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A Stochastic Knapsack Model with The Capacity Following an Additive form of Contagious Distribution

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ABSTRACT

We consider a dynamic stochastic knapsack problem with the aim of minimizing the cost of the selected items whose weights follow a mixture of Poisson and Exponential distribution using additive model. A graphical presentation of our contagious distribution with different values of both parameters and range of X is made to show their behaviors. We also propose an algebraic model for the problem.

KEY WORDS: Stochastic, Dynamic, deterministic, Contagious, Knapsack problem.

INTRODUCTION

The knapsack problem is an aspect of optimization problem that is well studied because of it various application in different aspect of human endeavor. It is applied in cutting stock problem, capital budgeting problem, cargo loading problem, etc. A situation where the parameters of the knapsack problem are known a priori constitute what is termed the deterministic knapsack problem, whereas, when any of the parameter is not made known before the commencement of the exercise is termed the stochastic knapsack problem. The knapsack problem can be static or dynamic. In static stochastic knapsack problems, a set of items is given, but the rewards and / or sizes are unknown. Whereas, in dynamic stochastic knapsack problems, the items arrive over time, and the rewards and/or sizes are unknown before arrival. Decisions are made sequentially as items arrive ; Kosuch & Lisser (2008). Literature has shown that much work has been done in the static aspect of the knapsack problem than the dynamic. Different authors have come up with different illustrations as a way of defining the knapsack problem; Martello & Toth (1990) illustrated that "the knapsack problem" can be likened to a hitch-hiker who intends to fill his knapsack by selecting from among various possible objects which will give him a maximum comfort. They formulated the problem mathematically by numbering the objects from 1 to n and introducing a vector of binary variable x_j $(j = 1,2,...,n)$. Where $x_j = \begin{cases} 1 & \text{if object } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$, then if p_j is a measure of the comfort given by object j, w_j its size and c the size of the knapsack, the problem will now be

$$Maximize \sum_{j=1}^{n} p_j x_j$$

$$Subject\ \ to\ \ \sum_{j=1}^{n} w_i x_i \ \le c$$

Whereas, Kosuch & Lisser (2008) in their own way defines knapsack problem as a combinatorial problem: each item is modeled by a binary decision variable $x \in \{0,1\}$ with x=1 if the item is chosen and 0 otherwise; He added that the knapsack problem is generally linear, that is both the objective function and the constraints are linear.

However, in this work , we are interested in a situation where the items we are selecting in order to minimize the cost in the firm comes from two distinct population, and we employ additive model of contagious distribution to address the problem. By minimizing the cost, we are maximizing the profit of our limited capacity. Our reason for the contagious distribution is that, we assume the random variable x takes up distinct values, $x_1, x_2,..., x_n$ with positive probabilities and also take up (assume) all values in an interval; say $a \le x \le b$. The probability

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distribution that will be obtained here will be as the result of combination of both discrete and continuous distribution; see Meyer (1965).

CONTAGIOUS DISTRIBUTION

From the brief overview of the mixed distribution mentioned in the last paragraph above, it is worth noting that this mixture of distribution can take any form; mixture of two or more continuous distribution or discrete distribution or even both through either multiplicative or additive model. According to Sandoval-Escalante (2007), the use of a mixture of probability distribution functions for modeling samples of data coming from two populations have been proposed long time ago by Mood et al(2006) as

$$\Pr(X \le x) = F(x) = PF_1(x) + (1-P)F_2(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where P is a factor used to weigh the relative contribution of each population $(0 < P < 1)$, and $F(x)$ is the composite exceedance probability. $F_1(x)$ and $F_2(x)$ are the component. He adopted the model in equation (1) and used it in his work on a mixed distribution with Extreme value distribution (EV1) and General Extreme Value component (GEV) component for analyzing heterogeneous samples as

$F(x) =$

……………………………….(2)

The assumption was that the first and second populations behave as Gumbel distribution (EV1) and (GEV) distributions respectively.

In line with our equation (1) by Mood et al, Stirzaker (1994) defines mixed distribution as let $f_1(x)$ and $f_2(x)$ be density functions, and let $f_3(x) = \lambda f_1(x) + (1-\lambda) f_2(x)$. Where $0 \le \lambda \le 1$, then $f_3(x) \ge 0$ and $\int f_3 = \lambda \int f_1 + 1 - \lambda \int f_2 = 1$.

Hence f_3 is a density function and is said to be a mixture of f_1 and f_2. There are other related works on this mixed (additive model) as shown in the literature review.

LITERATURE REVIEW

Willbaut & Hanafi (2008) presented several variants of knapsack problems mostly derived from the classical knapsack problem. They mentioned that the problems can be obtained by modifying the constraints or changing the objective function. Appropriate techniques that were found to be successful in solving these problems were briefly reviewed. They also briefly discussed hybrid methods that combine the strengths of different methods such as exact and heuristics. Argali & Geunes (2009) considered a stochastic resource allocation problem that generalizes the knapsack problem to account for random item weights that follow a poisson distribution. They provided a polynomial-time solution for the continuous relaxation of this problem and a customized branch – and – bound algorithm to solve the binary version of the problem. They carried out computational test on a set of randomly generated problem instances which showed that their algorithm performs favourably when compared with a commercial mixed-integer nonlinear solver.

Kosuch & Lisser (2008) studied, solved and compared two different variants of a stochastic knapsack problem with random weights. They applied a branch-and-bound algorithm to solved continuous sub-problems in order to provide upper bounds. They also used a stochastic gradient method for solving the continuous stochastic knapsack problem with simple recourse (SRKP) and a second order-cone-programming (SOCP) algorithm as well as a stochastic Arrow-Hurwics algorithm for solving the constrained version of the continuous knapsack problem. Fortz et al (2008) in their work on the knapsack problem with Gaussian weights stated that the main difficulty in two stage stochastic programming with real recourse is the number of scenarios to consider, resulting in a huge number of variables and constraints. They overcome this difficulty for the knapsack problem with penalty recourse by considering only the Gaussian random variables. They simplified the problem and avoided performing multiple integrations for evaluating the objectives using a summation property. Then, they gave a complexity results for a stochastic version of the sub-set-sum problem, and for the general problem with constant weights and capacity uniformly distributed.

Kleywegt & Papastavrou (1998) in their work on the dynamic and stochastic knapsack problem (DSKP) were able to define and analyzed the problem. For the infinite horizon case it was shown that a stationary deterministic threshold policy is optimal among all history-dependent deterministic policies. For the finite horizon case, it was shown that a memoryless deterministic threshold policy is optimal among all history-dependent deterministic policies. Also, the general characteristic of the optimal policies and optimal expected values were derived for different cases.

Kress et al (2007) presented a new knapsack related combinatorial problem termed minmax multidimensional knapsack problem (MKP) that was motivated by a military logistics problem. The logistic problem was to determine an optimal deployment of inventories that satisfy certain operational requirement in a two days scenario. They showed that the resulting two-period stochastic programming problem can be solved by solving a series of multidimensional knapsack problems. They developed a practically efficient algorithm for solving the multidimensional knapsack problem.

However, Cohn & Barnhart (1998) reported that the study of stochastic knapsack problems is limited almost exclusively to two cases in which the value of the objects are random, and in which the objects themselves arrive as part of a stochastic process. For more detailed coverage on these problems, see; Carraway et al (1993), Steinberg & Parks (1979), Henig (1990), Righter (1989), Kleywegt & Papastavrou (1998),etc. it was on this note that we decided to expand on this area by introducing the contagious distribution to solve the problem.

Researchers have made use of contagious distribution to solve different problems. For instance, Thamerus (1996) considered the case where a latent variable X cannot be observed directly and instead a variable W = X + U with a heteroscedastic measurement error, U was observed. It was assumed that the distribution of the true variable X is a mixture of normals and a type of the EM algorithm was applied to find approximate maximum likelihood estimates of the distribution parameters of X.

