

A Finite Element Study on the Mutual Influences of Two Stress Raisers

H. Cheraghali¹, M. H. Hojjati²

¹Sanandaj Vocational, Technical University, Sanandaj, Iran

²Department of Mechanical Engineering, Babol Noshirvani University of Technology, Babol, Iran

ABSTRACT

Machine parts often fail due to exceeding the actual magnitude of stress from the allowable values while the nominal stresses are still in the safe region. This happens because of the effects of existence of different discontinuities in the shape of the parts, which act as stress raisers. A finite element study has been carried out to determine the influences of existence of one stress raiser (a circular hole) nearby another discontinuity (a fillet) in a steel flat bar. The analysis reveals that stress concentration factors increases. The amount of increasing depends on the distance of the two discontinuities. The results are promising for further investigations.

KEY WORDS: Stress concentration, Stress raiser, Finite element analysis

1. INTRODUCTION

Basic stress equations normally used to describe the state of the stress at the mechanical parts rarely represent the actual value of the stress. This is due to the existence of discontinuities in machine part shapes caused by oil holes, key slots, shoulders, fillets and so on which act as stress raisers. A theoretical or geometrical stress - concentration factor k_t is used to relate the actual maximum stress σ_{max} at the discontinuity to the nominal stress. The factor is defined by the equation:

$$K_t = \frac{\sigma_{Max}}{\sigma_0} \quad (1)$$

where σ_0 is the stress calculated by using the elementary stress equation and the net cross section [1]. Although in certain geometrical shapes, it is possible to determine the values of stress concentration factors by the theory of elasticity [2], experimental methods such as photoelasticity, grid, brittle coating, brittle model and strain gauge method are extensively used to determine the stress concentration factors which have been fully described in the related texts [1,2,3,4]. In recent years, the finite element method has provided a very powerful tool to determine the stress concentration factors in a wide variety of cases. The results of the above mentioned studies are normally presented as tables and graphs of stress concentration factors caused by one of the stress raiser in the machine part under different loading conditions [5].

In this paper, the finite element method has been used to investigate the influences of existence of one stress raiser (a circular hole) on the stress concentration factor caused by another stress raiser (a fillet) nearby and vice versa. This problem which usually occurs in the machine parts is rarely addressed in the related literature.

2. FORMULATION OF PROBLEM

Figure 1 shows a stepped flat bar with a circular hole. The dimensions are also shown. The parameter “L” varies in different models to investigate the mutual influences of the two stress raiser in increasing stress concentration factors.

When “L” is large enough to consider two stress raiser separately, the theoretical stress concentration factor at a plate in tension with a circular hole is given by [6]:

$$k_t = 1.923 - 0.475 \ln \left(\frac{r}{a} + 0.1 \right) \quad (2)$$

for $(r/a) \leq 0.6$ and $r < b$, where r is the hole radius, a , half width of plate and b is the half - length of plate.

For the model shown in figure (1), $r = 10$ mm and $a = 30$ mm, so

*Corresponding Author: H. Cheraghali Sanandaj Vocational, Technical University, Sanandaj, Iran. Sanandaj Vocational, Technical University, Sanandaj, Iran. Email: m.zamani.n@gmail.com

$$k_t = 1.923 - 0.475 \ln \left(\frac{10}{30} + 0.1 \right) = 2.32 \quad (3a)$$

The theoretical factor of stress concentration caused by the fillet in a flat bar in tension is given by [5]:

$$K_t = 1 + \left[\frac{\left(\frac{D}{d} - 1 \right)}{2 \left(2.8 \frac{D}{d} - 2 \right)} * \frac{d}{r} \right]^{0.85} \quad (3b)$$

For the fillet in figure (1) with $(D/d) = 1.5$ and $(r/d) = (1/8)$, equation 3 yields to a value of $k_t = 2.05$.

The model in figure (1) is made of steel with a Young modulus of 207 Gpa, a Poisson ratio of 0.3. The plate thickness is 10 mm and plane stress condition is assumed. The area of cross section at right edge is 400 mm^2 . The net area at cross section with the hole is also 400 mm^2 .

