

## Applying Taper Function in Standard Volume Equation for the Past Volume Increment Analysis of Kinuyanagi Willow

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### ABSTRACT

Decision and policy makers as well as forest managers in practices require reliable and efficient estimations of volumes and biomass for the management of commercial and research scale plantings of short rotation woody crops. Taper functions have been used to describe the tree form, once they provide estimates for the height at any diameter or the diameter at any height. This study applied the Gompertz sigmoid function to predict the heights at any diameter or height for a stem section of Kinuyanagi willow (*Salix schwerinii* E. L. Wolf) and the predicted height were used as a tool in the standard volume equation to calculate the volumes accurately in different growing seasons. The analysis used a stem analyzed data set of 184 pieces of logs from 10 sample trees (five large-sized at the age of 11 years and five small-sized at the age of 4 years) cut from the experimental plot of willow plantations at Siikasalmi in Liperi, Finland. The linear functions were used to explain the variation in the estimated parameters from fitting the model to data from individual stems. The linear functions were also used to predict the parameter values of applied taper function to describe the shape of taper curves of sample trees. The applied models fitted the data set and predicted the height quite accurately for the taper curve fitting. In addition, the tool (predicted height) was reliable in standard volume equation to calculate the predicted sectional volumes in different growing seasons. However, in some growing seasons especially early growing seasons, taper curves of the stems did not fit properly and some differences between the predicted volumes and measured volumes were observed. The reason of such differences could be over/under estimations of measured data, due to the measurements errors since the visibility of the growth rings was not so clear, or a fault of the applied models simply did not fit the data.

**Keywords:** Willow, Gompertz sigmoid model, Standard volume equation, Taper curve, Volume and Age.

### 1.1 INTRODUCTION

The use of short rotation woody crop (SRWC) plantations for bioenergy production has become increasingly attractive as alternative sources of renewable energy are being explored (Ballard et al. 2000). There are many fast growing tree species, such as willow, suitable for SRWC in various regions of the world. They are easy to propagate vegetatively. Many of the species are adapted to a wide range of climatic and soil conditions. They are often integrated with agriculture, horticulture and apiculture (FAO 2005).

Environmental concerns over acidic deposition and global warming, in relation to the burning of fossil fuels, primarily coal, in industrial and utility plants, have sparked an interest in clean sources of renewable energy (e.g., CO<sub>2</sub>-neutral and low SO<sub>x</sub> and NO<sub>x</sub> emissions). Fast-growing plantations, such as willow or poplar have the potential to meet some of these needs in the world (Ballard et al. 2000). Co-firing willow biomass with coal results in consistently reduced SO<sub>x</sub> emissions, and reduced NO<sub>x</sub> emissions under certain conditions (Ballard et al. 2000). Both of these compounds are precursors of acid rain. The amount of CO<sub>2</sub> released in the burning of willow biomass is equivalent to the amount recaptured by growing willow trees, and therefore, can be considered a CO<sub>2</sub> - neutral representing carbon neutral energy source. Because of their rapid growth they are effective for carbon sequestration (Ballard et al. 2000).

The twenty-second session of the International Poplar Commission (IPC), held in Santiago, Chile from 28 November to 2 December 2004, focused on enhancing the contribution of poplars (*Populus* spp.) and willows (*Salix* spp.) to sustainable forestry and rural development, particularly in developing countries and those with economies in transition (FAO 2005). Remarkable research and development for SRWC has been carried out already in many countries. China, for example, has the largest and second largest area of planted willow, respectively 59 000 ha, planted to combat desertification and 21 000 ha (FAO 2005). At present around 14 600 ha of SRC willow are being cultivated in Sweden (according to the Swedish Board of Agriculture's statistics for 2006), and around 500 hectares are added every year in the form of new plantations. Every winter SRC willow is harvested delivering approximately 2 500 ha in the around 25 heating / power plants in central and southern Sweden.

Numerous research projects have begun in the U.S.A. demonstrating the environmental benefits and sustainability of the willow biomass system from an energy perspective. Poplar and willow resources are principally used for environmental purposes, for example, soil and water protection, riparian buffers management, phytoremediation, and some are used for carbon sequestration (Kuzovkina and Quigley 2005).

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In Canada, the Forest 2020 programme is establishing fast-growing plantations, of which poplar and willow are the major components, on previously non-forested lands (primarily agricultural lands) for carbon storage (FAO 2005). In Bulgaria and Chile, willows are planted along river banks to stabilize them and reduce sedimentation (Kuzovkina and Quigley 2005).

Zvereva *et al.* (1997) reported that willows, along with birches, were the only woody species found in those industrial areas of Europe with very high air pollution levels. They concluded that long-term and severe pollution by sulfur dioxide and heavy metals suppressed growth of most woody species, but caused no measurable stress response in willows, and may even stimulate growth of leaves and shoots of *Salix* species.

## 1.2 Background of stem analysis

Taper curve analysis is one way to analyze the annual growth rate of volume, height and diameter of stems in old forest and trees. Generally, stem taper is defined as the rate of change of diameter with height along the stem, where stem profile reflects the shape of the entire stem. A mathematical function, which predicts the diameter of the stem of an individual tree at any distance along its stem, is called taper function (West 2003). If the taper function is known at several points in time e.g., based on stem analysis, the past annual growth can be analyzed.

Modeling the stem taper or taper curve analysis has been a widespread effort in forestry background during the past century. Nowadays, the development of stem profile or taper and merchantable volume models is common in forestry related literature for the determination of merchantable height and volume between any two points along the stem (Muhairwe 1999 and Jordan *et al.* 2005). Benbrahim and Gavaland (2003) developed a three parameters stem taper function for individual trees as short-rotation coppice. The taper model used to estimate both diameter at a given height and height for a given top diameter.

There are not too many studies related to SRWC. Stem volume and taper models for Khasi Pine (*Pinus kesiya*) was calculated by Eerikainen (2001) in Finland. He developed a taper model for over and under bark diameters using the variable-exponent form of the bark-thickness and variable-exponent taper equation respectively. The models were comparatively more reliable than earlier models for volume and diameter predictors for Khasi Pine. Jiang *et al.* (2005) developed compatible segmented polynomial taper curve and volume functions for yellow-poplar (*Liriodendron tulipifera*). Twenty-six sample trees were analysed to develop taper curve and volume function. The analysis showed that the proposed equation has the lowest overall prediction error. Very recently, Wang *et al.* (2007) proposed taper modeling on Taiwan Plantation in Liukuei based on different types of taper functions. Malimbwi and Philip (1989) formulated a non-linear taper model equation of Mexican weeping pine in Tanzania for calculating the individual log volume. Brooks *et al.* (2008) developed a compatible stem volume and taper equations for different species in Turkey. The models fitted well to predict volume of the whole tree.

Numerous studies have been conducted on taper curve analysis for different species but few have studied on willow species and such kind of model for Kinuyanagi willow is not available yet in Finland. As presented above there are many numbers of applications in willow based green engineering. Consequently, more research and tool is needed for analyzing growth properties of tree species and clones. Forest research applies commonly taper functions for that target species based biomass development models to meet the demand of forest products in local, national and international wide. This way also the efficient utilization of genotype-environment interaction is possible in the future biomass production, as well as other environmental services.

In practice the foresters want to have tools for predicting the future volume or biomass for the management of plantations. We present willow as an example how to study past volume increments of SRWC trees.

A sigmoid function is mathematical function that produces a sigmoid curve generally 'S' shape. The Gompertz function is a sigmoid function. It is a type of mathematical model for a time series, where growth is slowest at the start and end of a time period (d'Onofrio 2005). In most of the forestry studies, the Gompertz sigmoid model is applied in the regression model. In this study, the Gompertz sigmoid model is generalized as non-linear logarithm regression equations with 3 parameters.

## 1.3 Research Objectives

The overall aim of the study is as follows:

- To investigate the annual growth rate (volume) of the stems of Kinuyanagi willow based on stem analysis (Gompertz sigmoid model and standard volume equation) in different growing seasons.
- To develop methodology and a tool on the basis of willow analysis to study past volume increments of other SRWC species for a limited number of sample trees.