Decarlo (2002) carried out an extension of signal detection theory (SDT) that incorporate mixture of the underlying distributions. The mixture was motivated by the idea that a presentation of a signal shifts the location of an underlying distribution only if the observer is attending to the signal; otherwise, the distribution was not shifted or was only partially shifted. Their mixture of SDT provided a general theoretical framework that offers a new perspective on a number of findings. It can also account for non-linear normal receiver operating characteristic curves.

Willmot (1986) considered the distribution of total claims payable by an insurer when the frequency of claims is a mixed Poisson random variable. He showed how in many cases, the total claims density can be evaluated numerically using simple recursive formula (discrete or continuous). He also showed how the results might be used to derive computational formulae for the total claims density when the frequency distribution is either from the Neyman class of contagious distribution, or class of negative binomial mixtures.

Arana and Leon (2004) considered the performance of a model of mixture normal distributions for dichotomous choice contingent valuation data, which allows the researcher to consider unobserved heterogeneity across the sample. The model was flexible and approaches a semi-parametric model, since any empirical distribution can be represented by augmenting the number of mixture distributions. They found that the mixture of normal model reduces bias and improves performance with respect to an alternative semi-parametric model, particularly when the sample is characterized by heterogeneous preferences.

Literature has shown that most studies on the static stochastic knapsack problem concentrates on normally distributed rewards, (Morton & Wood (1998), Goel & Indyk (1999), Sniedovich (1980), Carraway et al (1993)) because the normality assumption covers a wide range of practical applications and at the same time, makes the static problem more tractable. In this work we are going to deviate completely from the use of normal distribution to a mixture of Poisson and Exponential distribution.

THE CONTAGIOUS DISTRIBUTION MODEL

We employ the additive model of the contagious distribution as shown in Sandoval (2007).

$$\Pr(X \le x) = F(x) = PF_1(x) + (1-P)F_2(x)$$

Let $F_1(x)$ be a Poisson distribution and $F_2(x)$ be an exponential distribution. Therefore $F(x) =$

$$P\frac{e^{-\lambda}\lambda^{x}}{x!} + (1-P)\lambda e^{-\lambda x} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The graphical presentation of our new distribution with different values of $P, \lambda, and\ ranges\ of\ \ x$ is as shown below:

p=0.2;

