*J. Basic. Appl. Sci. Res.*, 2(8)7597-7603, 2012
© 2012, TextRoad Publication

ISSN 2090-4304
**Journal of Basic and Applied Scientific Research**
*www.textroad.com*

# Linear Relationship between Dynamic Pricing and Enterprise Profit

**Seyed Faramarz GHorani**

University of Allameh Tabatabaei, Faculty of Management

**ABSTRACT**

This paper discusses the dynamic pricing, variety of pricing and dynamic pricing strategies. One of the main problems in using the dynamic pricing method is its economical evaluation. To compare this method it has been evaluated by the enterprise profits. Then the hypothesis of relationship between corporate profitability and dynamic pricing is investigated and evaluated. Multi-attribute regression model is used to demonstrate the correlation between these two.

**KEYWORDS:** Dynamic pricing, dynamic pricing strategies, multi-attribute dynamic pricing, and power of dynamic pricing.

## 1 - INTRODUCTION

Dynamic pricing achievements [1] are well known since long ago in various industries such as airline companies, hotel management and power transmission companies and chain stores. This subject was introduced in the past decade in the revenue management framework and determines the price of products or services if are not sold after a specific period will be value less. The dynamic pricing aims to determine the sale price of these products during the periods prior that they loss their value so that the mathematical expectation of sales value maximizes. [2] In other words dynamic pricing involves the pricing of products in a way that reflects the time of changes in the supply cost. Dynamic pricing refers to the fluid pricing scheme between buyer and seller, rather than the traditional and fixed pricing. [3] Dynamic pricing, is a legacy from the past and its importance became more obvious with the advent of the industrial revolution and mass marketing and mass communication. Prior the Industrial Revolution, trading used to occur in markets with many buyers and sellers exchanging goods for goods. Current and evolving models for the dynamic pricing include: auction pricing system, pricing system group and reversed pricing system. In general these systems will get better if are involved with the actual value of product markets and reflect the need of additional work on the part of buyer demand. This system leads to more rapid and better evolution of intelligent standards agents for market development.

Time-based pricing refers to any offer or contract of service provider or products supplier in which price depends on the time of the offered services or delivered products. So it is expected that the reasonable pricing will be time-based or changes of supply and demand balance will be achieved over time. Time-based pricing is not limited by time; and the amount of energy and transport in dynamic pricing is obtained by reflecting the demand and supply situation or the difference in the delivery of a product regarding the delivery date. In most cases, time-based pricing refers to a specific act of a supplier. The typical time-based pricing, is the standard method of pricing in the tourism industry. Prices are higher during the season or are at their peak during an event in a particular period.

## 2 - Theoretical research:

In this paper, the dependence of dynamic pricing and profitability of an enterprise has been studied. The studied period is assumed to be discrete and the price set is assumed predetermined. Available inventory quantities which should be sold in a definite time period are clear. Demands for products are random and depend on the period and the proposed price. According to Poisson process, customers' visit for purchase is heterogeneous. At the end of the interval, not sold product will be worthless. Anyway the release of this assumption does not damage the model structure. The inventory is not renewable and the lost demands do not create any cost. The goal is to determine the amount of dependence between the dynamic pricing costs and the amount of organization profitability. In this section we define the types of pricing and its strategies.

### *2-1 Types of pricing*

- Pricing on a simple level

Pricing on a simple level, provides the applicant a list of all product categories and options to enable a pricing rule for a particular product category. [4]

***Corresponding Author:** Seyed Faramarz GHorani university of Allameh tabatabaei, Faculty of management.

- Advanced-level pricing

Advanced-level pricing allows access to the pricing rules configuration for a group, based on the shopping cart. By default, by using low prices and increasing the demand, you can access the different configurations of price tables or applying a set of rules, if necessary. This is useful if a set of customers receive a series of price adjustments, while others receive a different amount. [5]

***2-2 types of pricing strategies [6]:***

- The dynamic pricing strategy

In the dynamic pricing, the price alternates based on conditions, rather than being set firmly, such as increasing demand at certain times, and customer type which aims to change marketing conditions. This type of pricing strategy is common in certain types of businesses, especially firms that provide services such as air travel lines. [7]

- Segmented pricing

In Segmented pricing, some customers voluntarily tend to pay more for a service or product. Some customers are willing to pay more for faster service, higher quality and more features. For example, a product may have a guarantee at a certain price and at lower prices will be sold without warranty.