## 2 MATERIALS AND METHODS

The study area is an experimental plot located at Siikasalmi in Liperi, Finland (62° 30'N, 29° 30' E). The growth dynamic of variety clones of willow has been studied in this plot to establish the large scale plantations in different areas in eastern Finland. The experimental plot was established in an open-drained fallow field and soil type is silty and sandy where, PH ranges from 5.3 to 6.6 (Kuusela *et al.* 2004). Nitrogen content in the soil had a clear positive correlation with biomass production. The number of dead shoots correlated positively with increasing manganese (Mg) content and negatively with increasing Potassium (P) content in the soil (Tahvanainen 2004). The distances between the rows of planted cuttings were 0.8 m and between each cutting 0.5 m, making the planting density 15 000 cuttings ha<sup>-1</sup>. The climate in Finland is moist mid-latitude climate with cold winters. Tahvanainen (2004) stated, according to Köppen classification Finland includes in climate zone "D" and climate type "f", Boreal climate, where the coldest month has a mean temperature below -3° C and the warmest over +10° C, and the precipitation is sufficient all year-round.

## 2.1 Procedure of the data collection

The study is based on a stem analyzed data set of 184 pieces of logs from 10 sample trees (five large-sized and small-sized) harvested from the study area in 2008. Large-sized and small-sized sample trees were harvested separately in 11 and 4 years old plantations. In this sampling procedure, the 10 dominant sprouting based trees were selected separately from the 10 randomly selected individual stools. There were two or three sprouts in every individual stool and in this case, one dominant sprout was selected as a sample tree on the basis of dominant DBH (diameter at breast height) in the individual stool. The tree caliper device was used for the measurement of DBH. The numbers of rows were counted from any direction for the location of sample plots. The first 3 rows were not considered because they were in buffer zone. Then randomization samplings of rows were established. The total no. of plot was 2 in 11 and 4 years old plantations separately. The desired rows were the 3<sup>rd</sup> and 6<sup>th</sup> in the 11 years old plantation as well as, the 2<sup>nd</sup> and 8<sup>th</sup> in 4 years old plantation. The adjacent numbers of rows were not considered.

After random selection of sample trees (dominant sprouts), the felling operation was carried out and the stems were moved to the the university laboratory to cut stems into different sections. The desired length of most of the stem sections was 0.1m, 0.2 m, 0.5m, 1m and 1.5 m respectively. For the curve fitting at least 6 disks (every 0.1m), 5 disks (every 0.2m), 2 or 3 disks (every 0.5m), another 2 or 3 disks (every 1m), 1 or 2 disks (every 1.5 m) were taken from bottom to top part of the stem and details are shown in Figure 5. The average total height of large -sized (5 sample stems) and small -sized (5 sample stems) were 10.50 m and 5.85 m respectively. In both sized of sample trees, the disks were cut up to around 85 % of average total tree height, because the rate of taper changes frequently in the bottom and middle part of the stem and the uppermost part has a minimal influence on the total volume of the stem.

After sawing the cross-sectional discs, they were leveled with a sharp knife and then polished. The surface has treated with red aniline in order to increase the visibility of the annual rings. Windendro device was used for the measurements of annual rings and radiuses were calculated as a mean of four measurements in four directions (90°) and the direction was same at each measuring height. The height of disk, diameter at different heights up to around 85% of average total tree height of sample tree from bottom to top, diameter at the breast height and true volumes were measured and recorded. The volume of individual stem was calculated by applying the following equation and assuming that all stem sections have cylindrical shape.

$$V_i = \pi * r_i^2 * (h_i - h_{i-1}) \quad (1)$$

In equation (1),  $V_i$  is denoted as volume in  $\text{dm}^3$  for a stem section,  $r$  and  $h$  denote radius (mm) and height (m) for each section. The total sectional volumes of the individual stem up to around 85 % of total tree height were calculated by summing up the volumes of individual disks.

$$V = \sum_{i=1}^n \pi * r_i^2 * (h_i - h_{i-1}) \quad [\text{where, } i=1, 2 \dots n] \quad (2)$$

In equation (2),  $V_{is}$  denoted as stem volume in  $\text{dm}^3$  for a stem,  $r$  and  $h$  denote radius (mm) and height (m) for each section

## 2.2 Development of Non-linear and linear regression models

The statistical package sigma plot 8 was used for the analysis of regression equations in this study. The analysis used Gompertz sigmoid function and the developed non-linear model with parameters can be written as follows:

$$h = a * \text{EXP} (-\text{EXP} (-(-r - X0) / b)) \quad (3)$$

In equation (3),  $h$  and  $r$  denote sectional height (m) and radius (mm) and  $a$ ,  $b$  and  $X0$  represented the model coefficients. The coefficient values of  $a$ ,  $b$  and  $X0$  with their standard error in addition  $R^2$  values in the proposed model are presented in Table 1. The applied non-linear function represented one constant sigmoid shape for the whole set of sample trees in the different growing seasons and the change of model coefficient values described varying shapes of tapers. Three linear functions (Eq. (4), Eq. (5) and Eq. (6)) were fitted to data set to predict  $\hat{a}$ ,  $\hat{b}$  and  $\hat{X0}$  to influence the shape of taper curves in the different growing seasons as well as, to explain the variation in the estimated parameters from fitting the model to data from individual stems

### Sub-model 1

Due to the limited data set a linearity pattern was seen for non-linear regression parameter ( $a$ ) over the variable of  $\Delta d$  ( $\Delta d = r_{1.3} - r_2$ , where  $r_{1.3}$  and  $r_2$  denote as radius at 1.3 m and 2 m height respectively).  $\Delta d$  can be justified on the basis of practical measuring reasons. The linear trend was not strong but can be used in model development as the first approximation (Figure 1 (A)). The fitted linear equation can be written as follows:

$$\hat{a} = P (\Delta d + Q) \quad (4)$$

In equation 4,  $P$  and  $Q$  used as constant values, which were presented in Table 2.

### Sub-model 2

Same kind of linearity pattern was also seen for non-linear regression parameter ( $b$ ) over the variable of  $\Delta d$  (Figure 1 (B)). The fitted linear equation can be written as follows:

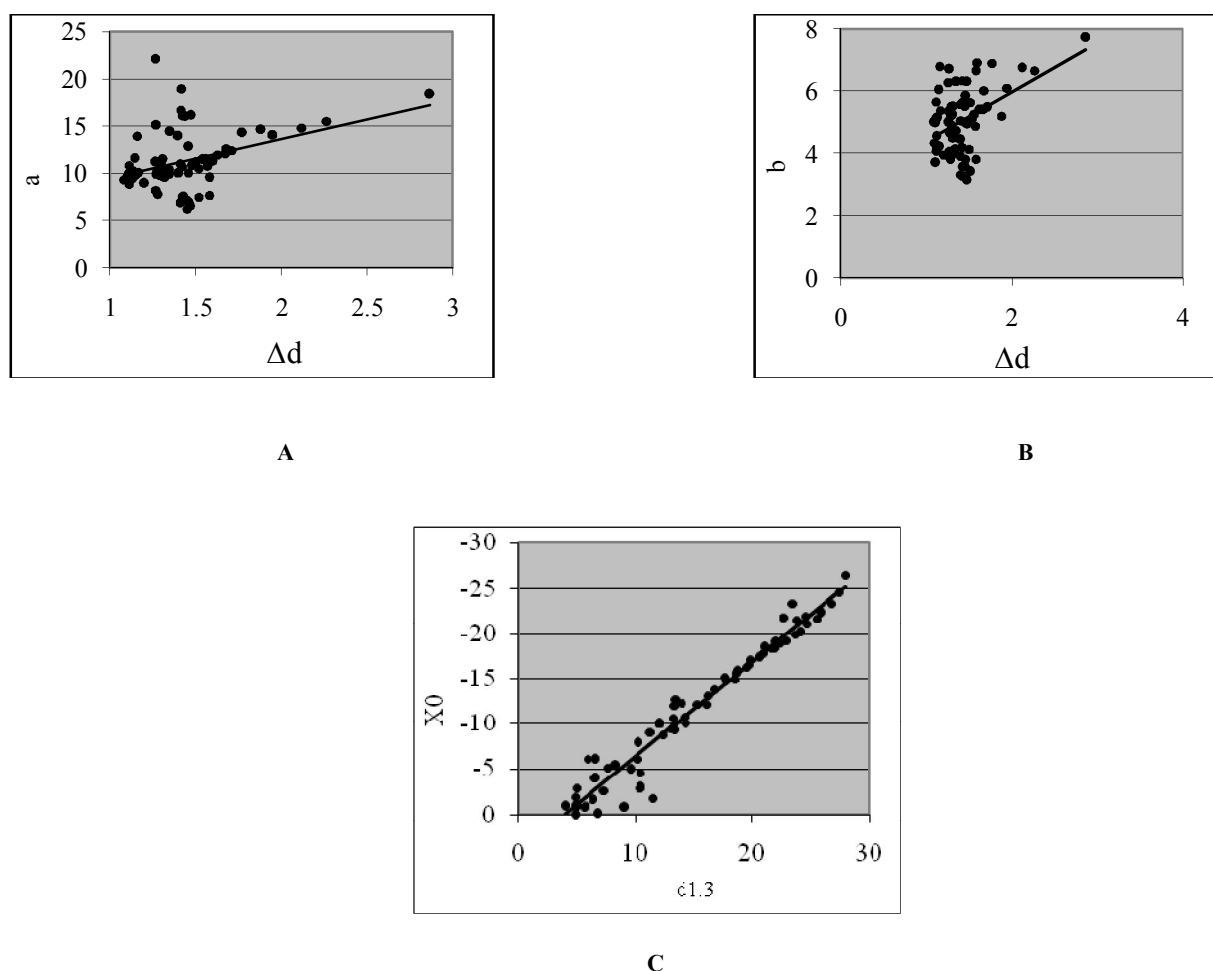
$$\hat{b} = R (\Delta d + S) \quad (5)$$

In equation 5,  $R$  and  $S$  used as constant values which were also presented in Table 2.