f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

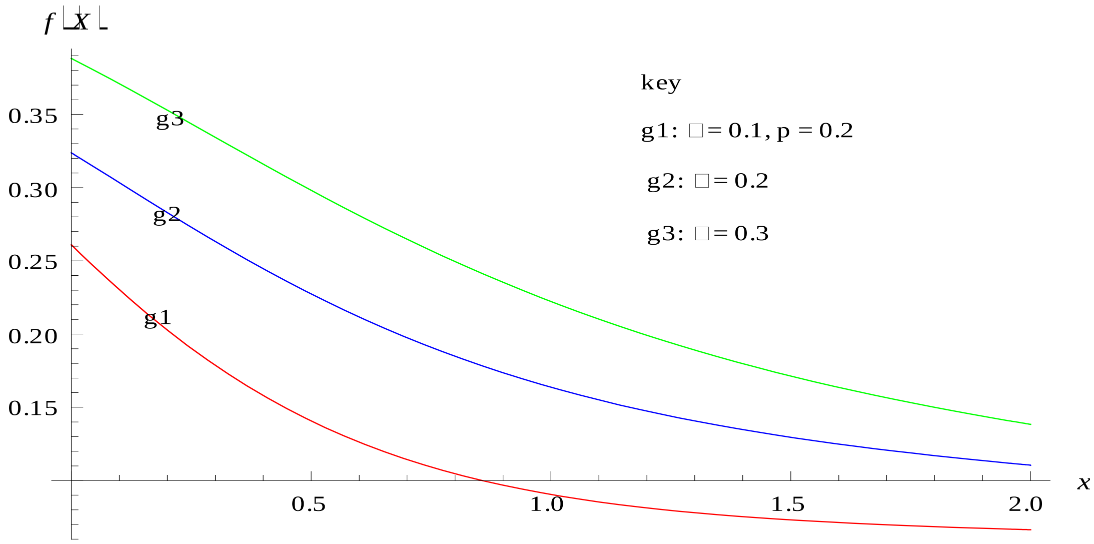

Fig. 1
p=0.5;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

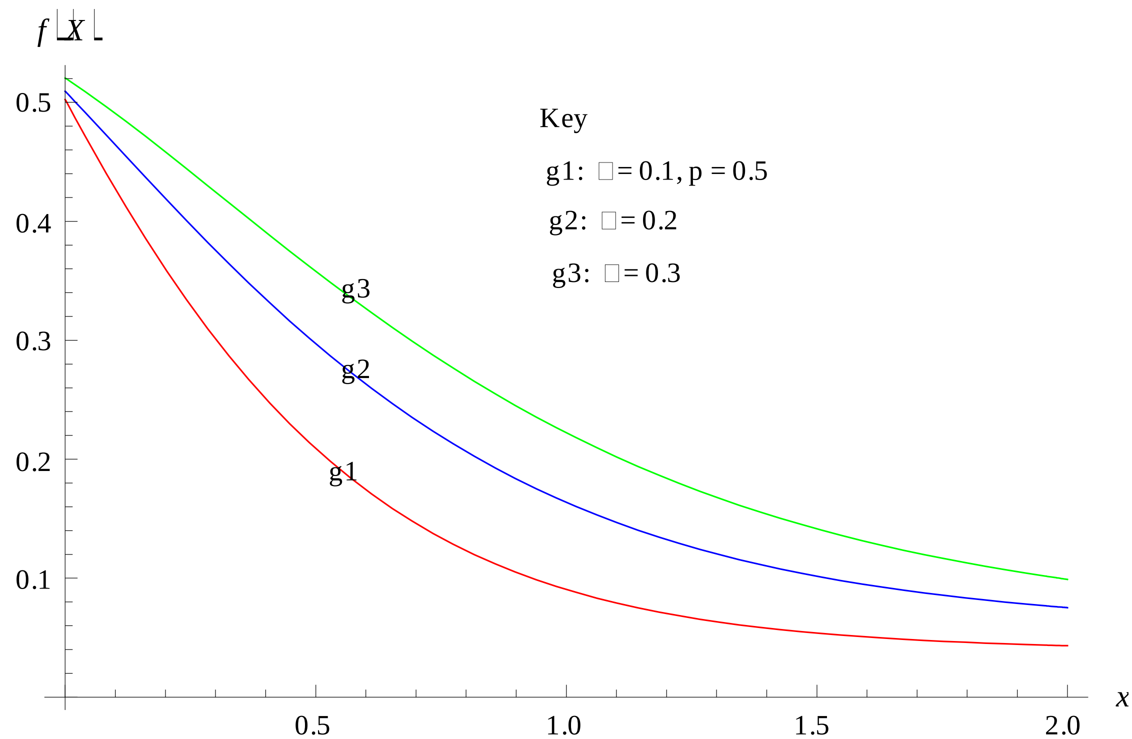

Fig. 2
p=0.8;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

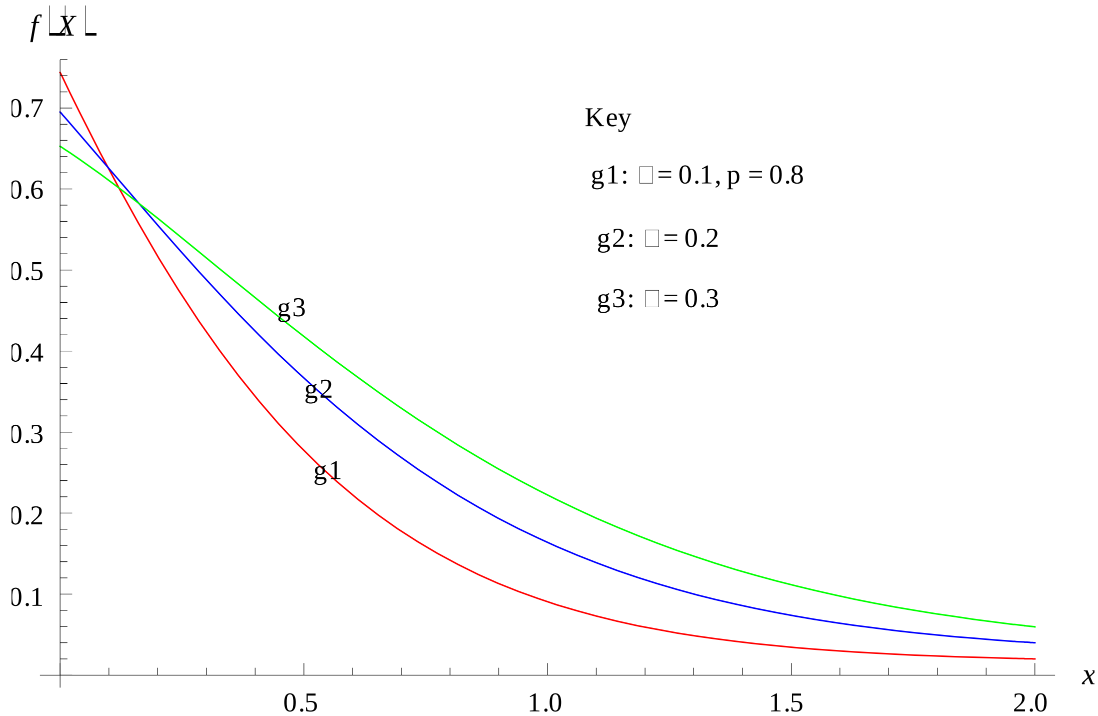

Fig. 3

p=0.2;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

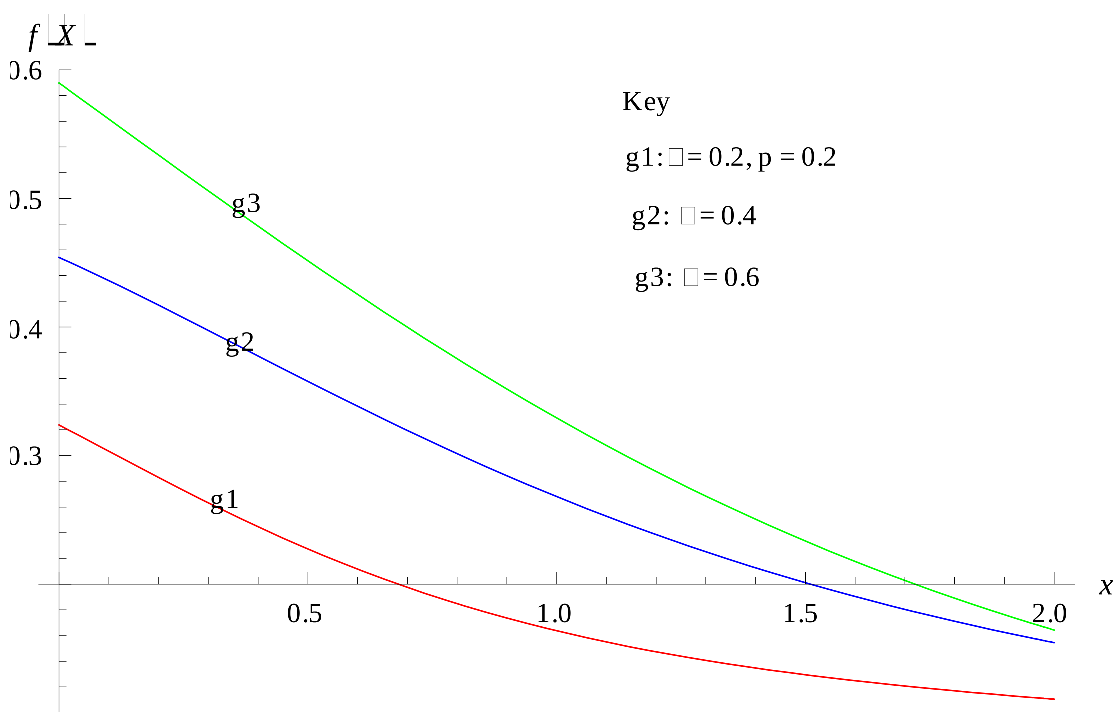