- The maximum pricing

Maximum pricing strategy is common in transportation and handling. For example, airlines and some of train companies charge higher prices for travel during rush hour on working days than holidays.

- Service time

Another dynamic pricing strategies lead to more charges for faster service. For example, cleaning services are more costly during the day than at night. It is argued that this strategy can increase customer loyalty without sacrificing profit margins.

- Time of purchase

Some dynamic pricing strategies offer different prices to customers based on when they buy. Again, most airlines are using this strategy. The price of economy class seats on a particular flight may be fluctuating over time. For example, an airline trying to fill seats, might lead passengers to night flights by lowering prices of night flights, or to fill business class seats try to increase the economy class ticket prices. [8]

- Change of circumstances

According to research, using dynamic pricing strategy can increase profit under certain market conditions. The researchers found that this versions work for dynamic pricing of products. When there is a great uncertainty in the market, retailers can increase the profit by lowering prices as a sales and then by increasing the demand, price rises again to its maximum. [9]

***2-3 Mathematical model of dynamic pricing***

This model which is derived from the dynamic pricing article with periodic review and discrete prices, is called the basic mathematical model [10]. It is assumed that C is the unit of the commodity at the T time interval offered for sale in n periods. During this period, prices should be revised in each period. Reversed periods are numbered backwards, for example, the first period is the last revise period and the n period is the first pricing period of products in the beginning of sell. On the other hand, selectable prices for offering products in each range are selected from the prices set: Sp = {p1, p2, ..., pK}. In the presented model, each distribution in each period can be used to predict the next period using the data from the previous period therefore the model is not limited to a particular distribution function for customer's entry. In most models presented in the dynamic pricing literature, Poisson distribution is used to model the purchasing process, but in the proposed model, this assumption is more generalized and further range of distribution can be applicable. Assume that customers' entry to purchase products has a heterogeneous Poisson distribution with arrival rate of $\lambda (t)$ where $t$ represents the time remaining until the end of the selling range. The maximum price that each customer is willing to pay for the products is also a random variable with certain distribution function. If $p$ is the price of products supplied in a range then at time $t$ within the interval, the possibility of a customer buying this product is equal to $1 - Ft (p)$. Therefore the purchase prices of products at time $t$ can be $\lambda (t) * (1-Ft (p))$. To determine the price of products' supply at the beginning of each $Ti$ interval, the probable dynamic programming is used with the backward movement. According to the contract: $Vi (ci)$ is the maximum mathematical expectation of profit in $i$ periods in the future if the balance at beginning of period $i$ is equal to $ci$.

Not sold products at the end of the sell range are considered to have a zero scrap value, as a result the following boundary conditions can be considered for the model:

$$V_i(0) = 0 \quad \forall i$$
$$V_0(c_0) = 0 \quad \forall c_0$$

Given the heterogeneous Poisson distribution for customers entry and the reservation price distribution, the average range of products sold during any decision making rang is calculated as follows:

$$m_i(p_k) = \int_{T_i}^{T_i - 1} \lambda(t) * (1 - F_t(p_k)) d_t$$

In this equation, *Ti* shows the beginning time of $i^{th}$ sell interval and $T_{i-1}$ is the end of sell time in this interval. According to the obtained average of the distribution function, the number of purchases in each interval is presented as follows:

$$pr\{X_i(p_k) = j\} = \exp(-m_i(p_k)) * m_i(p_k) \wedge j / j!$$

In the above equation, $X_i(p_k)$ is the random variable of the number of purchases in the $i^{th}$ interval, when the products are offered with the price of $p_k$.
Based on the recursive relationship of dynamic programming, the maximum mathematical expectation of obtainable profit in the first pricing period, when it has $c_1$ remaining products at the beginning of the first interval can be expressed as:

$$V_1(c_1, p_k) = \sum_{j=0}^{c_1} pr(X_1(p_k) = j) * jp_k + pr(X_1(p_k) \rangle c_1 * c_1 p_k \qquad (1)$$

