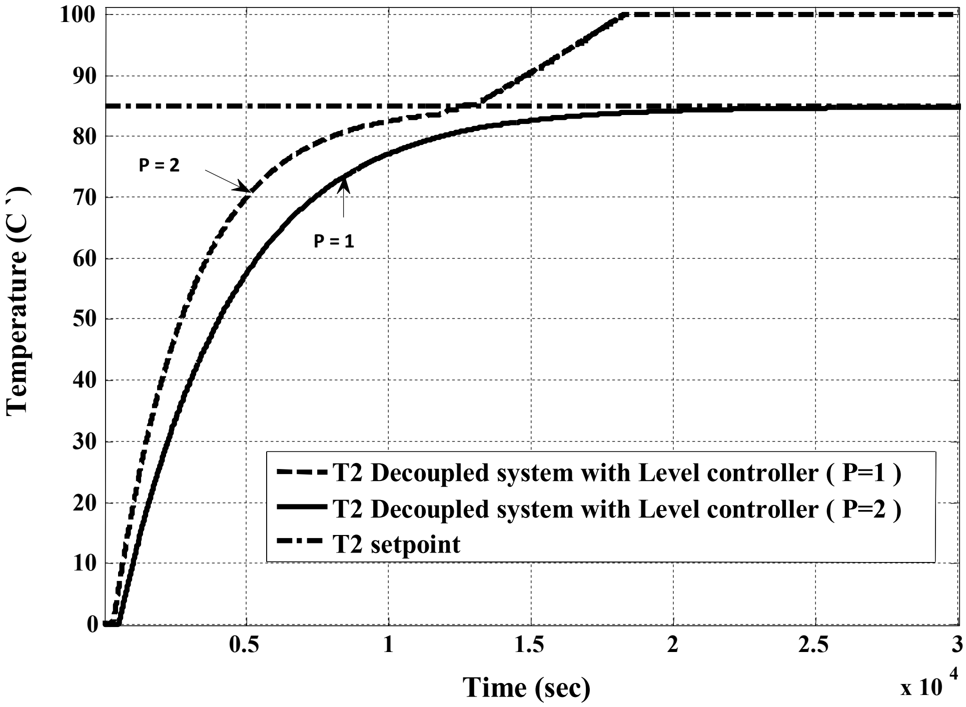


Figure 8: Decoupled system response

Even though, the decoupled system response, shown in Fig. 8, shows a more stable temperature system with less steady state error, better solutions are required to improve system's accuracy and response time since the system is showing a slow response taking up to 5×105 seconds to reach the steady state. A P and a PI controller are placed in the system loop.