Fig. 4
p=0.5;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

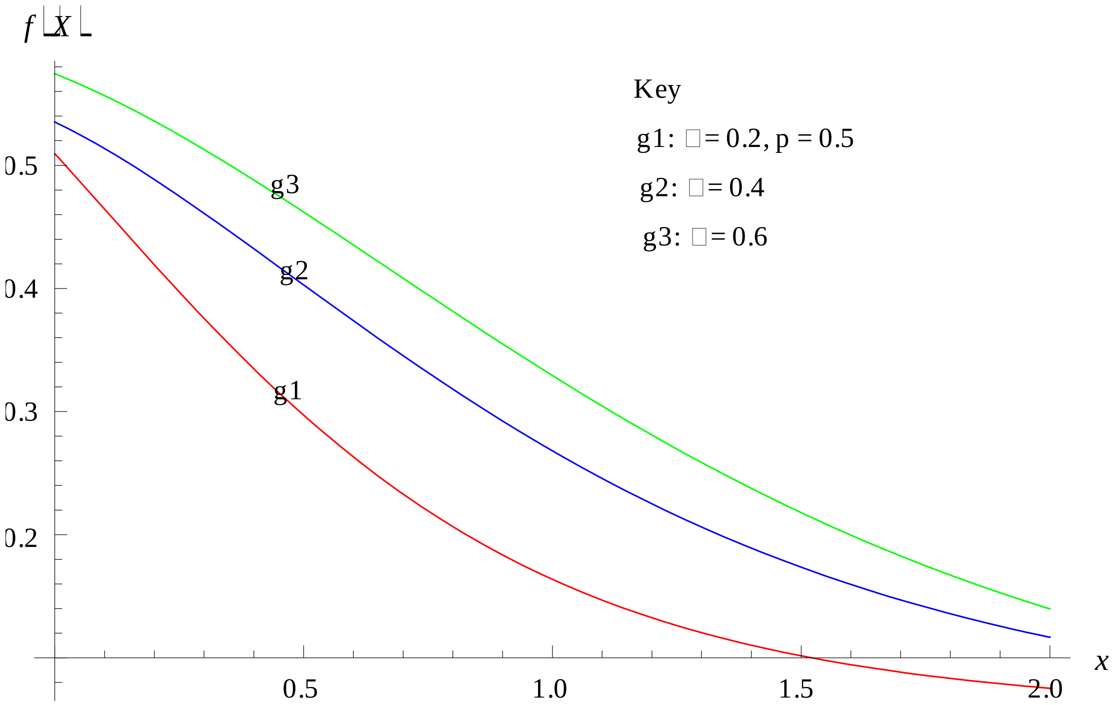

Fig. 5

p=0.8;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

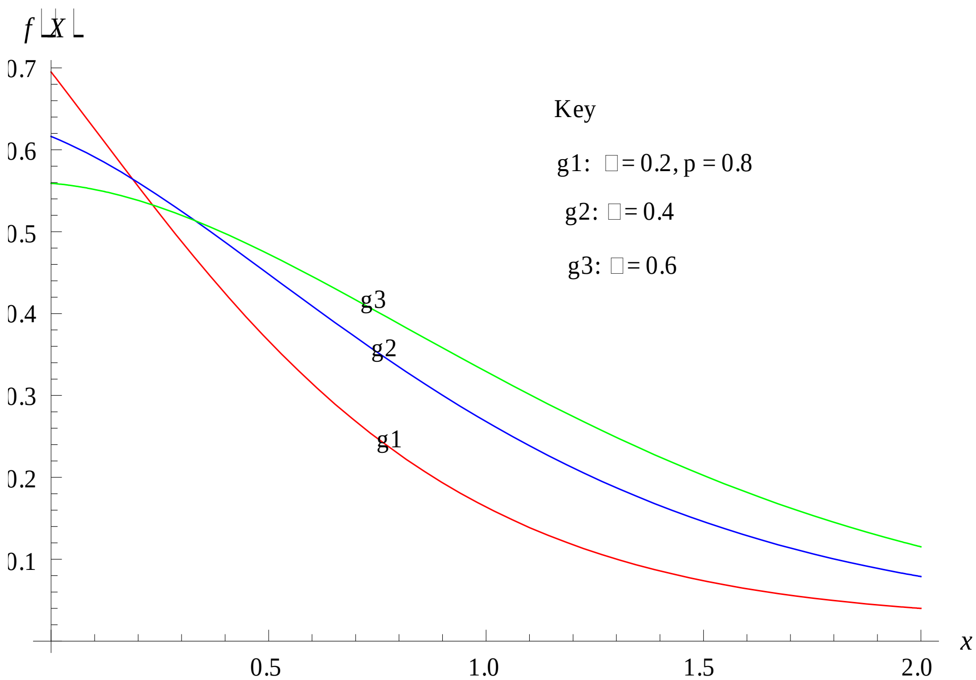

Fig. 6
p=0.2;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

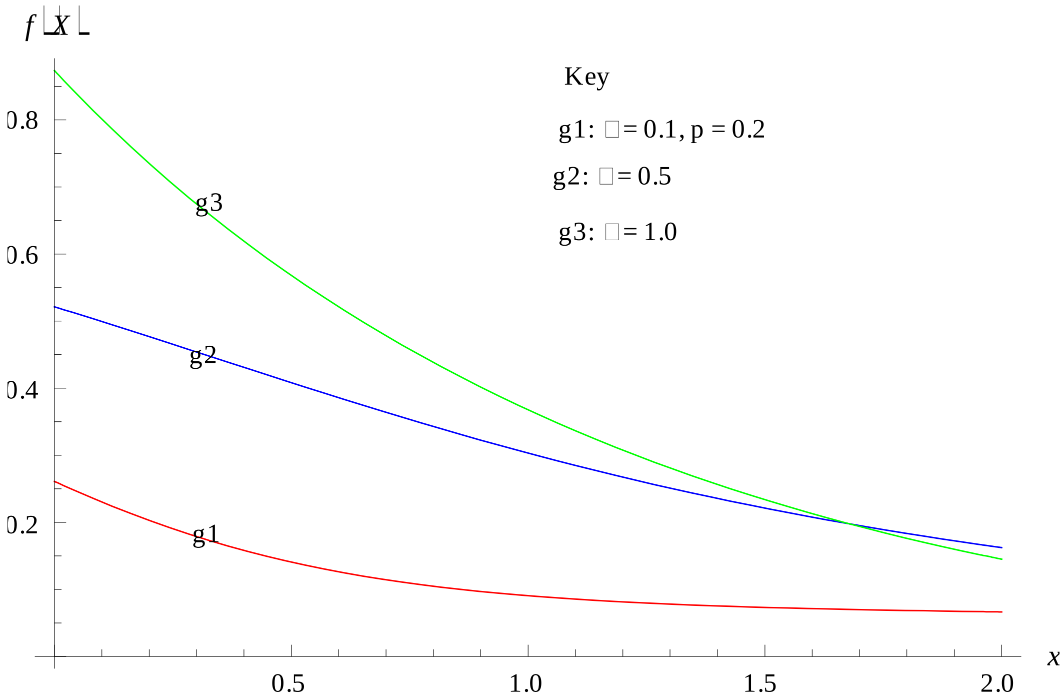

Fig.7
p=0.5;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