In the equation (1) $0 \le c_1 \le C$ is the state variable and $V_i(c_1, p_k)$ represents the maximum mathematical expectation of the profit in the last period if $c_1$ is the inventory at the beginning of the last period and the delivered price of product is $p_k$. Then we have:

$$V_i(c_1) = \underset{p_k}{Max}\{V(c_1, p_k)\} \qquad (2)$$

Using equation (2) the maximum mathematical expectation of profit and also the $p_k$ i.e. the optimum price of supplying product presentation during the final period is obtained. Now assuming the $i^{th}$ desirable interval, we have:

$$V_1(c_1, p_k) = \sum_{j=0}^{c_1} pr(X_i(p_k) = j) * (jp_k + V_{i-1}(c_i - j)) + pr(X_i(p_k) \rangle c_i * c_i p_k \qquad (3)$$

For this interval, equation (4) is assumed:

$$V_i(c_i) = \underset{p_k}{Max}\{V_i(c_i, p_k)\} \qquad (4)$$

Solving the presented recursive equations to the $n^{th}$ period and assuming $C = C_n$, the maximum mathematical expectation of the profit from selling *C* units of product during the period *T*, and also the optimal cost of product delivery at the beginning of the sell interval are obtained. Starting the sell at the beginning of the $T_n$ time interval and the end of $n^{th}$ interval and the beginning of *n-1* decision making interval, we will have $C_{n-1}$ balance unit. According to the solved recursive equations using the following equation, the optimal price of products in the relevant interval is obtained:

$$V_{n-1}(c_{n-1}) = \underset{p_k}{Max}\{V_{n-1}(c_{n-1}, p_k)\} \qquad (5)$$

**3 - The research literature and background**

The dynamic pricing issue has shown high efficiency in industries such as garment industry which follows fashion and is short-lived or in services such as airplane or hotel management. In Galgo and Van Ryazan (1994) further discussions about the development and application of dynamic pricing can be seen. Kincaid and Darling (1963) for the first time introduce the dynamic pricing. Galgo and Van Ryazan (1994) have demonstrated the following properties for the optimal policy: [11]

1 - At any moment of time, the optimal price increases with reduce of not sold products.
2 - At every level of inventory, the optimal price decreases over time.

Bytran and Mondschein (1997) has investigated the two above properties through heterogeneous Poisson demand distribution and show that if the reservation price distribution is not dependent on time, these properties are established. [12] Some other works done in this field are Bitran et al (1998), Fung and Galgo and Lazear (1986). The main assumptions of the models presented in these articles are as follows:

1 - The seller works in a non-competitive environment.
2 - The sale time interval is limited.
3 - The seller has a certain *n* inventory and cannot review the inventory during the sales interval.
4 - The investment made in inventory is considered as lost costs.
5 - The demand decreases with the price increase.
6 - Not sold products will have scrap value.

In the model of Bytran and Mondschein (1997) the time interval between reviews are pre-specified. They modeled their case within the framework of dynamic programming in which each stage has become a nonlinear programming. In this paper, dynamic pricing is defined for specified costs and then a model is provided for minimizing the costs of dynamic pricing. This paper aims to show the dependency of minimum amount of dynamic pricing in each period, to company's profitability. Based on this premise it is assumed that the product policy is made through dynamic pricing according to maximizing the sales and reducing the amount of the lost value and on the other hand due to policy of increasing the amount of company's profit, the cost of dynamic pricing shall be optimal.

