Seasonal Transmission Model of Malaria by Age Group of Population

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ABSTRACT

Malaria, one of the most well known diseases and leading cause of death in many countries, the most affected groups are young children and pregnant women. Malaria is an infectious disease that is transmitted between people through biting of an infected female Anopheles mosquito. In Thailand, season and age of patients influence to the transmission of this disease. In this study, we construct the transmission model of Malaria by including effects of age and season into the model. Steady state solutions of our model are found. The conditions for local stable of steady state solutions are given. The infection rate of this disease in each age group of human is given. The trajectories of solutions are shown to compare the different situations.

KEYWORDS: Anopheles mosquito, Age group, Anopheles mosquito, local stable, Malaria, seasonal transmission model.

INTRODUCTION

Malaria is transmitted between human by the biting of the Anopheles mosquitoes. There are 480 species of Anopheles, only about 50 species can transmit malaria, with every continent having its own species of these mosquitoes [1,2]. This type of mosquito becomes infected with one of the four plasmodium parasites that cause malaria in humans, through a previous blood meal from an infected person. Four types of human malaria, namely, Plasmodium falciparum, Plasmodium vivax, Plasmodium malariae and Plasmodium ovale. Malaria with Plasmodium falciparum causes the most serious illness and it is the most common cause of infection in Africa and South East Asia, and is responsible for about 80% of all malaria cases and about 90% of deaths [3,4]. The general symptoms of Malaria include headache, nausea, fever and flu. These symptoms can vary depending on the type of plasmodium that caused infection. Symptoms usually appear between 10 and 15 days after the infected mosquito bite. If it is not treated, malaria can quickly become life-threatening by disrupting the blood supply to vital organs [3]. Not only human are the host (vertebrate) of human but also the Anopheles mosquitoes are also a host (invertebrate). This disease can also be transmitted accidentally through blood transfusion when the donor has the malaria parasite. This is one of the reasons why people who have been infected with disease can never donate blood. Congenital infection of a newborn from an infected mother also happens, but it is comparatively rare [5]. Generally, transmission of malaria depends on the presence of the relationship between the three basic factors: the host, the agent, and the environment. Temperature and humidity are the most important environment factors for the growth of Malaria parasites. Malaria parasites stop developing in the mosquitoes when the temperature is below 16 °C. The best condition for the development of disease is within the range of 20 – 30 °C and the average relative humidity is at least 60% [6]. A high relative humidity lengthens the life of the mosquito and it enables them to live long enough to transmit the infection to several persons. From the data of Malaria in Thailand during 2003 and 2010 as shown in fig.1 and fig.2, we can see that age and season effect to the transmission of this disease[7].

Fig. 1. Situation of Malaria cases classified by age group of human [7].

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Fig. 2. Situation of Malaria cases classified by month [7].

The transmission of malaria was first explained by the Ross-MacDonald (RM) model [8]. However, this model is only suitable for the transmission of *Plasmodium falciparum* malaria since it does not consider the possibility of relapses of the illness. In 2007, we introduced the mathematical model for the transmission of *Plasmodium vivax* and considering the relapse of *Plasmodium vivax* but age group and season are not considered in our model[9]. We has introduced a mathematical model [10] to describe the transmission of malaria by separating the human into juvenile and adults groups but we did not consider age group of human and season in Thailand. To develop the model more appropriate in Thailand, age of human and season should be considered in this study.

**MATHEMATICAL MODEL**

Age group of human and season are considered in our model. We separate population into two groups, ie. human and vector populations. Human is divided into 4 classes such that susceptible, infectious, dormant and recovered classes. Vector is divided into 3 classes such that susceptible, infectious and recovered classes. Each human class is subdivided into 6 age groups such as 0-4 years old, 5-9 years old, 10-14 years old, 15-24 years old, 25-34 years old and more than 35 years old. In Thailand, there are three seasons: The dry (or cool season) is between November and February. The hot season is between March and June. The rainy season is between July and October. The transmission model can be described based on the diagram in Fig 3.
The variables and parameters of our model are defined in Table 1.