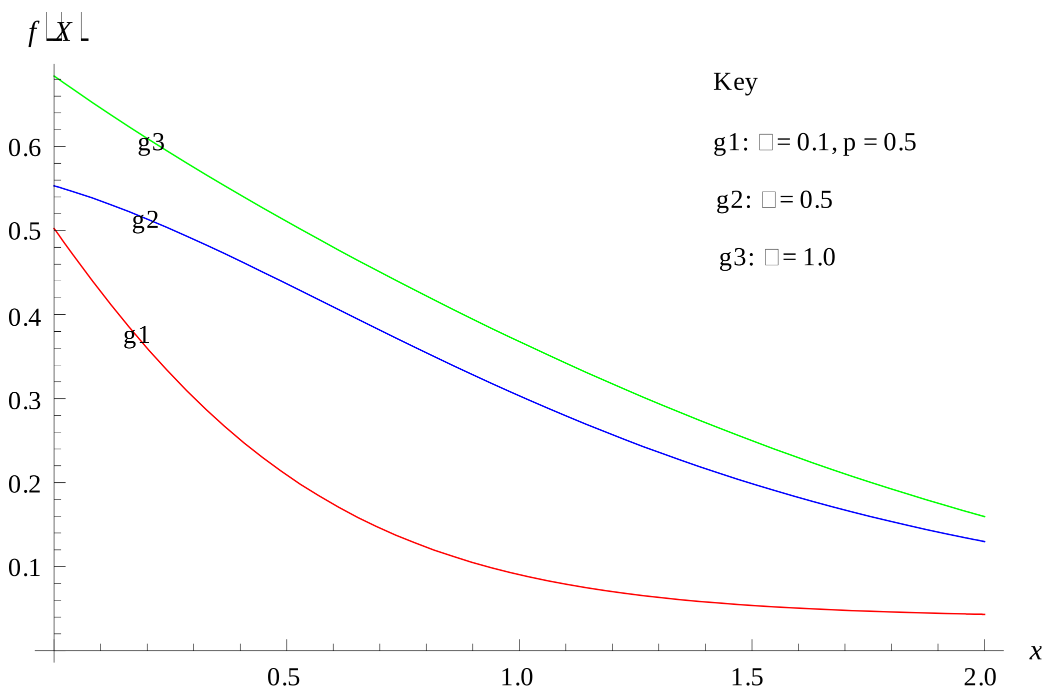

Fig. 8
p=0.8;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

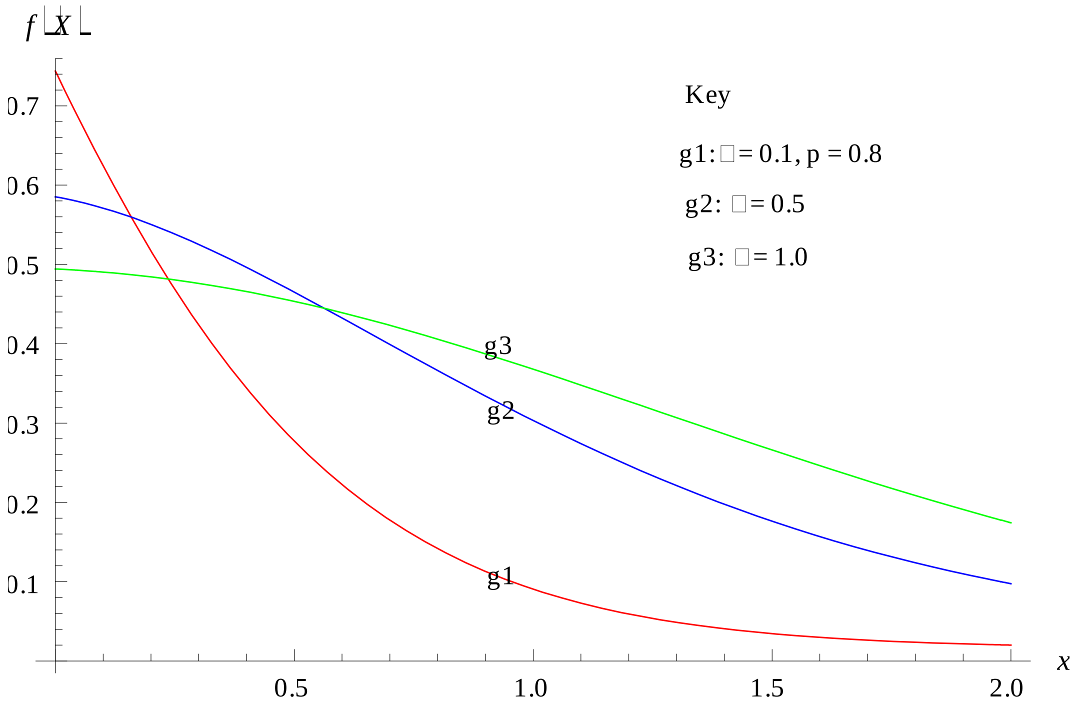

Fig.9

p=0.2;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

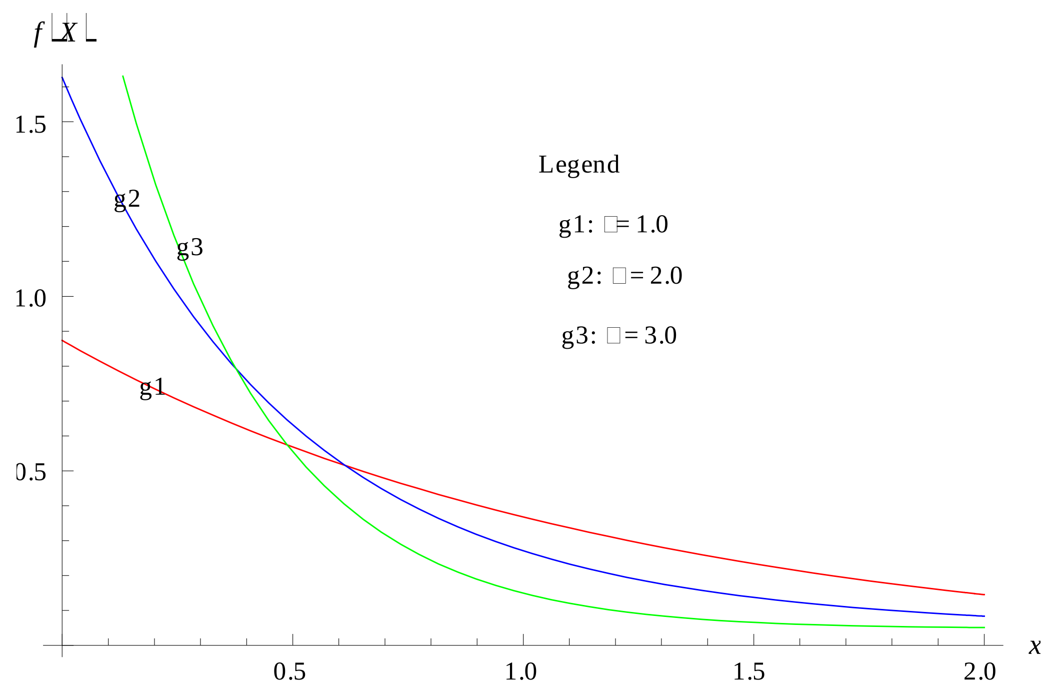

Fig. 10
p=0.5;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

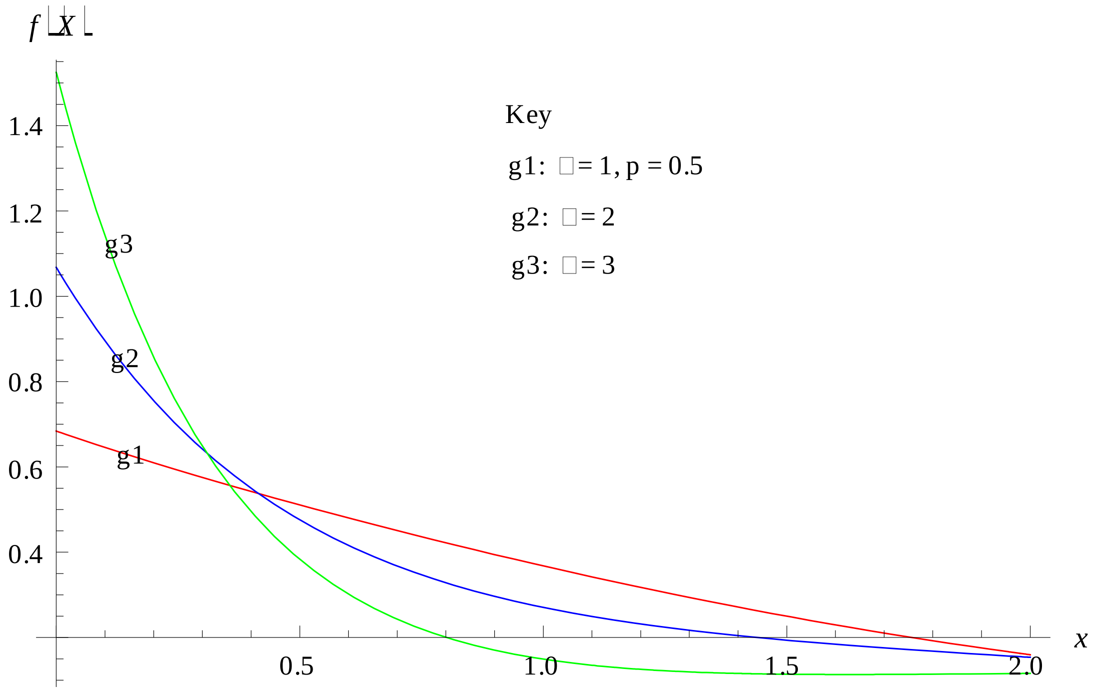

Fig. 11

p=0.8;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