(a) Temperature system response whit(out) level controller

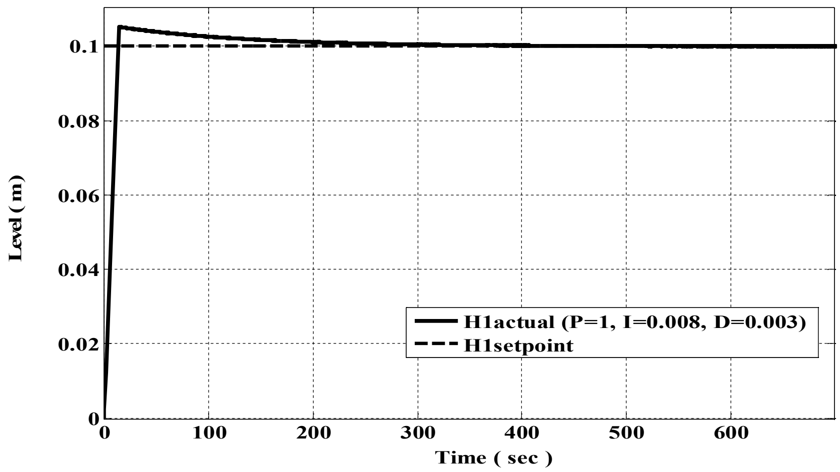


(b) Temperature system response with level controller

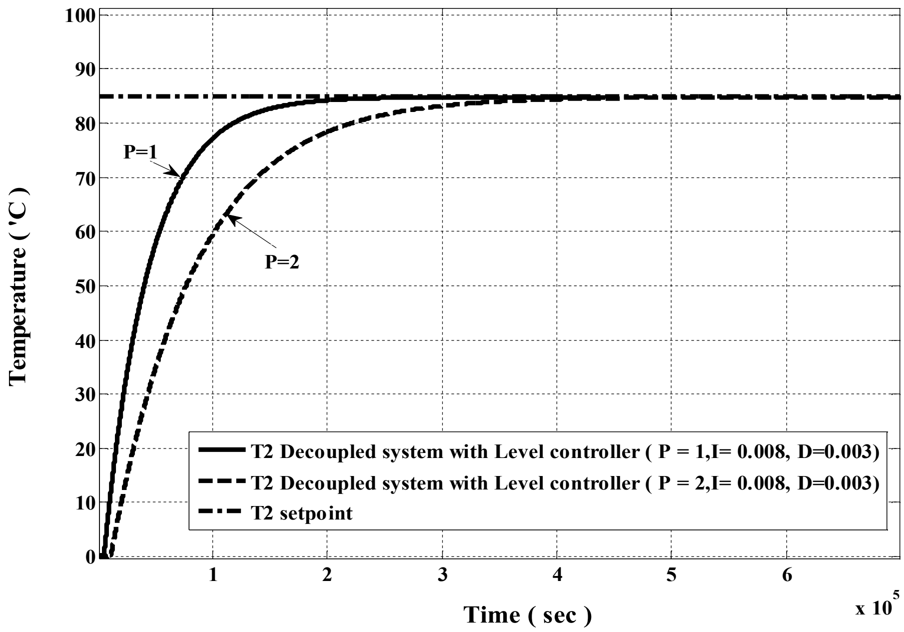
Figure 9: Decoupled system response with P controller

Fig. 9 shows the step response applied on the set point of the system using P controller of different values compared to the closed loop system with(out) controller (P=1). The P controller has shown a huge improvement in the system response speed almost 10times the speed of the closed loop system with no controller. Fig. 9-a shows the response with the level control loop set to one. The P controller with a low value produced a very small (almost zero) steady state error with no overshoot. While as shown in Fig. 9-b, the increase of the level loop gain increased the steady state error in the temperature output. If the interaction between the two loops was completely removed, the system would produce a zero steady state error response with no overshoot; but because of the system limitation, the decoupling block was not able to eliminate the interaction between the two loops completely.

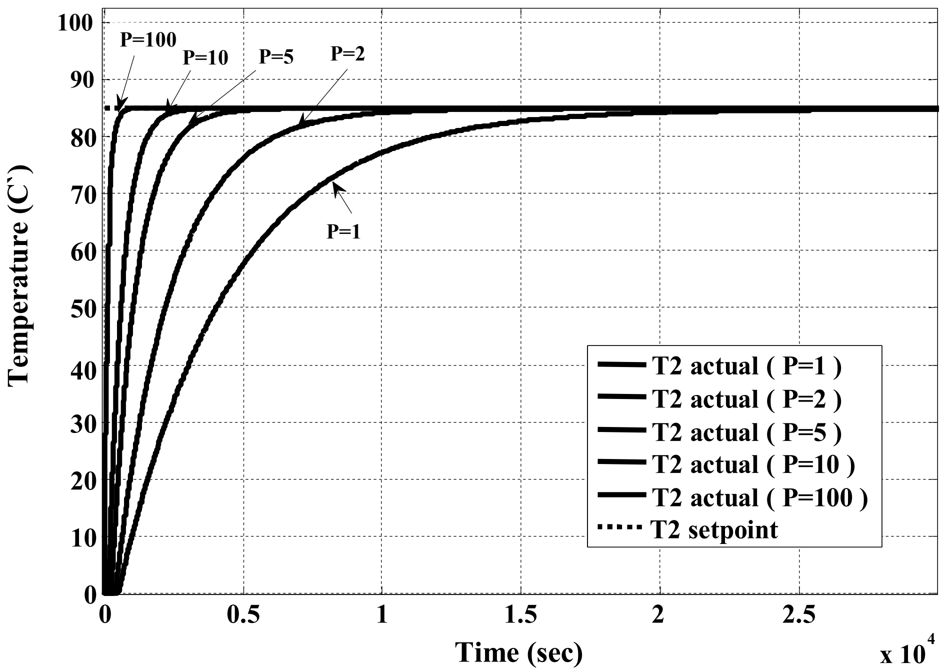
By adding Integral gain and Derivative gain into the Level controller, the interaction effect can be removed.