<table>
<thead>
<tr>
<th>Variables/Parameters</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_h$</td>
<td>Birth rate of human population.</td>
</tr>
<tr>
<td>$\mu_h$</td>
<td>Death rate of human population.</td>
</tr>
<tr>
<td>$\gamma_i'$</td>
<td>Rate at which human change from one age group to the another age group.</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Rate at which human change from infected status to be dormant status.</td>
</tr>
<tr>
<td>$\beta_{vj}$</td>
<td>Infection rate of Malaria from vector to human age group j.</td>
</tr>
<tr>
<td>$\frac{1}{S_j}$</td>
<td>Rate at which infectious human in whom some hypnozoites remain in the liver</td>
</tr>
<tr>
<td>$S_j(t)$</td>
<td>Number of susceptible human of group age j; $j = 1, 2, 3, 4, 5, 6$</td>
</tr>
<tr>
<td>$I_j(t)$</td>
<td>Number of infectious human of group age j; $j = 1, 2, 3, 4, 5, 6$</td>
</tr>
<tr>
<td>$D_j(t)$</td>
<td>Number of susceptible human of group age j; $j = 1, 2, 3, 4, 5, 6$</td>
</tr>
<tr>
<td>$R_j(t)$</td>
<td>Number of recovered human of group age j; $j = 1, 2, 3, 4, 5, 6$</td>
</tr>
<tr>
<td>$S_{ij}(t)$</td>
<td>Number of susceptible vector at season i; $i=1,2,3$</td>
</tr>
<tr>
<td>$E_{ij}(t)$</td>
<td>Number of exposed vector at season i; $i=1,2,3$</td>
</tr>
<tr>
<td>$I_{ij}(t)$</td>
<td>Number of infectious vector at season i; $i=1,2,3$</td>
</tr>
</tbody>
</table>

Table 1. Definition of variables/parameters in our model.

The rate of change for human population can be described as following equations:

**For the first age group:**

\[
\frac{d}{dt}S_{1j}(t) = b_hN_T + (1 - \varepsilon)S_{1j}I_{1j}(t) - \beta_{v1j}S_{1j}(t)I_{1j}(t) - (\gamma_1 + \mu_h)S_{1j}(t) + S_1D_{1j}(t) + \theta R_{1j}(t),
\]

**For age group $j; j = 2, 3, 4, 5$:**

\[
\frac{d}{dt}S_{hj}(t) = S_3D_{hj}(t) - \beta_{hj}S_{hj}(t)I_{hj}(t) + (1 - \varepsilon)S_{hj}I_{hj}(t) + \gamma_{j-1}S_{hj-1}(t) - \gamma_jS_{hj}(t) + \mu_hS_{hj}(t) + \theta R_{hj}(t),
\]

\[
\frac{d}{dt}R_{hj}(t) = \beta_{hj}I_{hj}(t) - (\theta + \gamma_j + \mu_h)R_{hj}(t),
\]
\[
\frac{d}{dt} R_{h,j}^i (t) = \beta I_{h,j}^i (t) - (\theta + \gamma_j + \mu_h) R_{h,j}^i (t) + \gamma_j R_{h,j-1}^i (t). \tag{8}
\]

For the last age group:
\[
\frac{d}{dt} S_{h,6}^i (t) = \gamma_5 S_{h,5}^i (t) + S_{h,6}^i D_{h,6}^i (t) - \beta_{h,6} S_{h,6}^i (t) I_{h,6}^i (t) - (1 - \epsilon) S_{h,6}^i (t) F_{h,6}^i (t) - \mu_h S_{h,6}^i (t) - \theta R_{h,6}^i (t). \tag{9}
\]
\[
\frac{d}{dt} I_{h,6}^i (t) = \beta_{h,6} S_{h,6}^i (t) I_{h,6}^i (t) - (1 - \epsilon) S_{h,6}^i (t) I_{h,6}^i (t) + \gamma_5 I_{h,5}^i (t) + \gamma_5 S_{h,6}^i (t) + S_{h,6}^i D_{h,6}^i (t) - (\mu_h + \beta) I_{h,6}^i (t). \tag{10}
\]
\[
\frac{d}{dt} D_{h,6}^i (t) = \epsilon S_{h,6}^i (t) ( t - (S_2 + S_3 + \mu_h) D_{h,6}^i (t) + \gamma_5 D_{h,6}^i (t), \tag{11}
\]
\[
\frac{d}{dt} R_{h,6}^i (t) = \beta I_{h,6}^i (t) + \gamma_5 R_{h,5}^i (t) - (\theta + \mu_h) R_{h,6}^i (t). \tag{12}
\]