## 4 - RESEARCH METHODOLOGY

### *4-1 Specifications of the statistical sample and population and research method*

In this scientific study the descriptive survey method is used. Statistical population data for this study consisted of a 5-year period of enterprise performance. Data was collected using periodic recorded data. Data classification is determined based on variables defined in the course of study.

### *4-2 Variables*

Dynamic Pricing variables include the storage cost, energy costs, transportation and communication and finally product distribution costs. Dependent variable in this study is that the amount of profitability in a period of 24 years which are shown with *P1* ... *P24* symbol. Independent variables are dynamic pricing variables that are shown with the *K1* ... *K4* symbol.

### *4-3 The main research hypothesis*

$H_0$: There is no direct correlation between the dynamic pricing and profitability of companies.
$H_1$: There is a direct correlation between the dynamic pricing and profitability of companies.

### *4-4 Specification of mathematical model and conceptual model*

Since the main assumption is based on determining the amount of dependence of organization's past events (done pricing) and the organization's profitability (performance values in the past), in this paper it is assumed that dynamic pricing is at the optimal point. Hence to prove the linear relationship between dependent and independent variables, multiple regression method is used. In a simple definition regression is the defensive reaction to some of the accepted motives. Statistical methods for estimating the relationships between variables are called regression analysis.

According to data collection of the statistical unit $\left\{ y_i, x_i, \ldots, x_{ip} \right\}_{i=1}^{n}$, a linear regression model assumes that the relationship between the dependent variable $y_i$ and the vector $P$ are linear returns of $x_i$. The model can be expressed as follows.

$$y_i = \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + \varepsilon_i = x_i^T \beta + \varepsilon_i, \qquad i = 1, \ldots, n$$

*T* denotes the transpose so that $x_i^T\beta$ is the inner product between the vectors $x_i$ and $\beta$. Most of these *n* piece equations in vector form are written as follows.

$$y = X\beta + \varepsilon$$

Where,

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} \mathbf{x}_1^{\mathrm{T}} \\ \mathbf{x}_2^{\mathrm{T}} \\ \vdots \\ \mathbf{x}_n^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ x_{21} & \cdots & x_{2p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_p \end{pmatrix}, \quad \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}.$$

$y_i$ is called exogenous variable, the response variable, measurement variable or dependent variable.
$x_i$ are called endogenous variables, explanatory variables, auxiliary variables, input variables, the predictor variables or independent variables.
β is the p-dimensional vector variable. Its elements are called effect or regression coefficients. Statistical estimation and inference in linear regression is focused on β.
$\varepsilon_i$ is called the error term or disturbance term. This variable includes the set of all other factors that affect the dependent variable $y_i$ and the auxiliary variable $x_i$. [13]

The following formula is used to prove the null hypothesis of linear relationship.

$$F = \frac{R^2_{y.12\ldots k} / k}{(1 - R^2_{y.12\ldots k}) / (N - k - 1)}, \quad df = k,\ N - k - 1$$

Where $R_{y.12\ldots k}$ is multiple correlation coefficient and shows how much *Y* can be predicted by the *X* Series.

## 5 - Case Study

### *5-1 Data of Model*

24 periods of economic profitability of α enterprise plus the costs of dynamic pricing in 4 parts of: storage costs, energy costs, transportation and communication and finally the distribution costs of product are listed in table 1.