### Sub-model 3

A strong linearity pattern was seen for non-linear regression parameter ( $X0$ ) over the variable of  $d_{1.3}$ , where  $d_{1.3}$  denote radius (mm) at 1.3 m of stem height (Figure 1 (C)). The linear equation of sub-model 3 with 2 parameters ( $\beta 0$  and  $\beta 1$ ) can be written as follows:

$$\hat{X0} = \beta 0 + \beta 1 * d_{1.3} \quad (6)$$



**Figure 1.** Scatter plots for linear regression equations: (A), “ $\Delta d$ ” versus co-efficient “ $a$ ” and (B), “ $\Delta d$ ” versus co-efficient “ $b$ ”. The radius (inside bark) at 1.3 m ( $d_{1.3}$ ) plots for linear regression equation: (C), “ $d_{1.3}$ ” versus coefficient ( $X0$ ).

The coefficient values  $a$ ,  $b$  and  $X0$  were replaced by their predicted values of  $\hat{a}$ ,  $\hat{b}$  and  $\hat{X0}$  and the new equation can be written as follows:

$$\hat{h}_i = \hat{a} * \text{EXP}(-\text{EXP}(-(-r_i - \hat{X0})/\hat{b})) \quad (7)$$

In equation 7,  $\hat{h}_i$  denoted as predicted sectional height in m where,  $i=1, 2, 3, \dots, n$  and  $i$  indicated as number of section in a tree,  $r$  represented as radius (mm).

The equation (1) can be replaced by the following equation to predict the stem sectional volumes of sample trees

$$\hat{V}_i = \pi * r_i^2 * (\hat{h}_i - \hat{h}_{i-1}) \quad (8)$$

In equation 8,  $\hat{V}_i$  means predicted sectional volume in  $\text{dm}^3$  for a stem section in a tree,  $r_i$  and  $\hat{h}_i$  indicated as radius (mm) and predicted height (m) where,  $i = 1, 2, 3, \dots, n$  and  $i$  is the number of stem section. The total predicted sectional volumes of the individual stem were calculated by summing up the volumes of individual disks (equation 9). Using the model it is possible to apply Riemann's sum method for calculating the stem volumes since it is possible to greatly increase the number of disks.

$$\hat{V} = \sum_{i=1}^n \pi * r_i^2 * (\hat{h}_i - \hat{h}_{i-1}) \quad (9)$$

In equation 9,  $\hat{V}$  denote predicted stem volume ( $\text{dm}^3$ ) in a sample tree,  $r_i$  and  $\hat{h}_i$  indicated as radius (mm) and predicted height (m) where,  $i = 1, 2, 3, \dots, n$  and  $i$  is the number of stem section.

To further evaluate the accuracy of the volume model the percent bias (%Bias) was calculated on the basis of the data set using the following equation:

$$\% \text{Bias} = \left[ \sum_{i=1}^n \frac{V_i - \hat{V}_i}{V_i} \right] \times \frac{100}{n} \quad (10)$$

Where  $V_i$  and  $\hat{V}_i$  denote observed and predicted volume of individual section in a sample tree and  $n$  is the total no. of observations.

### 3 RESULTS

The Gompertz sigmoid function (Eq. (2)) was used to describe the relationship between sectional height and sectional radius in the shoot, and the model explained (92-99) % of the variation in the predicting height with the response of radius. The applied non-

linear function was fitted to the data sets accurately and described the relationship between predicted sectional height and radius throughout the different growing season, and the function kept itself within the standard error limits for the constant value  $a$ ,  $b$  and  $X0$  (Table 1). The estimated coefficient values were presented in the Table 1. In addition, three linear functions (sub-model 1, sub-model 2 and sub-model 3) were fitted to describe the relation between non-linear regression parameter values  $a$  and variable  $\Delta d$ , non-linear regression parameter values  $b$  and variable  $\Delta d$ , and non-linear regression parameter values  $X0$  and variable  $d_{1,3}$  in addition to explain influence of aging of trees. In the fitted linear regression model (Eq. (4), Eq. (5) and Eq. (6)),  $\bar{X0}$  explained 95% variation with response of  $d_{1,3}$  but  $\hat{a}$  and  $\hat{b}$  explained only 15% and 18% variation with response of  $\Delta d$ . The constant values of regression models are also presented in Table 2.

Table 1: Constants values estimates for the function  $h = a * \text{EXP} (-\text{EXP} (- (-r - X0) / b))$  on samples data.