\( j = 1 \) means group of human age 0-4 years old.
\( j = 2 \) means group of human age 5-9 years old.
\( j = 3 \) means group of human age 10-14 years old.
\( j = 4 \) means group of human age 15-24 years old.
\( j = 5 \) means group of human age 25-34 years old.
\( j = 6 \) means group of human age more than 35 years old.

We define \( N_T \) is the total human population. This means that
\[
N_T = \sum_{j=1}^{6} \left( S_{h,j}^i (t) + I_{h,j}^i (t) + D_{h,j}^i (t) + R_{h,j}^i (t) \right).
\]

Because we suppose that \( N_T \) is the constant number, thus the dynamical change for \( N_T \) equals to zero. From
\[
\frac{d}{dt} N_T = 0; \text{ we can have } b_h = \mu_h \text{ or birth rate and death rate of human are equivalent.}
\]

The rate of change for vector population can be described as following equations:
\[
\frac{d}{dt} S_{v,i}^j (t) = M_i - \beta_{v,i} S_{v,i}^j (t) I_{v,i}^j (t) - \mu_v S_{v,i}^j (t). \tag{13}
\]
\[
\frac{d}{dt} E_{v,i}^j (t) = \beta_{v,i} S_{v,i}^j (t) I_{v,i}^j (t) - \lambda_v E_{v,i}^j (t) - \mu_v E_{v,i}^j (t). \tag{14}
\]
\[
\frac{d}{dt} I_{v,i}^j (t) = \lambda_v E_{v,i}^j (t) - \mu_v I_{v,i}^j (t). \tag{15}
\]

We define \( N_V = (S_{v,i}^j (t) + E_{v,i}^j (t) + I_{v,i}^j (t)) \) is the total vector and supposed it to be constant; this means that\[
\frac{d}{dt} N_V = 0 \text{ where } S_{v,i}^j (t) = \sum_{i=1}^{3} S_{v,i}^j (t), \quad E_{v,i}^j (t) = \sum_{i=1}^{3} E_{v,i}^j (t) \quad \text{and} \quad I_{v,i}^j (t) = \sum_{i=1}^{3} I_{v,i}^j (t). \tag{16}
\]

From setting the rate of change for vector population equals to zero, we obtain
\[
N_V = \frac{3 M_i}{\mu_v}.
\]

We suppose that number of vectors is different in each season. We define \( i = 1 \) means the dry or cool season, \( i = 2 \) means the hot season and \( i = 3 \) means the rainy season.

We reduce eqs.(1) to (15) by letting
\[
s_{h,j} = S_{h,j}^i / N_{T,j}, \quad i_{h,j} = I_{h,j}^i / N_{T,j}, \quad d_{h,j} = D_{h,j}^i / N_{T,j}, \quad r_{h,j} = R_{h,j}^i / N_{T,j}, \tag{17}
\]
\[
s_{v,i} = S_{v,i}^j / N_{V,i}, \quad e_{v,i} = E_{v,i}^j / N_{V,i}, \quad i_{v,i} = I_{v,i}^j / N_{V,i}.
\]

where \( N_{T,j} \) is the total human population for age group \( j(1,2,3,4,5,6) \) and \( N_{V,i} \) is the total vector for season \( i(1,2,3) \). Thus, The normalized equations become:

**For human population:**

The first age group:
\[
\frac{d}{dt} s_{h,1} (t) = b_h + (1 - \epsilon) S_{h,1} (t) - \beta_{h,1} s_{h,1} (t) (t \epsilon v_1 (t) N_{V,2} + i v_2 (t) N_{V,3}) - (\gamma_1 + \mu_h) s_{h,1} (t) + S_{h,1} (t) + \theta s_{h,1} (t), \tag{17}
\]
\[
\frac{d}{dt} i_{h,1} (t) = \beta_{h,1} s_{h,1} (t) (t \epsilon v_1 (t) N_{V,1} + i v_2 (t) N_{V,2} + i v_3 (t) N_{V,3}) - (1 - \epsilon) S_{h,1} (t) - \gamma_1 i_{h,1} (t) - \epsilon S_{h,1} (t) + S_{h,1} (t) - (\beta + \mu_h) i_{h,1} (t), \tag{18}
\]
\[
\frac{d}{dt} n_i(t) = \beta_i(t) - (\theta_i + \mu) n_i(t),
\]
\[
\frac{d}{dt} h_j(t) = \beta_j h_j(t) - (\theta_j + \mu) h_j(t),
\]
\[
\frac{d}{dt} S_j(t) = S_j \frac{\beta_j S_j h_j(t)}{s_j(t)} + \gamma_j - \theta_j \frac{n_j(t)}{s_j(t)} - \theta_j h_j(t) - \mu h_j(t).
\]

**Age group: j = 2, 3, 4, 5:**

\[
\frac{d}{dt} s_{h_j}(t) = S_j \frac{\beta_j h_j(t)}{s_j(t)} + \gamma_j - \theta_j \frac{n_j(t)}{s_j(t)} - \theta_j h_j(t) - \mu h_j(t).
\]

**The last age group:**

\[
\frac{d}{dt} n_6(t) = \beta_6 n_6(t) + S_6 \frac{\beta_6 h_6(t)}{s_6(t)} - \gamma_6 - \theta_6 \frac{n_6(t)}{s_6(t)} - \theta_6 h_6(t) - \mu h_6(t).
\]

**For vector population:**

\[
\frac{d}{dt} s_{v_i}(t) = (M_i / N_v) - \beta_{v_i} s_{v_i}(t) \sum_{j=1}^{6} \frac{1}{i_j} N_{T_j} - s_{v_i}(t).
\]

\[
\frac{d}{dt} c_{v_i}(t) = \beta_{v_i} s_{v_i}(t) \sum_{j=1}^{6} \frac{1}{i_j} N_{T_j} - \lambda_v c_{v_i}(t) - \mu c_{v_i}(t).
\]

\[
\frac{d}{dt} i_{v_i}(t) = \lambda_v c_{v_i}(t) - \mu i_{v_i}(t).
\]

Let 

\[
S_j(t) = \sum_{j=1}^{6} s_{h_j}(t), i_j(t) = \sum_{j=1}^{6} i_{h_j}(t), d_h(t) = \sum_{j=1}^{6} d_{h_j}(t), \eta_j(t) = \sum_{j=1}^{6} \eta_j(t),
\]

\[
s_v(t) = \sum_{i=1}^{3} s_{v_i}(t), c_{v}(t) = \sum_{i=1}^{3} c_{v_i}(t), i_{v}(t) = \sum_{i=1}^{3} i_{v_i}(t).
\]

\[
\frac{d}{dt} s_{h_i}(t) = \mu h_i + (1 - \varepsilon) S_j h_j(t) - i_{v_i} (M / \mu) \frac{\beta_i s_{h_i}(t)}{s_{h_i}(t)} + S_2 d_{h_i}(t) - \mu h_i(t) + \theta (1 - s_h - d_h).
\]

\[
\frac{d}{dt} i_{h_i}(t) = i_{v_i} (M / \mu) \frac{\beta_i h_i(t)}{h_i(t)} - S_j h_j(t) + S_2 d_{h_i}(t) - (\beta + \mu) i_{h_i}(t).
\]