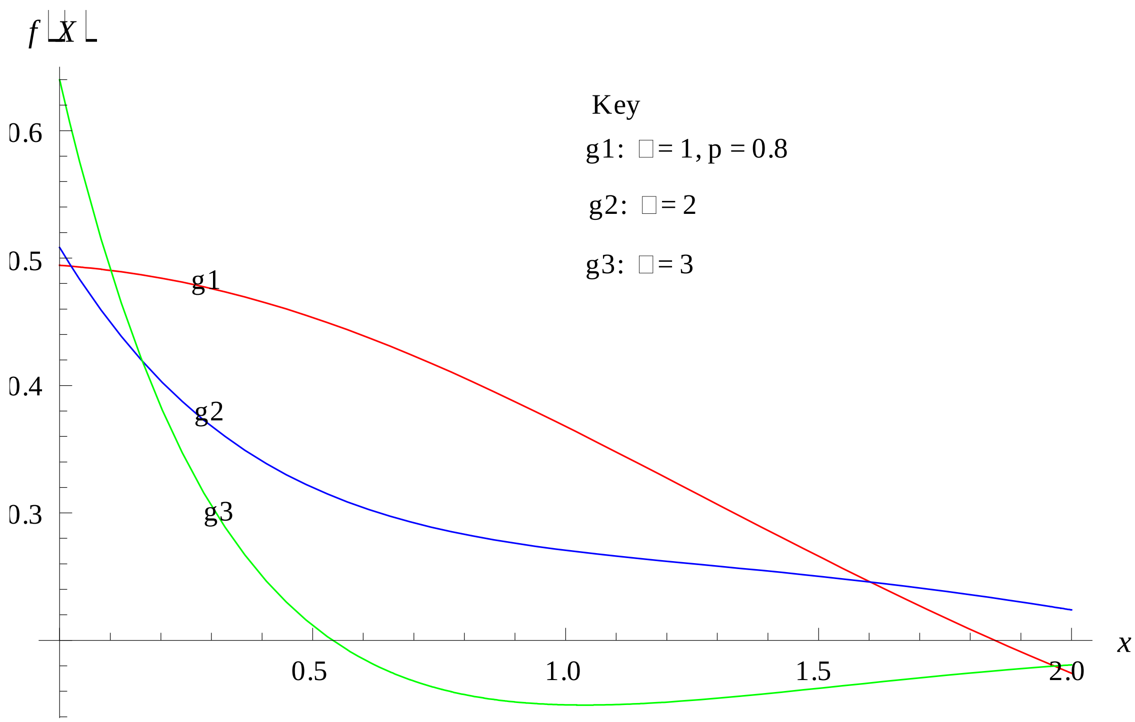

Fig. 12
p=0.2;
f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

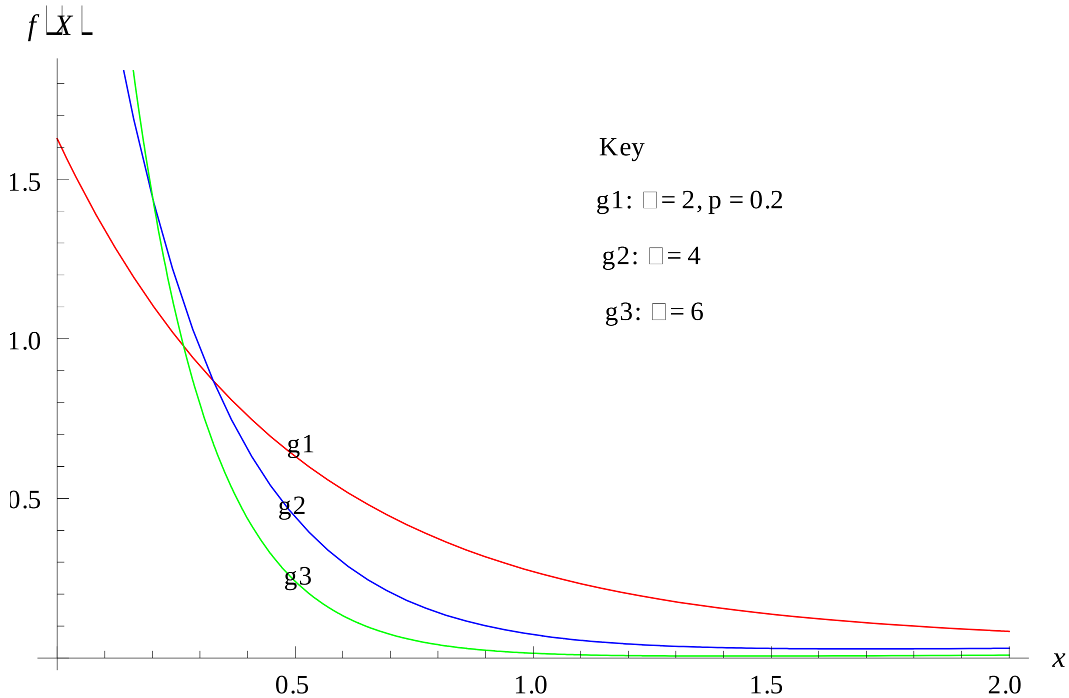

Fig. 13

p=0.5;

f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

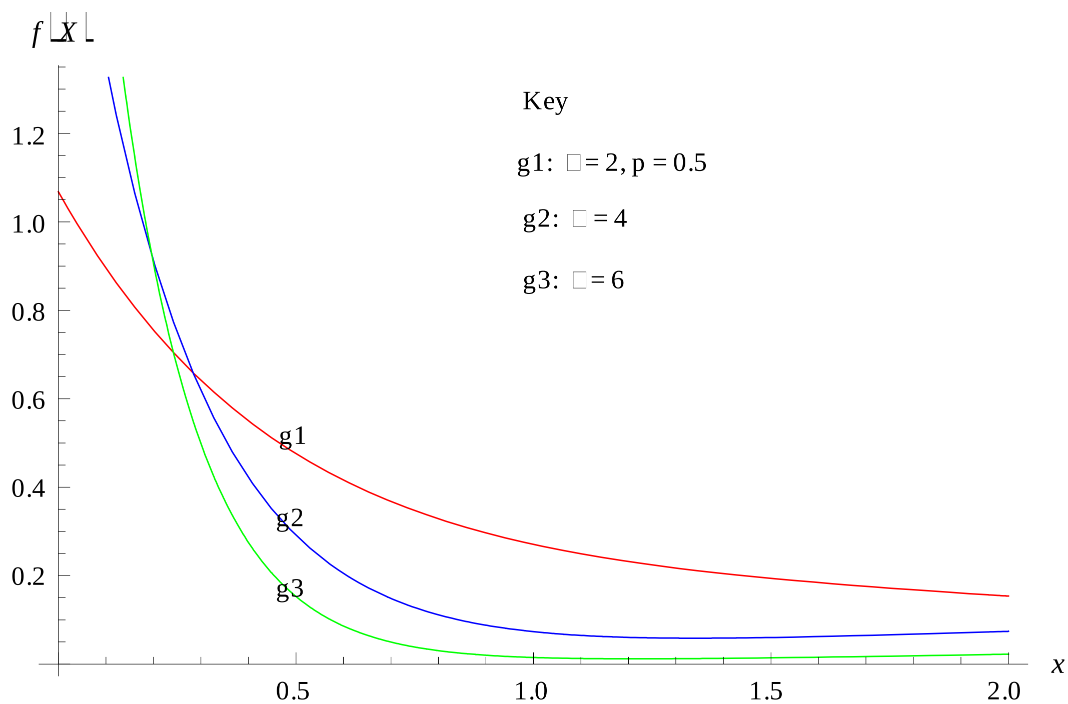

Fig. 14

p=0.8;

f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

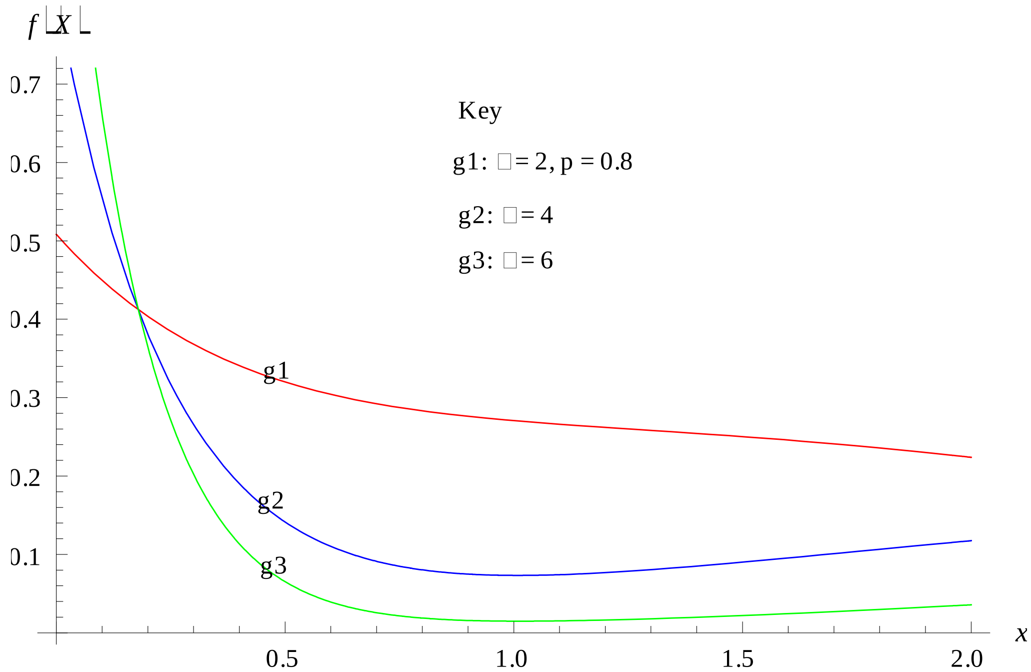

Fig. 15

p=0.2;