Tree no.	Age	$d_{1,3}$	$d_2$	$\Delta d = d_{1,3} - d_2$	Random coefficients values			SE of coefficients values			RSE	R <sup>2</sup>
					$a$	$b$	$x0$	$a$	$b$	$x0$		
1	11	23,81	22,23	1,58	9,52	3,81	-21,48	0,36	0,20	0,18	0,15	0,99
1	10	22,96	21,55	1,41	10,99	5,06	-19,17	0,49	0,25	0,27	0,13	0,99
1	9	21,85	20,55	1,30	10,13	4,92	-18,39	0,49	0,30	0,29	0,16	0,99
1	8	20,65	19,36	1,29	9,71	4,67	-17,44	0,43	0,27	0,26	0,17	0,99
1	7	19,56	18,25	1,31	9,70	4,59	-16,25	0,40	0,25	0,24	0,16	0,99
1	6	17,65	16,45	1,20	8,94	3,96	-15,17	0,30	0,21	0,18	0,16	0,99
1	5	15,32	14,20	1,12	9,56	4,57	-12,16	0,47	0,31	0,29	0,18	0,99
1	4	12,36	11,25	1,11	10,81	4,98	-8,69	0,79	0,41	0,43	0,21	0,99
1	3	9,61	8,21	1,40	13,93	5,58	-4,99	0,89	0,61	0,78	0,37	0,93
1	2	6,42	5,07	1,35	10,43	4,73	-1,70	0,92	0,71	0,81	0,67	0,92
1	1	4,91	2,79	2,12	14,78	6,75	-0,02	0,99	0,82	0,79	0,47	0,93
2	11	24,13	22,87	1,26	11,18	5,01	-20,18	0,98	0,45	0,51	0,16	0,99
2	10	23,45	22,30	1,15	9,83	4,23	-20,29	0,23	0,13	0,12	0,08	0,99
2	9	22,72	21,59	1,13	9,47	4,16	-19,75	0,38	0,22	0,21	0,14	0,99
2	8	21,97	20,85	1,12	9,36	4,08	-19,10	0,39	0,23	0,22	0,15	0,99
2	7	21,07	19,96	1,11	8,84	3,70	-18,53	0,24	0,16	0,13	0,12	0,99
2	6	18,72	17,61	1,10	9,55	4,33	-15,63	0,39	0,22	0,23	0,11	0,99
2	5	16,27	14,95	1,32	9,53	3,99	-13,13	0,47	0,26	0,25	0,17	0,99
2	4	13,22	11,82	1,40	10,08	4,45	-9,62	0,75	0,39	0,40	0,21	0,99
2	3	10,44	8,67	1,77	14,26	6,89	-3,06	0,89	0,61	0,78	0,37	0,99
2	2	7,30	5,63	1,67	12,06	5,28	-2,63	0,96	0,71	0,81	0,67	0,93
2	1	5,63	3,36	2,27	15,50	6,65	-0,79	0,99	0,92	0,88	0,69	0,92
3	11	21,65	20,15	1,50	11,20	4,13	-18,36	1,00	0,45	0,49	0,25	0,93
3	10	20,96	19,47	1,48	10,78	4,06	-17,83	0,84	0,37	0,39	0,22	0,99
3	9	19,89	18,44	1,46	10,03	3,81	-17,13	0,64	0,31	0,33	0,20	0,99
3	8	18,81	17,41	1,40	10,03	3,91	-15,90	0,50	0,25	0,27	0,16	0,99
3	7	17,75	16,41	1,34	9,90	4,12	-14,80	0,29	0,16	0,15	0,10	0,99
3	6	16,10	14,81	1,30	11,21	5,22	-12,08	0,99	0,49	0,56	0,21	0,99
3	5	14,27	12,95	1,32	10,37	4,49	-10,73	0,76	0,39	0,41	0,21	0,99
3	4	11,51	10,24	1,27	15,07	8,70	-1,73	0,99	1,00	0,97	0,27	0,99
3	3	9,02	7,75	1,26	22,05	7,28	-0,87	1,52	0,98	1,11	0,28	0,99
3	2	5,71	4,00	1,71	12,39	5,49	-0,88	0,98	0,92	0,98	0,45	0,99
3	1	4,01	2,47	1,54	11,44	5,13	-0,99	1,00	0,87	0,99	0,31	0,93
4	11	26,10	24,64	1,46	12,83	5,84	-12,83	0,73	0,32	0,37	0,13	0,92
4	10	25,55	24,24	1,31	11,55	5,50	-21,48	0,65	0,34	0,36	0,15	0,93
4	9	24,56	23,26	1,30	10,79	5,24	-20,91	0,58	0,35	0,34	0,17	0,99
4	8	23,74	22,44	1,30	10,97	5,51	-19,85	0,62	0,37	0,37	0,16	0,99
4	7	22,42	21,12	1,29	10,60	5,22	-18,89	0,56	0,34	0,33	0,16	0,99
4	6	19,78	18,51	1,28	10,14	4,68	-16,59	0,33	0,21	0,19	0,12	0,99
4	5	16,78	15,52	1,27	9,88	4,34	-13,75	0,30	0,18	0,16	0,12	0,99
4	4	13,37	12,11	1,26	11,18	5,35	-9,28	0,53	0,29	0,30	0,14	0,99
4	3	10,39	8,97	1,42	18,92	7,34	-2,83	0,88	0,62	0,76	0,34	0,99
4	2	5,96	4,39	1,57	10,75	4,87	-6,03	0,96	0,71	0,81	0,67	0,99
4	1	4,98	3,10	1,88	14,60	5,19	-1,15	0,99	0,93	0,88	0,79	0,99
5	11	28,01	26,85	1,16	13,93	6,80	-22,44	1,55	0,64	0,80	0,19	0,93
5	10	27,52	26,37	1,14	11,58	6,05	-23,08	0,84	0,46	0,50	0,18	0,92
5	9	26,83	25,70	1,13	10,17	5,14	-23,30	0,40	0,26	0,24	0,14	0,93
5	8	25,89	24,76	1,12	9,91	5,16	-22,41	0,49	0,33	0,31	0,18	0,99
5	7	24,67	23,55	1,11	10,02	5,66	-21,06	0,59	0,42	0,40	0,20	0,99
5	6	22,19	21,11	1,08	9,29	5,00	-18,92	0,45	0,35	0,30	0,19	0,99
5	5	18,59	17,42	1,17	10,09	5,37	-14,90	0,66	0,43	0,42	0,21	0,99
5	4	14,30	12,74	1,56	11,53	5,26	-10,02	0,73	0,35	0,38	0,16	0,99
5	3	10,46	9,11	1,35	14,41	6,30	-4,50	1,78	0,62	0,82	0,19	0,99
5	2	6,59	3,73	2,87	18,37	4,72	-6,11	1,98	0,82	0,92	0,19	0,99

5	1	4,93	2,98	1,95	14,00	6,09	-1,85	1,68	0,61	0,81	0,19	0,99
6	4	12,99	11,36	1,63	11,94	4,41	-9,34	1,32	0,72	1,40	0,13	0,99
6	3	10,18	8,66	1,52	10,47	4,63	-6,06	3,17	0,96	1,43	0,16	0,93
6	2	6,78	5,31	1,47	16,13	6,30	-0,15	6,36	1,28	2,21	0,13	0,92
6	1	4,94	3,53	1,41	16,68	5,58	1,10	10,61	1,78	3,12	0,18	0,93
7	4	13,30	11,85	1,45	6,18	2,51	-11,97	0,53	0,27	0,29	0,12	0,99
7	3	10,27	8,75	1,52	7,43	3,42	-7,90	0,77	0,34	0,42	0,11	0,99
7	2	6,57	5,12	1,45	7,11	3,58	-4,13	0,89	0,45	0,54	0,13	0,99
7	1	4,99	3,55	1,44	15,98	5,66	-0,94	7,62	1,39	2,41	0,14	0,99
8	4	14,01	12,55	1,46	6,83	3,00	-12,29	0,67	0,32	0,36	0,12	0,99
8	3	12,04	10,63	1,41	6,82	3,31	-9,90	0,87	0,45	0,52	0,15	0,99
8	2	7,70	6,42	1,28	7,76	3,80	-5,09	1,17	0,53	0,66	0,13	0,99
8	1	4,97	3,55	1,42	10,68	4,19	-1,23	2,00	0,57	0,83	0,11	0,99
9	4	13,40	11,80	1,60	11,25	1,91	-12,69	0,27	0,18	0,15	0,13	0,99
9	3	12,04	10,57	1,47	6,50	3,15	-10,12	0,66	0,37	0,41	0,14	0,93
9	2	6,55	5,12	1,43	7,47	3,56	-3,96	0,64	0,30	0,36	0,09	0,92
9	1	4,99	3,57	1,42	7,34	3,26	-2,87	0,78	0,34	0,41	0,10	0,93
10	4	13,25	11,57	1,68	12,50	4,01	-10,56	0,99	0,38	0,50	0,09	0,99
10	3	11,25	9,67	1,58	7,63	3,65	-8,95	0,61	0,28	0,34	0,08	0,99
10	2	8,26	6,99	1,27	8,19	3,92	-5,42	0,96	0,40	0,52	0,09	0,99
10	1	4,92	3,50	1,42	16,05	5,63	-0,94	8,70	1,58	2,74	0,16	0,99

$d_{1,3}$ : radius in mm at 1.3 m height,  $d_2$ : radius in mm at 2 m height,  $\Delta d$ : differences between  $d_{1,3}$  and  $d_2$ , S.E: standard error; RSE: residual standard error, Age (years).

Table 2: Parameter estimates for the function  $\hat{a} = P(\Delta d + Q)$ ,  $\hat{b} = R(\Delta d + S)$  and  $\widehat{X0} = \beta_0 + \beta_1 d_{1,3}$

Sub-model	Constant values	S.E	R <sup>2</sup>
sub-model 1	$P = 5.6$	1.652	0.15
	$Q = 0.5$	1.143	
sub-model 2	$S = 0.9$	0.548	0.18
	$R = 2.1$	0.378	
sub-model 3	$\beta_0 = 5.4$	0.518	0.95
	$\beta_1 = 1.1$	0.031	

S.E: Standard error

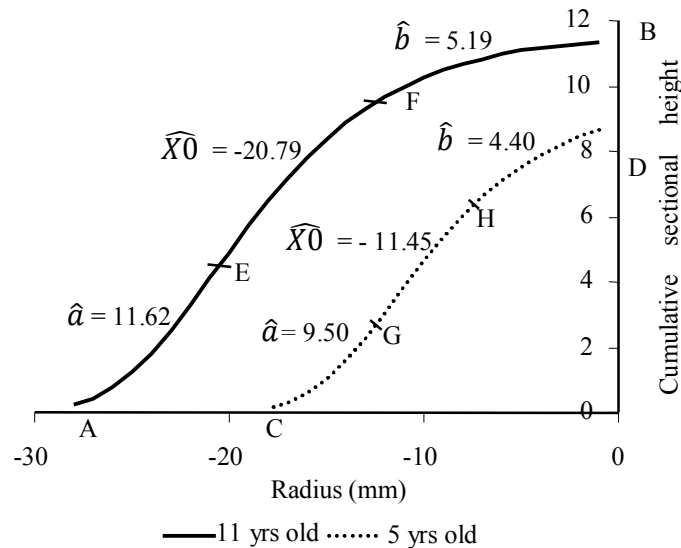
The shape of the predicted taper curve is mainly dependent on the fitted parameter values of ( $\hat{a}$ ,  $\hat{b}$  and  $\widehat{X0}$ ) in the taper model equation. The preliminary analysis of the data showed that the variable  $\Delta d$  was changing with aging. The fitted parameters values  $\hat{a}$ ,  $\hat{b}$  and  $\widehat{X0}$  were also the changing with the changes of  $d_{1,3}$  and  $\Delta d$  in different growing seasons (Table 3). The fitted parameter values had big influences in the rate and shape of taper stems. The fitted parameters values are presented in Table 3.