\[
\frac{d}{dt} d_{h_i}(t) = \varepsilon S_j h_j(t) - (S_2 + S_3 + \mu) d_{h_i}(t).
\]

\[
\frac{d}{dt} s_v(t) = \mu v - i_{h_i} N_T (\beta_v s_{v}(t) - \mu v s_v(t)),
\]

\[
\frac{d}{dt} e_v(t) = i_{h_i} N_T (\beta_v s_v(t) - \mu v e_v(t)),
\]

where 

\[
\beta_h = \frac{6}{6} \frac{\beta_i h_i(t)}{s_{h_i}(t)}, \beta_v = \frac{3}{3} \frac{\beta_i v_i s_{v_i}(t)}{s_{v_i}(t)}, \eta = \frac{6}{6} \frac{\eta_j}{N_T}, \mu_v = \frac{3}{3} \frac{\mu v}{N_v}.
\]

and 

\[
s_h + i_h + d_h + \eta = 1, s_v + e_v + i_v = 1.
\]
MODEL ANALYSIS

The steady state solutions \((s_{h_j}^*, i_{h_j}^*, d_{h_j}^*, r_{h_j}^*)\) for \(j = 1, 2, 3, 4, 5, 6\) and \((s_{v_i}^*, e_{v_i}^*, i_{v_i}^*)\) for \(i = 1, 2, 3\) are obtained by setting the right hand side of eqs. (17) to (31) equals to zero. The steady state solutions are as follows:

**For human population:**

The first age group:

\[
s_{h_1}^* = \frac{b_h + (1 - e)S_1i_{h_1}^* + S_d^* i_{h_1}^* + \theta_{h_1}^*}{(\beta_{hl}(i_{v_1}^* N_{v_1}^* + i_{v_2}^* N_{v_2}^* + i_{v_3}^* N_{v_3}^*) + \gamma_1 + \mu_h)},
\]

\[
i_{h_1}^* = \frac{\beta_{hl} s_{h_1}^* (i_{v_1}^* N_{v_1}^* + i_{v_2}^* N_{v_2}^* + i_{v_3}^* N_{v_3}^*) + S_2 d_{h_1}^*}{((1 - e)S_1 + \gamma_1 + eS_1 + (\beta + \mu_h))},
\]

\[
d_{h_1}^* = \frac{eS_1 i_{h_1}^*}{(S_2 + S_3 + \gamma_1 + \mu_h)}.
\]

\[
r_{h_1}^* = \frac{\beta_{hl}^*}{(\theta + \gamma_1 + \mu_h)}.
\]

**Age group j; j = 2,3,4,5:**

\[
s_{h_j}^* = \frac{S_3 d_{h_j}^* + (1 - e)S_1 i_{h_j}^* + \gamma_{j-1} s_{h_j-1}^* + \theta_{h_j}^*}{(\beta_{hl}(i_{v_1}^* N_{v_1}^* + i_{v_2}^* N_{v_2}^* + i_{v_3}^* N_{v_3}^*) + \gamma_j + \mu_h)},
\]

\[
i_{h_j}^* = \frac{\beta_{hl} s_{h_j}^* (i_{v_1}^* N_{v_1}^* + i_{v_2}^* N_{v_2}^* + i_{v_3}^* N_{v_3}^*) + S_2 d_{h_j}^* + \gamma_{j-1} i_{h_j-1}^*}{(S_1 + \gamma_j + \mu_h + \beta)}.
\]

\[
d_{h_j}^* (t) = \frac{eS_1 i_{h_j}^* (t) + \gamma_{j-1} d_{h_j-1}^* (t)}{(S_2 + S_3 + \gamma_j + \mu_h)}.
\]

\[
r_{h_j}^* (t) = \frac{\beta_{hl}^* (t) + \gamma_{j-1} r_{h_j-1}^* (t)}{(\theta + \gamma_j + \mu_h)}.
\]