3. RESULTS AND DISCUSSION

Figure 2 shows the basic finite element model. It consists of 1116 (plan82-Ansys) plane stress elements. A uniform tension stress of 10 Mpa has been applied in the right edge of the plate while the left edge has been restrained to move. In the basic model, $L = 60 \text{ mm}$ and the two stress raisers are considered to be well far from each other. Different mesh densities have been used to select a model that gives stress values at two critical cross sections (hole and fillet) close to the expected values of corresponding theoretical stress concentration factors multiplied by the nominal stress of 10 Mpa.

Table 1 compares the theoretical stress concentration factors with the finite element analysis ones which shows a good agreement and verifies the FE model.

Having established an acceptable and verified model, by varying the value of L in figure 1, different models have been analyzed and for each of them the stress concentration factors at the hole and at the fillet have been calculated by dividing the computed maximum stress by the nominal stress of 10 Mpa. Table 2 summaries the results.

Figures 3 and 4 show the stress contours for the two cases of $L = 60$ and 10 mm respectively. They clearly show the concentration of stress around the discontinuities.

Figures 5 shows the variation of K_t at circular hole vs different L values. It shows that for $L < 40 \text{ mm}$ there is a considerable increase in stress concentration factor at the hole.

Figure 6 shows the variation of K_t at the fillet for different L which shows the same trend as in figure 5.

In order to better demonstration of the mutual influences of two stress raisers in increasing individual K_t , let 's consider the following relations which define the amount of increase in each coefficient in terms of α percent of the K_t caused by the other stress raiser nearby:

$$(K_t)'_{hole} = (K_t)_{hole} + \alpha (K_t)_{fillet} \quad (4)$$

$$(K_t)'_{fillet} = (K_t)_{fillet} + \alpha (K_t)_{hole} \quad (5)$$

where $(K_t)'$ s are the factors when the two stress raiser considered separately and to be far away. (K_t) s are the factors when they are affected by the interaction caused by closing the two stress raisers. Table 3 shows the α values. By considering the fact that the depth of the flat bar at the left side is also 60mm, it can be concluded that when L is equal or greater than the depth of the machine part, α can be assumed zero and the stress raisers act separately. Figure 7 shows α values vs L values. As it might be expected as L increases α approaches zero.

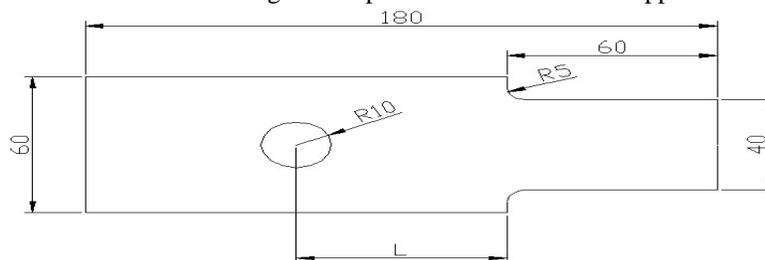


Fig. 1. Dimensions of the stepped flat bar with a circular hole.

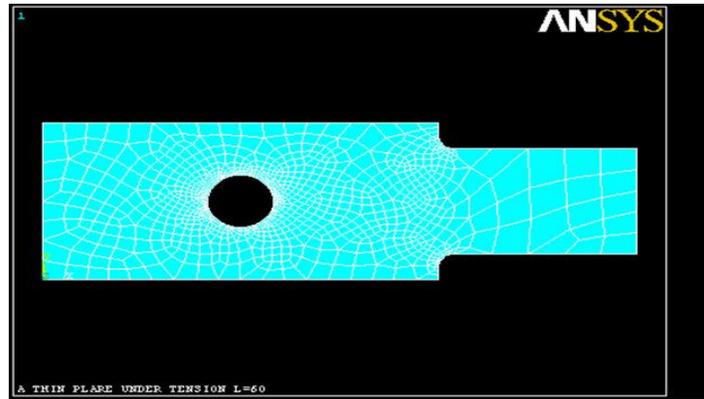


Fig. 2. The basic finite element model of the stepped flat bar with circular hole for the case $L = 60$ mm.