Table 3: Fitted parameters values for the function  $\hat{h}_t = \hat{a} * \text{EXP}(-\text{EXP}(-(-r_t - \widehat{X0})/\hat{b}))$ .

Tree_id	Age	$d_{1,3}$	$\Delta d$	$\hat{a}$	$\hat{b}$	$\widehat{X0}$
1	11	23,81	1,58	11,62	5,19	-20,79
1	10	22,96	1,41	10,70	4,85	-19,86
1	9	21,85	1,30	10,08	4,62	-18,64
1	8	20,65	1,29	10,02	4,60	-17,32
1	7	19,56	1,31	10,14	4,64	-16,12
1	6	17,65	1,20	9,52	4,42	-15,02
1	5	15,32	1,12	9,50	4,40	-11,45
1	4	12,36	1,11	9,85	4,54	-8,20
1	3	9,61	1,40	10,62	4,82	-5,17
1	2	6,42	1,35	10,33	4,72	-1,66
1	1	4,91	2,12	14,69	6,72	-0,02
2	11	24,13	1,26	9,87	4,54	-21,15
2	10	23,45	1,15	9,26	4,31	-20,40
2	9	22,72	1,13	9,15	4,27	-19,60
2	8	21,97	1,12	9,10	4,25	-18,77
2	7	21,07	1,11	9,02	4,22	-17,78
2	6	18,72	1,10	8,98	4,21	-15,19
2	5	16,27	1,32	10,19	4,66	-12,50
2	4	13,22	1,40	10,65	4,83	-9,15
2	3	10,44	1,77	12,72	5,61	-6,08
2	2	7,30	1,67	12,16	5,40	-2,63

2	1	5,63	2,27	15,50	6,65	-0,79
3	11	21,65	1,50	11,21	5,04	-18,42
3	10	20,96	1,48	11,09	5,00	-17,65
3	9	19,89	1,46	10,96	4,95	-16,48
3	8	18,81	1,40	10,64	4,83	-15,29
3	7	17,75	1,34	10,32	4,71	-14,13
3	6	16,10	1,30	10,06	4,61	-12,31
3	5	14,27	1,32	10,17	4,65	-10,29
3	4	11,51	1,27	9,91	4,56	-7,27
3	3	9,02	1,26	9,88	4,54	-4,52
3	2	5,71	1,71	12,39	5,49	-0,88
3	1	4,01	1,54	11,44	5,13	-0,99
4	11	26,10	1,46	10,97	4,95	-23,31
4	10	25,55	1,31	10,13	4,64	-22,70
4	9	24,56	1,30	10,09	4,62	-21,61
4	8	23,74	1,30	10,07	4,62	-20,71
4	7	22,42	1,29	10,04	4,61	-19,26
4	6	19,78	1,28	9,95	4,57	-16,36
4	5	16,78	1,27	9,90	4,55	-13,06
4	4	13,37	1,26	9,86	4,54	-9,31
4	3	10,39	1,42	10,75	4,87	-6,03
4	2	5,96	1,57	11,60	5,19	-1,15
4	1	4,98	1,88	13,33	5,84	-0,08
5	11	28,01	1,16	9,28	4,32	-25,41
5	10	27,52	1,14	9,21	4,29	-24,87
5	9	26,83	1,13	9,15	4,27	-24,12
5	8	25,89	1,12	9,09	4,25	-23,08
5	7	24,67	1,11	9,04	4,23	-21,73
5	6	22,19	1,08	8,87	4,17	-19,01
5	5	18,59	1,17	9,33	4,34	-15,04
5	4	14,30	1,56	11,54	5,17	-10,33
5	3	10,46	1,35	10,35	4,72	-6,11
5	2	6,59	2,87	14,00	6,09	-1,85
5	1	4,93	1,95	13,75	5,99	-0,02
6	4	12,99	1,63	11,92	5,31	-8,89
6	3	10,18	1,52	11,30	5,08	-5,79
6	2	6,78	1,47	11,04	4,98	-2,06
6	1	4,94	1,41	10,71	4,86	-0,03
7	4	13,30	1,45	10,92	4,94	-9,23
7	3	10,27	1,52	11,30	5,08	-5,90
7	2	6,57	1,45	10,92	4,94	-1,83
7	1	4,99	1,44	10,86	4,91	-0,09
8	4	14,01	1,46	10,98	4,96	-10,01
8	3	12,04	1,41	10,70	4,85	-7,84
8	2	7,70	1,28	9,97	4,58	-3,07
8	1	4,97	1,42	10,76	4,88	-0,07
9	4	13,40	1,60	11,76	5,25	-9,34
9	3	12,04	1,47	11,03	4,98	-7,84
9	2	6,55	1,43	10,81	4,89	-1,81
9	1	4,99	1,42	10,75	4,87	-0,09
10	4	13,25	1,68	12,21	5,42	-9,18
10	3	11,25	1,58	11,65	5,21	-6,98
10	2	8,26	1,27	9,91	4,56	-3,69
10	1	4,92	1,42	10,75	4,87	-0,01

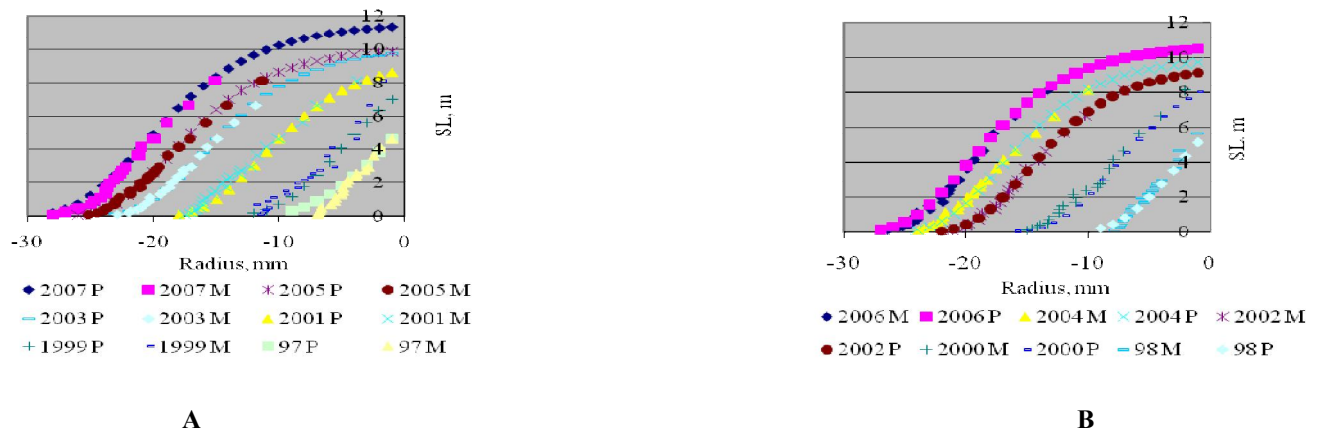
In figure 2, AB and CD is two predicted taper curves at the age of 11 and 5 years of sample tree no.1. AB is divided into three parts and CD is also divided into three parts. In AB, three segmented parts are AE, EF and FB respectively. AE represents the bottom part of the taper curve. EF and FB represent the middle and upper part of the taper curve. Similarly in CD, CG represents the bottom part and, GH and HD represent the middle and upper part of the taper curve. In the bottom part, the shape of taper curve is influenced by the changes of parameter value  $\hat{a}$ . In the middle part, the shape of taper curve tends to be influenced with the changes of parameter value  $\hat{X}0$ . In the upper part, the shape of taper curve is influenced when the changes of parameter value  $\hat{b}$ . How the shapes of taper curves were influenced with the changes of parameter values, were shown in the below (Figure 2), but more details of others sample trees in (Figure 3). In AB, the parameter values  $\hat{a} = 11.62$ ,  $\hat{b} = 5.19$  and  $\hat{X}0 = -20.79$ . In CD, the parameter values  $\hat{a} = 9.50$ ,  $\hat{b} = 4.40$  and  $\hat{X}0 = -11.45$ .