**The last age group:**

\[
s_{h_6}^* = \frac{\gamma s_{h_5}^* (1) + S_3 d_{h_6}^* (1) + (1 - e)S_1 i_{h_6}^* (1) + \theta_{h_6}^* (t)}{(\beta_{hl}(i_{v_1}^* (1) N_{v_1}^* + i_{v_2}^* (1) N_{v_2}^* + i_{v_3}^* (1) N_{v_3}^*) + \mu_h)}.
\]

\[
i_{h_6}^* = \frac{\beta_{hl} s_{h_6}^* (0)(i_{v_1}^* (1) N_{v_1}^* + i_{v_2}^* (1) N_{v_2}^* + i_{v_3}^* (1) N_{v_3}^*) + \gamma s_{h_5}^* (0) + S_2 d_{h_6}^* (1)}{(\mu_h + \beta + S_1)}.
\]

\[
d_{h_6}^* (t) = \frac{eS_1 i_{h_6}^* (t) + \gamma s_{h_5}^* (1)}{(S_2 + S_3 + \mu_h)}.
\]

\[
r_{h_6}^* (t) = \frac{\beta_{hl}^* (t) + \gamma s_{h_5}^* (1)}{(\theta + \mu_h)}.
\]

**For vector population:**

\[
s_{v_i}^* (t) = \frac{M_i (N_i)}{(\beta_{vi}(i_{h_j} N_{v_j}) - \mu_v)},
\]

\[
e_{v_i}^* (t) = \frac{\beta_{vi} s_{v_i}^* (t)(i_{h_j} N_{v_j})}{(\lambda_v + \mu_v)},
\]

\[
i_{v_i}^* (t) = \frac{\lambda_v e_{v_i}^* (t)}{\mu_v}.
\]

From (32)-(36), the steady state solutions are given by
\[ s^*_h = \frac{\mu_v (\lambda_v + \mu_v) (\mu_v + \beta > \lambda_v N_T) (\mu_v^2 + (\beta + \mu_v) (\mu_v + S_2 + S_3) + S_1 (S_2 (1 - \epsilon) + S_3))}{\beta \mu_v > \lambda_v MN_T (\mu_v + S_2 + S_3)} \]  
(52)

\[ d^*_h = \frac{\beta}{\mu_v + S_2 + S_3} \]  
(53)

\[ s^*_v = \frac{\mu_v}{\beta + \mu_v > \beta > \lambda_v N_T} \]  
(54)

\[ e^*_v = \frac{\beta}{\mu_v + \lambda_v} \]  
(55)

\[ l^*_h = \frac{W (\mu_v + \theta)}{G} (B_0 - 1) \]  
(56)

\[ W = \mu_v^2 (\lambda_v + \mu_v) (\beta + \mu_v + S_1) (\mu_v + S_2 + S_3) \]

\[ G = (\beta) N_T ((\beta) \lambda_v M (\beta + \mu_v + S_1 + S_2 + S_3) + \mu_v (\lambda_v + \mu_v) (\mu_v + S_1 + S_2 + S_3) + S_1 (S_2 (1 - \epsilon) + S_3)) (\mu_v + \theta) \]

\[ + \beta (\mu_v + S_2 + S_3) ((\beta) \lambda_v M + \mu_v (\mu_v + \theta)) \]

(57)

and

\[ B_0 = \frac{1}{\beta + \mu_v + S_1} \left( \frac{\beta}{\mu_v} \lambda_v M N_T + \frac{\varepsilon S_2}{\beta N_T} \right) \]

(58)

It can be seen from the above equations that the steady state solution is positive for \( B_0 > 1 \).

A. Local Stable

We determine the local stable of steady state solutions from the signs of eigenvalues of Jacobian matrix of the right hand side of eqs. (32) to (36). If the eigenvalues have negative real part, then the steady solution will be locally stable[11].