f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

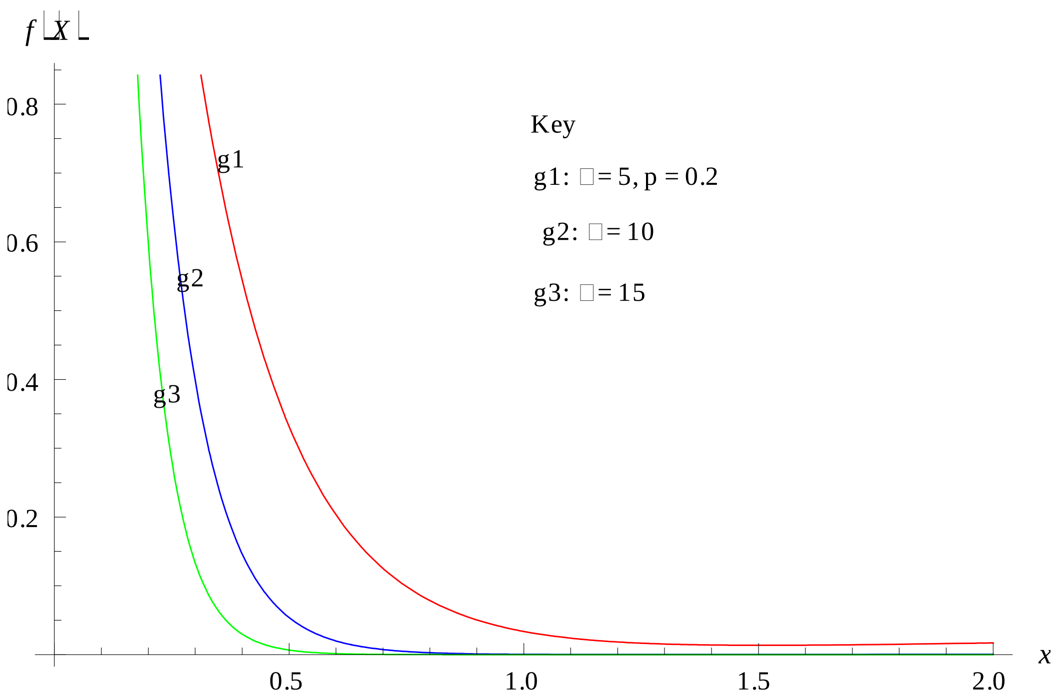

Fig. 16

p=0.5;

f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

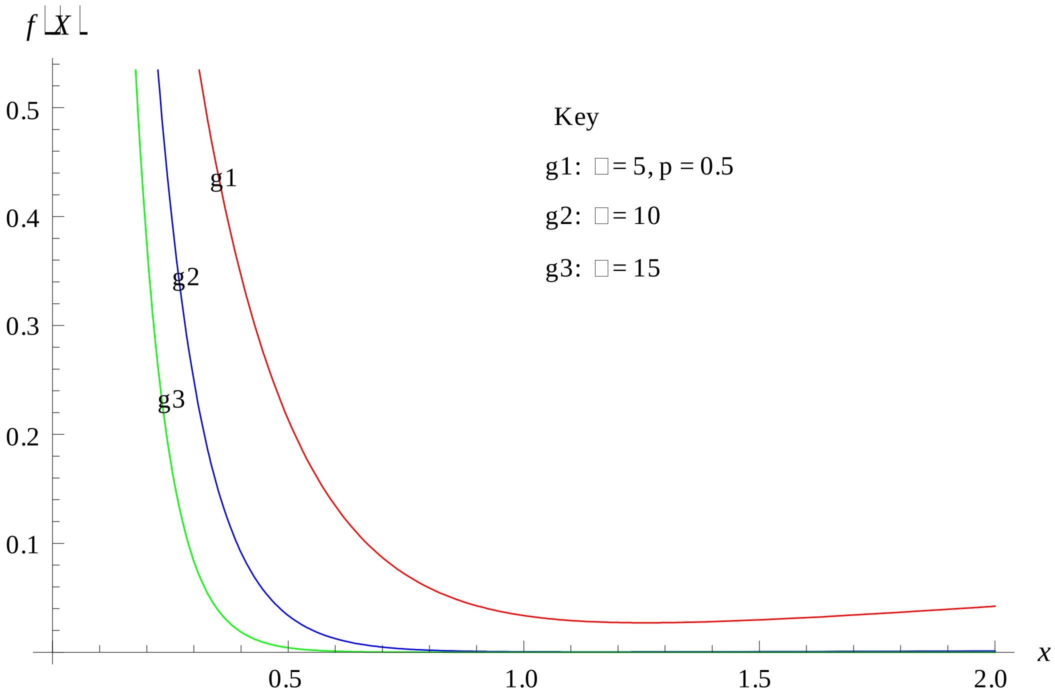

Fig.   17

p=0.8;

f[□ _,X_]=((p*Exp[-□]*□ ^x)/x!)+((1-p)*□ *Exp[-□ *x]);

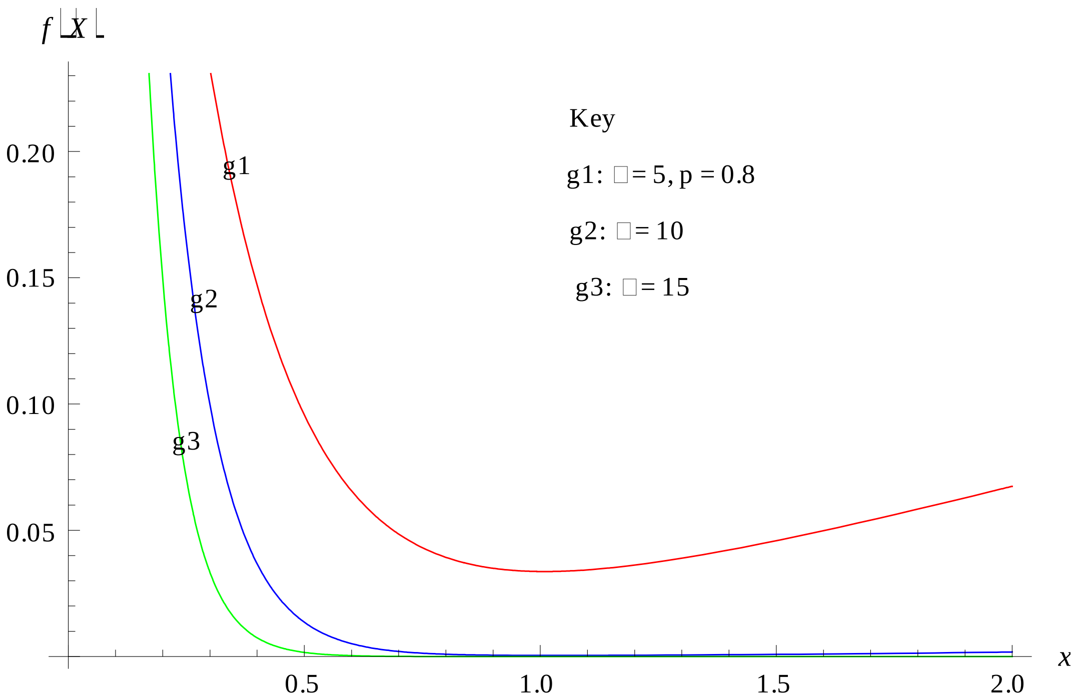

Fig. 18

From the above graphs (of mixed Poisson and Exponential distribution), it was observed that **figure 1 - 3** had the parameter λ being very small (0.1, 0.2, & 0.3) with the value of P being varied from 0.2, 0.5 and 0.8 respectively. The graphs are all decreasing function. They all decay as the variable x increases. **Figure 4 – 6** shows the graph of the same mixture of Poisson and Exponential distribution but the parameter λ is a bit increased (0.2, 0.4 & 0.6) with the weighing factor P being varied as 0.2, 0.5 & 0.8 respectively. **Figure 4** in particular possesses a sharp decrease whereas **figure 5 & 6** with higher P value possesses a decay function as the variable x increases.

In **figure 7 - 9,** the parameter λ was increased again (0.1, 0.5 & 1.0) and weighing factor P was varied as 0.2, 0.5 & 0.8 respectively. It was observed that, though the graphs were still decreasing function, but the rate of decay was just mild. In **figure 10 - 12,** the value of the parameter λ was made an integer (1, 2 & 3) and weighing factor P was varied as 0.2, 0.5 & 0.8 respectively. They were all decreasing function graphs, but the rate of decay was high as the weighing factor P increases. However in **figure 13 - 15** the parameter λ was increased to 2, 4, & 6 whereas the weighing factor P was varied as 0.2, 0.5 & 0.8 respectively. They were all decreasing function graphs, but the value of P affected the rate of decay in the sense that as P increases the rate or level of decay was reducing.