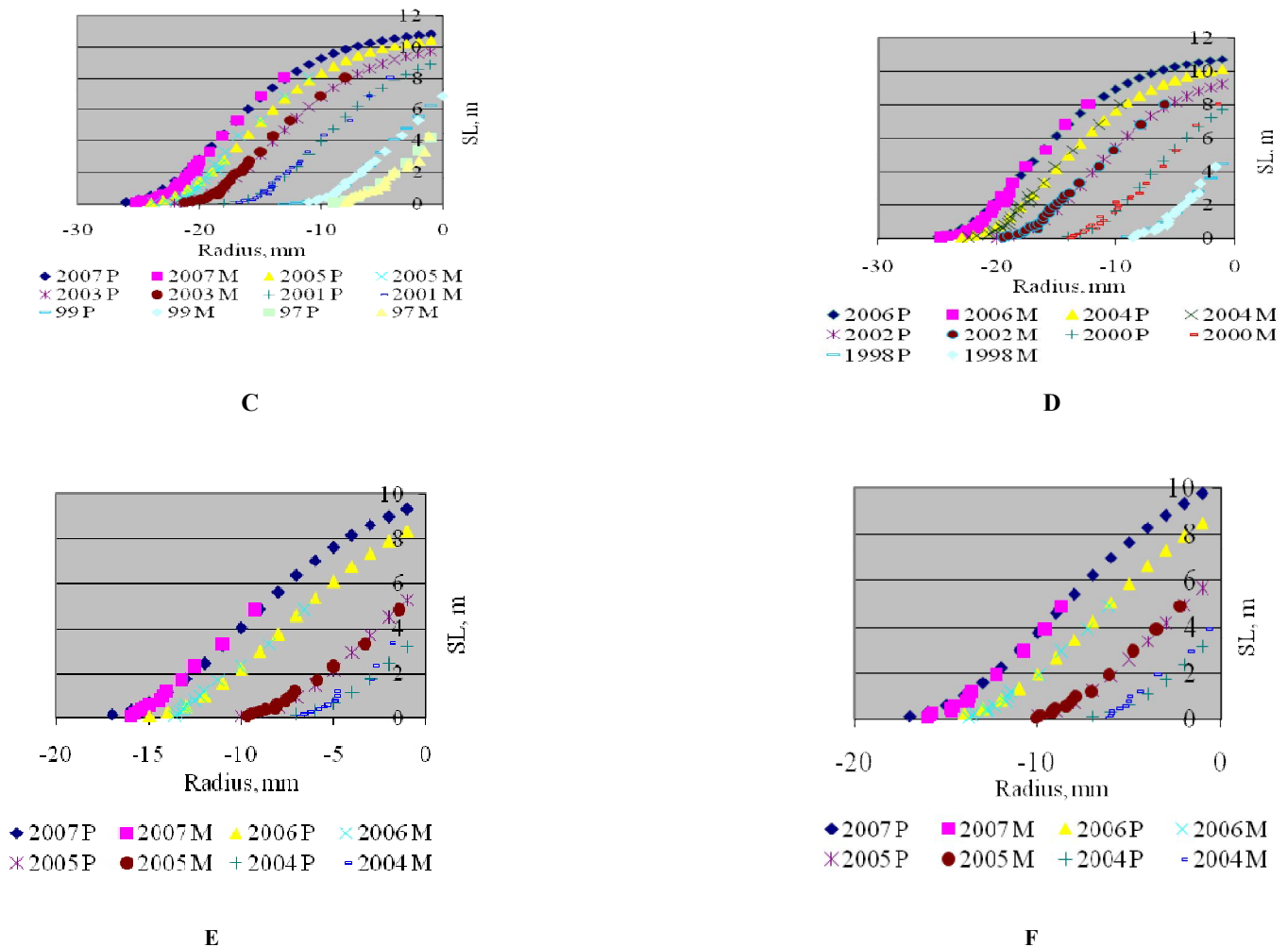
**Figure 2.** The influence of predicted parameter values ( $\hat{a}$ ,  $\hat{b}$  and  $\hat{X}0$ ) in the stem taper curves of 11 years and 5 years old of sample tree no. 1.

### 3.1 Taper curve fitting of sample trees

With the changes of  $d_{1.3}$  and  $\Delta d$  along the height, the parameter values changes, and influences on shape of taper curve. However, results showed that predicted taper curve and measured taper curve were fitted accurately in different growing seasons of both sized sample trees, but sometimes taper curves were not fitted especially in the early two or three growing seasons. There could be an over or under estimation of measured data due to the measurement errors because growth rings were not so distinct or applied model did not fit the measured data. But still the curve fitting for those growing seasons is acceptable (Figure 3). These data indicate that the taper model fitted accurately and we could use the applied taper model equation as a tool in standard volume equation to observe the growth performances of studied species in terms of height and volume in different growing seasons for the forest management. Nevertheless, the curve fitting of early two growing seasons might be improved, if the visibilities of growth rings could be distinct, or to do more research for the development of applied model.







**Figure 3.** Taper curve fitting of sample trees in different growing seasons, **A** and **B** represented sample tree **1**, **C** and **D** represented sample tree **3**, **E** represented sample tree **8**, and **F** represented sample tree **10**. SL denoted cumulative sectional height in m. **P** means predicted taper curve and **M** expressed as measured taper curve.

### 3.2 Volumes distribution of sample trees

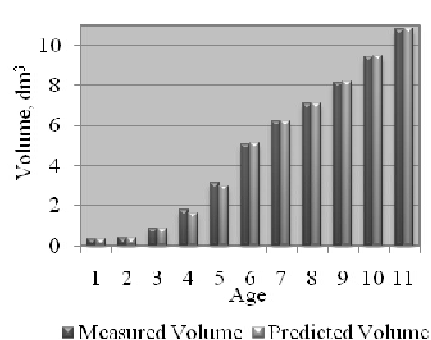
The total predicted sectional volumes and the true volumes of the sample trees for different growing seasons were almost the same. However, in big-sized sample trees, the highest volume was about  $14.09 \text{ dm}^3$  measured in the tree no. of 5 at the age 11, where the highest predicted volume was also found about  $14.08 \text{ dm}^3$  in the same tree at that age. For the small-sized sample trees, the highest volume about  $2.19 \text{ dm}^3$  was found in the tree no. of 8 at the age 4. On the other hand side, the highest volume was predicted about  $2.007 \text{ dm}^3$  in the tree no. of 8 at that age (Table 4a, Table 4b and Figure 4). In big-sized sample trees, the lowest volume was about  $0.236 \text{ dm}^3$  measured at the tree no. of 4 in the first growing season, but the lowest predicted volume was about  $0.229 \text{ dm}^3$  measured at the tree no. of 3 in that age. In small-sized sample trees, the lowest volume was about  $0.143 \text{ dm}^3$  measured at the tree no. of 6 in the first growing season, where the predicted lowest volume was about  $0.125 \text{ dm}^3$  in the same age and tree. The predicted and measured volumes of sample trees for others growing seasons is presented in the Table 4a, Table 4b and Figure 4. But in some growing seasons, measured volumes were slightly higher than the predicted volumes especially in the early growing seasons, but it was acceptable details were shown in the above (Table 4a, Table 4b and Figure 4). It could be due to measurement errors because the growth rings were not so clearly visible. Another reason could be that applied models did fit the measured data. Nevertheless, if we compare the predicted volumes and the measured volumes in different growing seasons, result shown that no big differences between them. These results indicate that the applied taper model worked appropriate as a tool in the standard volume equation for the estimation and calculation of sample trees of volumes in different growing seasons. The predicted volumes for the early growing seasons might be improved, if the visibilities of growth rings could be distinct, or to do more research for the development of applied model.

Table 4a: Estimations of true of volumes (based on measurements of disk volumes) of sample trees in different growing seasons.