For equations (32)-36), the Jacobian matrix evaluated at the steady state solution is given by

\[ J_{B_0} = \begin{pmatrix}
-\mu_h - (\beta_h) M/\mu_v (1 - s_v^* - e_v^*) & 0 & S_1 - 0 & S_2 - 0 & S_3 - 0 & \frac{\beta_h}{\mu_v} M s_h^* \\
0 & -\mu_h - S_1 & S_2 & S_3 & 0 & \frac{\beta_h}{\mu_v} M s_h^* \\
0 & -\varepsilon s_1 & -\mu_h - S_2 - S_3 & 0 & 0 & \frac{\beta_h}{\mu_v} M s_h^* \\
0 & -s_h^* N_T s_v^* & 0 & -\mu_v - (\beta_h) N_T s_h^* & 0 & \frac{\beta_h}{\mu_v} M s_h^* \\
0 & 0 & 0 & 0 & 0 & \frac{\beta_h}{\mu_v} M s_h^* \\
\end{pmatrix} \]

The eigenvalues are obtained by solving the characteristic equation; \( \det \left( J - \lambda I_4 \right) = 0 \) where \( I_4 \) is the identity matrix dimension 4x4. The characteristic equation for the steady state solution is given by

\[ \lambda^4 + Z_3 \lambda^3 + Z_2 \lambda^2 + Z_1 \lambda + Z_0 = 0 \]

(59)

where

\[ (-1 + B_0)^2 < \beta > \frac{\beta}{\mu_v} (\lambda_v + \mu_v) N_T \frac{\mu_v}{\beta_N W (\mu_v + \theta)} + \frac{G_1^2 (\beta_v + \mu_v \lambda_v) \frac{\mu_v}{\beta_N W (\mu_v + \theta)} + \frac{G_1^2 (\beta_v + \mu_v \lambda_v) \frac{\mu_v}{\beta_N W (\mu_v + \theta)}}{\beta > \frac{\beta}{\mu_v} (\lambda_v + \mu_v) N_T \frac{\mu_v}{\beta_N W (\mu_v + \theta)}} \]

(60)

\[ Z_0 = \frac{G_1}{G_1 \lambda_v (\lambda_v + \mu_v) G_1 \lambda_v (-1 + B_0) \beta > \frac{\beta}{\mu_v} (\lambda_v + \mu_v) N_T \frac{\mu_v}{\beta_N W (\mu_v + \theta)}} \]

(61)
\[\begin{align*}
\beta(\mu + S + S_1) &> \frac{\lambda_1}{\mu_1 + \mu + \mu_2} \left( \left( \lambda_1 M(\mu + S + S_1) + \lambda_2 \left( \mu + \mu_2 \right) \right) \left( \lambda_1 + \mu \right) \right) + \left( 1 - B_0 \right) \left[ \frac{\lambda_1}{\mu_1 + \mu + \mu_2} \left( \left( \lambda_1 M(\mu + S + S_1) + \lambda_2 \left( \mu + \mu_2 \right) \right) \left( \lambda_1 + \mu \right) \right) \right] \\
&+ \left( 1 - B_0 \right) \left[ \frac{\lambda_1}{\mu_1 + \mu + \mu_2} \left( \left( \lambda_1 M(\mu + S + S_1) + \lambda_2 \left( \mu + \mu_2 \right) \right) \left( \lambda_1 + \mu \right) \right) \right]
\end{align*}\]

We use Routh-Hurwitz criteria for determining signs of all eigenvalues. If they satisfy the following conditions, then a steady state solution will be local stable. The Routh-Hurwitz criteria is given by:

1. \( Z_1 > 0 \); \( i=0,1,2,3,4,5 \)
2. \( Z_3 Z_4 Z_2 \cdot (Z_2^2 + Z_3^2 Z_4) > 0 \)
3. \(( Z_4 Z_1 - Z_0 ) ( Z_4 Z_3 Z_2 \cdot (Z_2^2 + Z_3^2 Z_4) ) - ( Z_0 (Z_4 Z_3 - Z_2)^2 + Z_4 Z_2^2 ) > 0 \)

From our calculations, we found that i) to iii) are satisfied when \( B_0 > 1 \) where

\[ B_0 = \frac{1}{\beta + \mu + S_1} \left( \left( \frac{\lambda_1}{\mu + \mu_2} \right) \left( \lambda_1 M + \lambda_2 \right) \right) + \frac{\lambda_2}{\mu_1 + \mu + \mu_2} \left( \frac{\lambda_1}{\mu_1 + \mu + \mu_2} \left( \lambda_1 M + \lambda_2 \right) \right) \]

Thus the steady state solution is local stable for \( B_0 > 1 \).