In **figure 16 - 18,** the parameter λ was increased again to 5, 10 & 15 while the weighing factor P was varied as 0.2, 0.5 & 0.8 respectively. The largest value of λ decayed to zero as x tends to 0.6 whereas the graph of $\lambda = 10$ decayed to zero at $x = 0.9$ while $\lambda = 5$ decayed as x increases in **figure 16**. However, in **figure 17 & 18,** the graph with $\lambda = 15$ decayed to zero and got terminated at $x = 0.65$ while the one with $\lambda = 10$ decayed to zero at $x = 0.8$ and continued at that zero level all through. Whereas the graph with $\lambda = 5$ were decreasing function but did not decay to the level of zero but was continuous as x increases.

PARAMETER ESTIMATION FOR THIS MIXED DISTRIBUTION

We apply the maximum likelihood method of likelihood estimation as follows;

$$\frac{Pe^{-\lambda}\lambda^{x}}{x!} + (1-P)\lambda e^{-\lambda x}$$

$$L(\lambda; x_1, ..., x_n) = \prod_{i=1}^{n}\left[\frac{Pe^{-\lambda}\lambda^{x}}{x!} + (1-P)\lambda e^{-\lambda x}\right] \qquad \text{...} \qquad (2)$$

Let

$$\frac{Pe^{-\lambda}\lambda^{x}}{x!} = a$$

and

$$(1-P)\lambda e^{-\lambda x} = b$$

Recall that

$$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + ... + {}^nC_n a^{n-n} b^n$$

$${}^nC_0\left[\frac{Pe^{-\lambda}\lambda^{x}}{x!}\right]^n \left((1-P)\lambda e^{-\lambda x}\right)^0 + {}^nC_1\left[\frac{Pe^{-\lambda}\lambda^{x}}{x!}\right]^{n-1} \left((1-P)\lambda e^{-\lambda x}\right)^1 + ... + {}^nC_n\left[\frac{Pe^{-\lambda}\lambda^{x}}{x!}\right]^0 \left((1-P)\lambda e^{-\lambda x}\right)^n$$

$$= \quad {}^nC_0\left[\frac{Pe^{-\lambda}\lambda^{x}}{x!}\right]^n + {}^nC_1\left[\frac{Pe^{-\lambda}\lambda^{x}}{x!}\right]^{n-1} \left((1-P)\lambda e^{-\lambda x}\right)^1 + ... + \left((1-P)\lambda e^{-\lambda x}\right)^n \qquad \text{...........} \qquad (3)$$

At this point, we observe that the function is not differentiable with respect to λ .

THE MODEL

To formalize our model, let x_i equals to 1 if item i is selected and zero otherwise. Also, for convenience, let $R(x)$ denote our contagious distribution; that is

$$R(x) = P\frac{e^{-\lambda}\lambda^{x}}{x!} + (1-P)\lambda e^{-\lambda x}$$

Our proposed model will be similar to that of chapter three above. This is because we are still considering a case of mixed distribution for the capacity, though with additive form. Hence our objective function is stated as

Minimize f = $\sum_{i=1}^{n} k_i x_i + g\int_{L}^{U} P(C)u(c)dc + h\int_{L}^{U} P(c)v(c)dc$ .... (4)

S.t $\sum_{i=1}^{n} w_i x_i + u(c) - v(c) = C$ .... (5)

$L \le C \le U$; U(c).V(c) =0 ; $\int P(c)dc = 1$

$x_i \ge 0$ and integer

Where $U(c)$ is the slack variable with the capacity c

$V(c)$ is the surplus variable with respect to the capacity c

$P(c)$ is the probability distribution of the capacity c

L is the lower bound of the capacity

U is the upper bound of the capacity

g is the penalty cost of the slack variable

h is the penalty cost of the surplus variable

we defined the interval $Q \in [L, U]$

there exist $\sum_{i=1}^{n} w_i x_i \ge Q$ and

$\sum_{i=1}^{n} w_i x_i < Q + \Delta Q$

Where Q is a positive real number. Let $\sum_{i=1}^{n} w_i x_i = \alpha$

$\alpha \ge Q$; $\alpha < Q + \Delta Q$

Hence Q creates a neighbourhood for $Q = \sum_{i=1}^{n} w_i x_i$

Define $V(c) = \sum w_i x_i - c$ $\quad L \le C < Q$

$U(c) = C - \sum w_i x_i$ $\quad Q < C \le U$

Substituting these into our objective function we have

Min f = $\sum_{i=1}^{n} k_i x_i + g\int_{Q}^{U} P(c)(C - \sum_{i=1}^{n} w_i x_i)dc + h\int_{L}^{Q} P(c)(\sum_{i=1}^{n} w_i x_i - c)dc$ .... (6)

$$= \sum_{i=1}^{n} k_i x_i + g\int_{Q}^{U} CP(c)dc - g\int_{Q}^{U} P(c)(\sum_{i=1}^{n} w_i x_i)dc + h\int_{L}^{Q} P(c)(\sum_{i=1}^{n} w_i x_i)dc - h\int_{L}^{Q} CP(c)dc$$

S.t $\sum_{i=1}^{n} w_i x_i \geq Q$

$\sum_{i=1}^{n} w_i x_i < Q + \Delta Q$

$0 \leq x_i \leq d_i$; $x_i \geq 0$

Min f = $\sum_{i=1}^{n} k_i x_i + g\int_{Q}^{U} P(c)(C - \sum_{i=1}^{n} w_i x_i)dc + h\int_{L}^{Q} P(c)(\sum_{i=1}^{n} w_i x_i - c)dc$ .... (7)

= $\sum_{i=1}^{n} k_i x_i + g\int_{Q}^{U} CP(c)dc - g\int_{Q}^{U} P(c)(\sum_{i=1}^{n} w_i x_i)dc + h\int_{L}^{Q} P(c)(\sum_{i=1}^{n} w_i x_i)dc - h\int_{L}^{Q} cP(c)dc$ 4.8

(Here, we make P(c) = P(x), such that $\int P(c)dc = \int P(x)dx$)

$$= \sum_{i=1}^{n} k_i x_i + g\int_{Q}^{U} xP(x)dx - g\int_{Q}^{U} P(x)(\sum_{i=1}^{n} w_i x_i)dx + h\int_{L}^{Q} P(x)(\sum_{i=1}^{n} w_i x_i)dx - h\int_{L}^{Q} xP(x)dx$$

$$= \sum_{i=1}^{n} k_i x_i + g\int_{Q}^{U} x\left(\frac{Pe^{-\lambda}\lambda^{x}}{x!} + (1-P)\lambda e^{-\lambda x}\right)dx - g\int_{Q}^{U}\left(\frac{Pe^{-\lambda}\lambda^{x}}{x!} + (1-P)\lambda e^{-\lambda x}\right)(\sum_{i=1}^{n} w_i x_i)dx$$

$$+ h\int_{L}^{Q}\left(\frac{Pe^{-\lambda}\lambda^{x}}{x!} + (1-P)\lambda e^{-\lambda x}\right)\left(\sum_{i=1}^{n} w_i x_i\right)dx - h\int_{L}^{Q} x\left(\frac{Pe^{-\lambda}\lambda^{x}}{x!} + (1-P)\lambda e^{-\lambda x}\right)dx$$

S.t $\sum_{i=1}^{n} w_i x_i \geq Q$

$\sum_{i=1}^{n} w_i x_i < Q + \Delta Q$

$0 \leq x_i \leq d_i$; $x_i \geq 0$

CONCLUSION

In this paper, we consider a form of production process in which items (goods) arrive stochastically and the weights of these items are made known on its arrival. The profits made with respect to the items accepted for processing is maximized and the weight of those items is assumed to follow a contagious distribution of Poisson and Exponential distribution using additive model. The graphs of our new distribution are observed to be a decreasing function graph generally. However, when the values of the parameter λ gets larger, (a positive integer e.g 5, 10, & 15), the graph possesses a sharp decrements as $0 \leq x \leq 0.5$ and then continued to decay at almost zero level as x increases. However, when λ is small (0.1, 0.2, & 0.3) and (0.1, 0.5 & 1.0), the graph is a decreasing function graph. Also, an algebraic stochastic knapsack model is proposed for the item weight following a contagious distribution of Poisson and exponential distribution using an additive model.

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