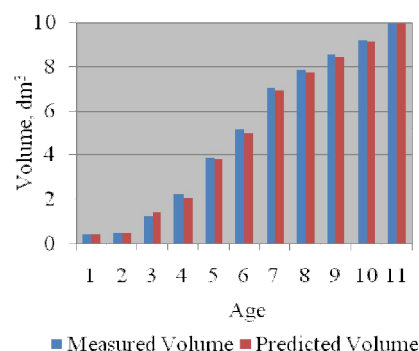
Tree no.	Age (years)										
	1	2	3	4	5	6	7	8	9	10	11
1	0.326	0.376	0.848	1.767	3.14	5.044	6.20601	7.075238	8.157001	9.462631	10.77994
2	0.436	0.475	1.272	2.232	3.87	5.16	7.04721	7.865596	8.54208	9.208438	10.08251
3	0.238	0.285	0.632	1.447	2.7	3.694	4.97337	5.879495	6.943787	7.828466	8.404225
4	0.236	0.307	1.177	2.253	4.15	6.515	8.84045	10.00661	11.10816	12.09477	12.99507
5	0.252	0.517	1.087	2.831	5.2	7.941	10.506	11.77964	12.80445	13.60739	14.09903
6	0.144	0.337	0.995	1.761							
7	0.15	0.315	1.021	1.884							
8	0.151	0.399	1.445	2.2							
9	0.163	0.299	1.484	2.082							
10	0.153	0.48	1.334	1.97							

Table 4b: Estimations' of predicted of volumes of sample trees in different growing seasons.

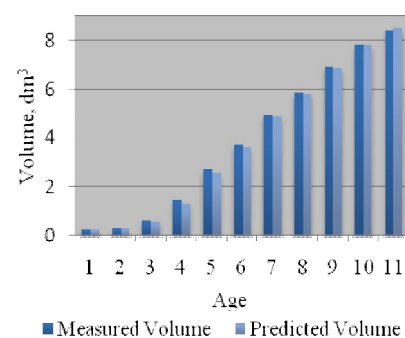
Tree no.	Age (years)										
	1	2	3	4	5	6	7	8	9	10	11
1	0.347	0.351	0.834	1.619	2.97	5.138	6.21835	7.11567	8.209048	9.476206	10.83024
2	0.466	0.484	1.4	2.077	3.76	4.992	6.92768	7.757102	8.439857	9.122095	9.973704
3	0.229	0.269	0.613	1.313	2.58	3.621	4.87675	5.805062	6.844649	7.828415	8.492467
4	0.238	0.254	1.074	2.056	4.04	6.403	8.90827	10.26376	11.10602	12.09204	12.99842
5	0.254	0.511	1.038	2.964	5.08	7.932	10.5066	11.77021	12.67903	13.12542	14.08189
6	0.125	0.321	1.006	1.827							
7	0.134	0.287	1.031	1.842							
8	0.129	0.381	1.408	2.008							
9	0.13	0.279	1.433	1.925							
10	0.125	0.467	1.348	1.96							



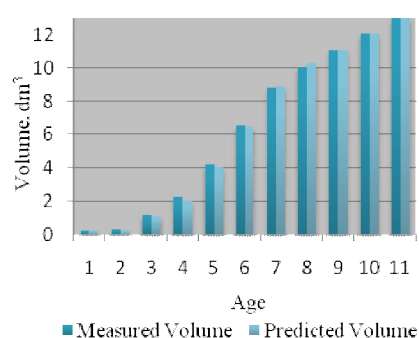
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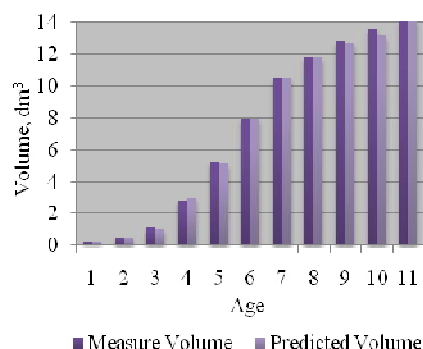
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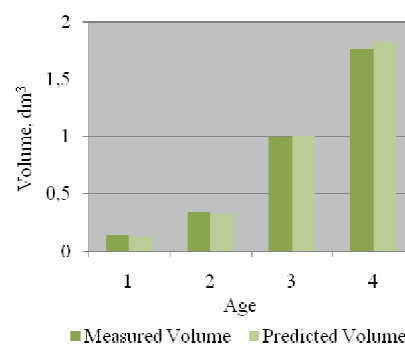
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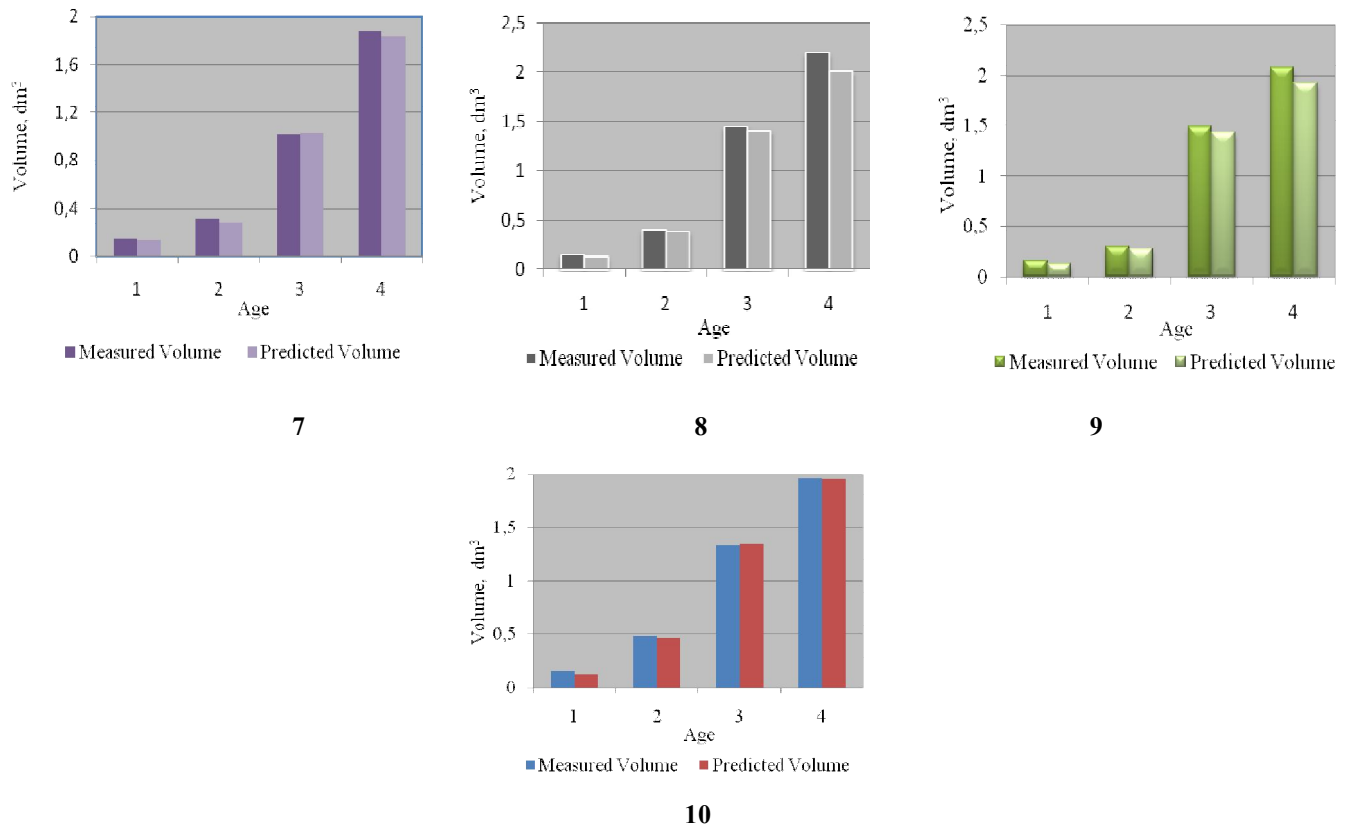
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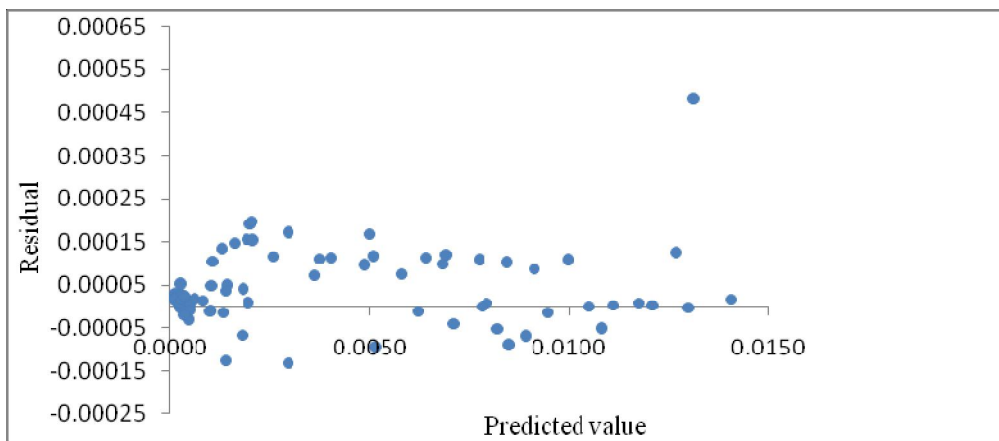
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**Figure 4.** The comparisons of predicted and measured volumes of 10 samples trees in different growing seasons.