We simulate eqs. (32)-(36) by using numerical method. The parameters satisfy the real life observations. The trajectories of the solutions will lead to a steady state solution.
The value of parameters are $\mu_h = 1/(365 \times 70)$, $\varepsilon = 0.8$, $M = 100,000,000$, $\mu_v = 1/25$, $\langle \beta_h \rangle = 0.000000001$, $\langle \beta_v \rangle = 0.000001$, $S_1 = 1/30$, $S_2 = 1/(365 \times 2)$, $S_3 = 1/(365 \times 3)$, $\theta = 1/(365 \times 5)$, $\beta = 1/30$, $N_i = 6000000$; $\lambda_v = 1/14$, $B_0 = 3600$. The solution oscillates to the steady state solution $(0.13, 0.013, 0.14, 0.71, 0.97, 0.01, 0.02)$.

We use real data of Malaria patients and simulate $\beta_{hj}$ for $j = 1, 2, 3, 4, 5$ by using eqs. (17) – (31). The results are shown in fig.6.
We can see that the highest infection rate of this disease is occurred in the human age group 10-14 years. This is corresponding to fig. 1.

**DISCUSSION AND CONCLUSION**

In this paper, we analyze the transmission of Malaria by considering age group of human and season in Thailand. The basic reproductive number is defined by

$$ B_0 = \frac{1}{\beta + \mu_h + S_1} \left( \frac{\beta}{\mu_v (\lambda_v + \mu_v)} + \frac{\varepsilon S_v S_2}{\mu_h + S_2 + S_3} \right). $$

If $B_0 > 1$, the steady state solution will be local stable, then we can reduce the outbreak of this disease. The basic reproductive number are used for reducing the outbreak of different diseases\cite{9-10,12-14}.

We compare the numerical solutions for the different basic reproductive numbers.
Fig. 7 Time series solution for the different basic reproductive number.

7a) The value of parameters are $\mu_h = 1/(365 \times 70)$, $\varepsilon = 0.8$, $M = 100,000,000$, $\mu_v = 1/25$, $\langle \beta_h \rangle = 0.0000001$, $\langle \beta_v \rangle = 0.00001$, $S_1 = 1/30$, $S_2 = 1/(365 \times 2)$, $S_3 = 1/(365 \times 3)$, $\theta = 1/(365 \times 5)$, $\beta = 1/30$, $N_t = 6000000$; $\lambda_v = 1/14$, $B_0 = 360000$.

7b) The value of parameters are $\mu_h = 1/(365 \times 70)$, $\varepsilon = 0.8$, $M = 100,000,000$, $\mu_v = 1/25$, $\langle \beta_h \rangle = 0.000000001$, $\langle \beta_v \rangle = 0.000001$, $S_1 = 1/30$, $S_2 = 1/(365 \times 2)$, $S_3 = 1/(365 \times 3)$, $\theta = 1/(365 \times 5)$, $\beta = 1/30$, $N_t = 6000000$; $\lambda_v = 1/14$, $B_0 = 3600$.

From the above results, if the basic reproductive number is higher, this means that one case can produce the greater number of secondary cases, and then the period of oscillation is shorter.

Furthermore, we simulate eqs. (17)-(31) by using the real life parameters to see the behavior solutions of human population.
Fig. 8 Numerical solutions for eqs. (17)-(31). The parameters are corresponding to the real life observations. The solutions converge to the steady state solution.

From comparing the results in fig. 5 and fig. 8, we can see that the periods of fluctuations in the number of individuals in each class are much shorter in the absence of any age group and season. This model should be better for describing the situation of Malaria in Thailand because age group of human and seasons affect to the number of Malaria patients. The results of this study should be the alternative way for controlling this disease.

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