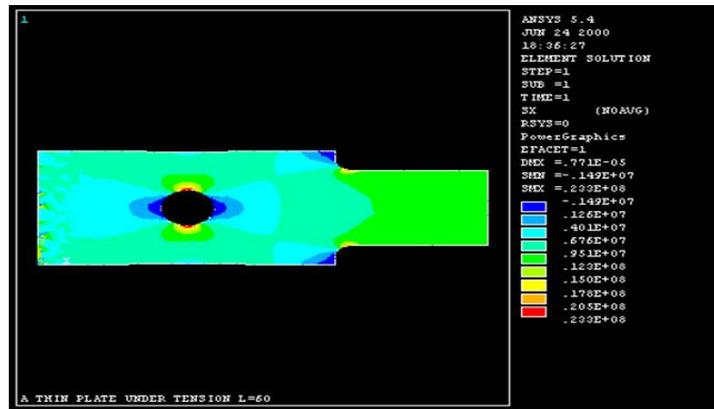


Fig. 3 Axial stress contours for the case $L = 60$ mm.

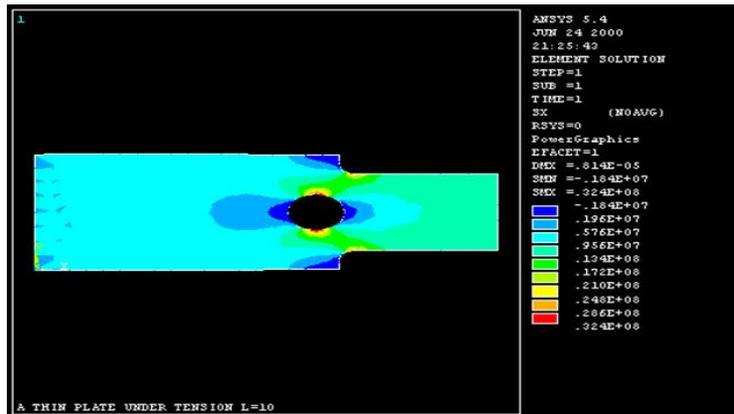


Fig. 4. Axial stress contours for the case $L = 60$ mm.

For different values of material inhomogeneity parameter n , radial displacement, radial stress, and circumferential stress along the radial direction are plotted in Figures 2-5.

In Figs. 2 and 3, distribution of the radial displacement and the radial stress along the radial direction for different values of n and $\eta = 1.2$ are shown. It is seen from the curves that at the same position ($0 < R < 1$), for higher values of n , radial displacement and radial stress decrease.

The circumferential stress along the radial direction for different values of n and $\eta = 1.2$ is plotted in Fig. 4. It must be noted from this figure that at the same position, almost for $R < 0.5$, there is an decrease in the value of the circumferential stress as n increases, whereas for $R > 0.5$ this situation was reversed. Besides, along the radial

direction for the positive magnitudes of n the circumferential stress increases, while for negative magnitude of n , the circumferential stress decreases.
 In Figs. 5 and 6, the stresses and radial displacement using values $n = -0.5$ and $\eta = 3.6$, is calculated and compared to those in a homogeneous disk ($n = 0$).

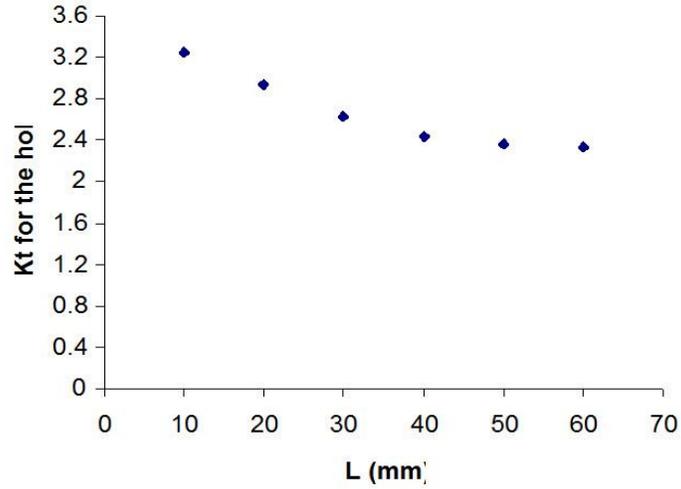


Fig. 5. Variation of K_t at the circular hole vs different L values.