### 3.3 Volumes Residual plot

The non-linear model that was considered ( $f = a * \text{EXP} (-\text{EXP} (-(-r - X0) / b)))$ ) fit the data well to predict the height which was further used as a tool in standard volume equations ( $\hat{V} = \sum_{i=1}^n \pi * r_i^2 * (\hat{h}_i - \hat{h}_{i-1})$ ) for the estimations of volumes in different growing seasons. The residual analysis did suggest that error variance was non-homogeneous (Figure 5). The estimator was not discarded since it performed reasonably well in predictions. The bias percentage was 0.94%, and which was less than 1%. This data indicates that applied regression models based on controlled variables i.e. (Ad) and DBH or  $d_{1.3}$  worked well in the volume function for the studied species to predict the volumes with lowest error (Figure 5). The applied taper model could be improved if we consider more variables in the sub-model 1 and sub-model-2 or do more research so that bias percentage reduced (less than 0.94%) in the volume function.



**Figure 5.** Residual plot (residuals versus predicted) of the applied functions for stems of volumes in different growing seasons.

## 4 DISCUSSION AND CONCLUSIONS

### 4.1 Discussion

One basic principle of a suitable model is that commonly available software can be used and the number of needed measurements is reasonable. In this study, no remarkable differences were found between the predicted volumes and the measured volumes of stems in different growing seasons. These results indicate that the applied functions were appropriate for the studied species to investigate the annual volume growth rate. In addition, the predicted taper curves which were developed from the model equation, fitted accurately to the measured taper curves of sample trees in different years. But in some growing seasons especially in the early growing seasons, the measured taper curves were not fitted accurately to the predicted taper curves, as well as little bit differences observed between the predicted volumes and the measured volumes, but still acceptable. The reason could be over or under estimation of data due to the measurement errors during the annual growth rings analysis. The growth rings were not so distinct. The form of a tree usually is not as ideal as models predict. Due to this reason it is realistic that applied model would not fully fit the measured data.

The main model consists of three sub-models. Three sub-models were developed based on two variables i.e. ( $\Delta d$ ) ( $\Delta d = r_{1.3} - r_2$ , where  $r_{1.3}$  and  $r_2$  denote as radius at 1.3 m and 2 m height respectively) and  $d_{1.3}$ . The sub-models are based on the theoretical properties of the applied Gompertz model. The empirical data gave evidence for the sub-models even if due to the limited data the relation between two sub-model parameters and radius based variables were poor. The resultant  $R^2$  values were 15%, 18% and 95% respectively in three sub-model equations. The basic assumption of the sub-models 1 and 2 (difference of radius at 1.3 m and 2 m height) is most likely too simple and for describing the complexity of taper changes more measured information is needed. Consequently, we could think another variables or higher variables especially for sub-model 1 and sub-model 2 in order to increase  $R^2$  value, as well as to develop the more efficiency of sub-models and main model so that they can fit the measured data accurately for the taper curve fitting and also more efficiency in the volume function for the estimation of volumes in different years. On the basis of the model based operational analysis it is well concluded that increment of sample trees in terms of volume, DBH, height and diameter is faster. It could be determined from the study of increment curve of volume on the basis of Current Annual Increment (CAI) theory that the studied species is a very fast growing short rotation species.

Benbrahim and Gavaland (2003) developed the stem taper function for individual trees of two poplar hybrid clones grown on a short-rotation coppice. The function is being used to estimate both diameter at a given height and height for a given top diameter but the model could not be integrated to calculate volumes (total volume, merchantable volume), which were estimated by numerical integration. Different types of taper functions have been used to describe the stem profile, and to measure DBH, diameter at different height and volume of trees (Wang et al. 2007). Very recently, Wang et al. (2007) proposed taper modeling on Taiwania Plantation in Liukuei based on different types of taper functions. They found that sigmoid-form approach could not predict the rate of taper in the bottom part of the stem but very well prediction in the middle part or canopy end of stem axis. In this study, Gompertz sigmoid functional form was applied, and it functioned well for the taper curve fitting and the prediction of total volumes with reasonable bias in the volume function in different growing seasons. The parameter values in the taper model had positive co-relation with the shape of taper curve. The use of parameters could be further developed for minimizing practical field measurement and for finding relation between properties of stem development and taper curves.

Taper curve method has widely used for estimating log properties of long rotation tree species. Malimbwi and Philip (1989) fitted a non-linear taper model equation of Mexican weeping pine in Tanzania for calculating the individual log volume. They concluded that developed taper model equation has very low prediction error for the volume of trees. Brooks et al. (2008) developed a compatible stem volume and taper equations for different species in Turkey. The models provided needed merchantable stem volume and diameter estimates in the bole of the tree based on the 10 relative height classes examined for the species. In this study, applied Gompertz sigmoid models also fitted better to provide sectional height which was used as a reliable tool in volume function to estimate the stem volume, where radius was considered as predictor variable.

However, most of the researchers has tried to explain the shape of tree form, rate of taper in diameter inside or outside bark along the height, calculation of merchantable log volume by taper functions in current growing season. The applied taper models of short rotation species (willow) to predict the height and further the volume of stems has not been used in the forestry research. The opportunity to analyze past volume development on the basis of annual growth ring measurements using such common software tools as Windendro and Excel would provide a method to make growth predictions even if the number of trees would be limited. The development of this kind of methodology is important if the task is to find new promising fast growing species for short rotation plantations. The applied taper model gives good preconditions for further development since it seems to be accurate in past growth analysis and we could calculate or estimate the volume increment of existing willow stands by applying taper functions in the volume functions

### 4.2 Conclusions

Short rotation forests (SRF) will provide multifunctional environmental services such as bioenergy for climate change mitigation, phytoremediation (i.e. taking up heavy metals to purify polluted soils) of degraded sites, nutrient management, living snow fences, stream bank stabilization, rehabilitation of fragile ecosystems (including combating desertification) and forest landscape restoration, bioengineering (water and wind erosion, and protective structures) in various geographic regions in the World. Fast growing species can be bred and selected for highest biomass production within short time. Willows are typical examples suitable for

SRWC plantations in temperate and boreal zone. Wasteland and marginal land (including abandoned agricultural fields and set-aside peat harvesting and mining areas) are challenges for improved wood biomass production in many countries. To get maximum utilization of wasteland resources without conflict of agricultural production land and afforestation of industrial sites new species, sub-species and clones need to be studied intensively for fulfilling the challenging targets of green engineering based bio-economy.

Growth and yield modeling for stand development has been a widespread effort in forestry during the past century. Forest managers, planners and policy makers forecast the outcomes of different types of forest use for making wise decisions for sound and balanced forest ecosystem management. The efficient and readily understandable models of growth and yield are invaluable tools for forest managers. They make it possible to predict future yields, and to explore the effect of silvicultural options. The past growth analysis applying taper function is used mainly to predict volumes inside and/or outside the bark of logs, tops, and stumps on standing trees. This well known analysis should be developed for the use of SRF improvement for meeting the future needs of wood biomass production.

In addition, it is expected that the study can be focused to encourage the local people to participate in SRF biomass development in the various regions and to take benefits bio-fuel productions. The finding of the study will address to sustainable management of SRF in context of economic and ecological considerations and develop a new environmentally friendly production system.

The regression equations performed very well in the volume functions to estimates total sectional volume with minimum prediction error. The applied model fitted the measured data accurately. However, taper curve fitting of sample trees in some growing season especially in early two or three growing seasons in addition comparison of total sectional predicted volumes and measured volumes are not perfectly satisfactory. Therefore, still needs more research to develop applied model and emphasis on sub-model of linear regression for increasing the  $R^2$  value in regression equation for best fitting the measured data as well as lowest bias in the volume residual plot.

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