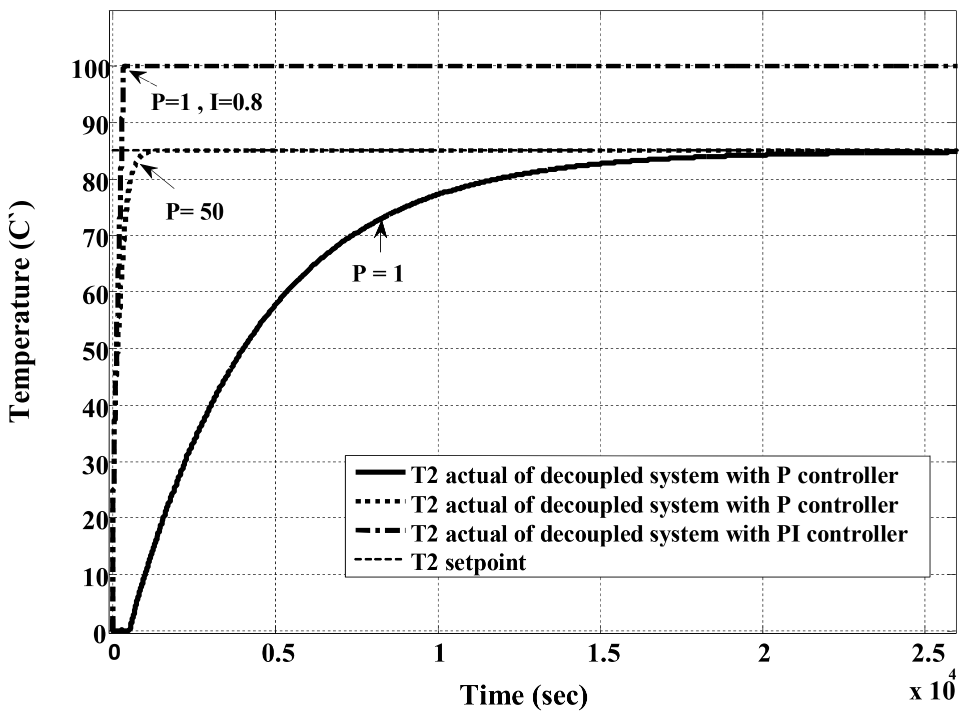
(a) Level system response to PID controller in level



(b) Temperature system response with PID controller in level



(c) Temperature system response to P controller



(d) Temperature system response to P and PI controller

Figure 10: Decoupled system response with P and PI controllers

Fig. 10 shows the response of the system with no controller, P controller, and PI controller. The PI controller in temperature system shows a very fast response with a very small transient time; but the error increased instead of decreasing due to the system's constraints which were clipping the control signals.

5.CONCLUSIONS

The analytical modeling of the thermal process plant has been presented by using equations of mass and energy conservation and followed by the steady state equations and unsteady state equation. The MIMO system transfer functions have been resulted. In an interacting system a regular PID controller might not be enough alone, which cause for the usage of decoupling blocks. The decoupled system has shown improvement in the system steady state error, but due to the existence of the system constraints the decouple block did not terminate the coupling completely. The P controller produced a response with negligible steady state error in the output temperature with unity gain in the flow loop. Increasing the flow loop gain affected the temperature loop negatively and Integral and Derivative control elements, in the level loop, were needed to bring the system's error to zero.

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