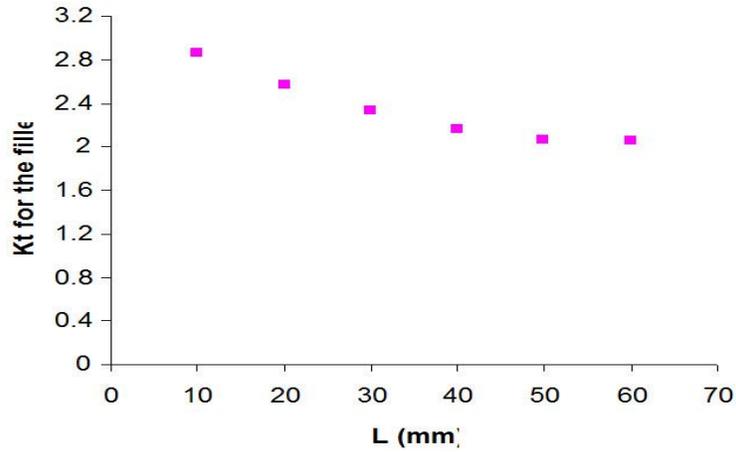


Fig. 6. Variation of K_t at the fillet vs different L values.

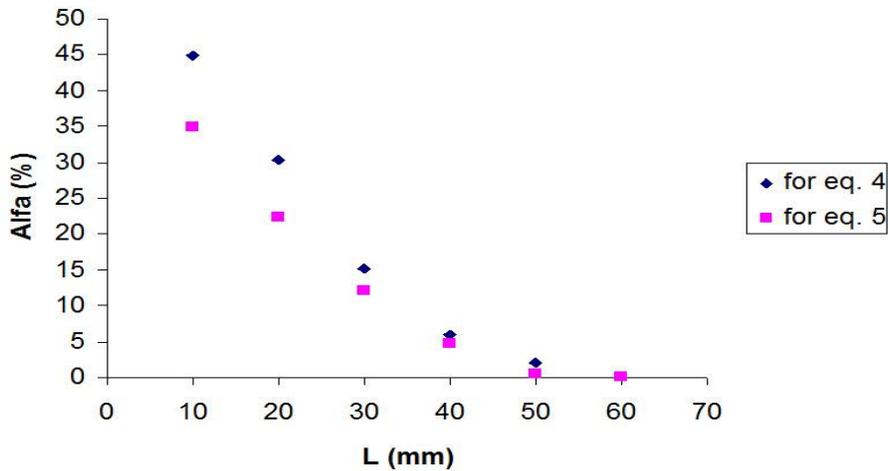


Fig. 7. α values vs L values

Table 1. Comparison of K_t

	At hole	At fillet
Theoretical	2.32	1.91
F.E. Analysis	2.33	2.05

Table 2. Stress concentration factors for different L values

L(mm)	K_t , at the hole	K_t , at the fillet
60	2.33	2.05
50	2.36	2.06
40	2.44	2.16
30	2.63	2.33
20	2.94	2.57
10	3.24	2.86

Table 3. α Values to be used in Eqs. (4) & (5) $\left[(K_t)_{hole} = 2.32 , (K_t)_{fillet} = 2.05 \right]$

L(mm)	60	50	40	30	20	10
α (percent) for Eq. 4	0.0	1.95	5.85	15.12	30.24	44.88
α (percent) for Eq. 5	0.0	0.43	4.74	12.07	22.41	34.91

4. CONCLUSION

- 1- The basic finite element model (with L=60mm) gives reliable stress concentration factors at the hole and the fillet which are in close agreement with the theoretical values. This verifies finite element analysis.
- 2- The finite element method provides a reliable tool to study stress concentration factors. This especially helps in parts with complex geometry and combined loading.
- 3- When two stress raisers are nearby, stress concentration factors are greater than those when considered separately.
- 4- Reducing the distance between two stress raisers increases the percentage of increase of each individual stress concentration factor. On the other hand, when the distance is equal or greater than of the depth of the part, they can be considered separately with no mutual interaction.
- 5- Two empirical relations have been developed for calculation of new stress concentration factors.

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