Table for comparison the costs and profits of the enterprise

| Year | Cost of inventory | Cost of energy | Cost of transport | Cost of distribution | Total profit |
|---|---|---|---|---|---|
| | $K_1$ | $K_2$ | $K_3$ | $K_4$ | P |
| **1987** | 77.6 | 86.64 | 40.3 | 93.73 | 78445.01 |
| **1988** | 86.6 | 96.87 | 39.8 | 91.41 | 82760.84 |
| **1989** | 97.0 | 104.31 | 39.4 | 90.41 | 87084.56 |
| **1990** | 104.4 | 117.4 | 41.8 | 92.53 | 93662.19 |
| **1991** | 111.7 | 126.33 | 50.2 | 92.80 | 100210.9 |
| **1992** | 116.0 | 126.16 | 56.0 | 91.85 | 102572.6 |
| **1993** | 115.4 | 108.46 | 53.7 | 93.89 | 97691.35 |
| **1994** | 104.1 | 96.13 | 49.4 | 97.37 | 91261 |
| **1995** | 92.0 | 68.5 | 36.6 | 99.33 | 77961.09 |
| **1996** | 77.6 | 49.29 | 10.7 | 101.05 | 62762.32 |
| **1997** | 67.6 | 32.94 | 0.9 | 100.08 | 52999.76 |
| **1998** | 67.6 | 33.07 | 0.3 | 100.30 | 52934.01 |
| **1999** | 63.9 | 31.04 | 2.4 | 95.93 | 50830.01 |
| **2000** | 58.2 | 39.88 | -3.8 | 98.33 | 50656.43 |
| **2001** | 56.6 | 42.2 | -7.4 | 99.78 | 50280.34 |
| **2002** | 56.2 | 51.02 | -12.8 | 98.28 | 50680.1 |
| **2003** | 57.8 | 61.46 | -14.3 | 98.03 | 53386.37 |

| | | | | | |
|---|---|---|---|---|---|
| **2004** | 58.5 | 56.16 | -21.0 | 99.60 | 50827.38 |
| **2005** | 61.1 | 62.8 | -14.8 | 99.54 | 54872.32 |
| **2006** | 64.2 | 60.98 | -12.9 | 98.68 | 55482.48 |
| **2007** | 64.6 | 67.43 | -01.8 | 98.53 | 60163.88 |
| **2008** | 62.7 | 69.43 | 18.4 | 98.80 | 65573.79 |
| **2009** | 63.4 | 66.33 | 27.2 | 98.83 | 67264.88 |
| **2010** | 65.3 | 69.85 | 26.3 | 99.03 | 68506.24 |

- Prices are based on the base year (1987) and are presented in dollar.

***5-2 Descriptive Statistics***

for the calculations by multivariate regression model, the STATGRAOHICS Plus software is used. Calculation results and the analysis are as follows.

**Parameter -** The independent variables are measured and the extent of their impact on the dependent variable is tested.

**Estimate -** Shows the impact of independent variables on the dependent variable. Negative numbers have an adverse impact and positive numbers have a direct impact.

**Standard Error -** The standard deviation shows the data dispersion of the arithmetic mean of responses.

**T Statistic –** Are the numbers test in the critical region.

**P-Value -** indicates the ability to generalize answers to the entire community.

**Durban - Watson statistic -** This test shows the independence or dependence of research data in the previous series with the current series. Numbers higher than 1.4 shows the independence and lower than 4.1 shows the independence of study results.

| Parameter | Estimate | Standard Error | T Statistic | P-Value |
|---|---|---|---|---|
| constant | -0.01499 | 0.01020 | -1.46974 | 0.1580 |
| K1 | 0.00380 | 3.74325 | 10159.1 | 0.0000 |
| K2 | -1.00012 | 0.00010 | -9877.4 | 0.0000 |
| K3 | -1.00013 | 0.00009 | -10932.1 | 0.0000 |
| K4 | -1.00017 | 0.00009 | -10185.2 | 0.0000 |

Analysis of Variance

| Source | Sum of Squares | DF | Mean Square | P-Value |
|---|---|---|---|---|
| Model | 243.955 | 4 | 60.989 | 0.0000 |
| Residual | 0.00001511 | 19 | 7.9517 | |
| Total (corr.) | 243.955 | 23 | | |

R-squared = 100 percent
Standard Error of Est. = 0.000891726
Mean absolute error = 0.000653426
Durbin-Watson statistic = 2.10978

## 5-3 ANALYSIS OF THE RESULTS

As P-Value in ANOVA (ANOVA) variance table was less than 0.1 there is a 99% statistically significant relationship between variables. R-Square statistic indicates that the model shows the 100% variability of the dependent variable. The standard deviation of the estimation represents the residual standard deviation which is equal to 0.000891726. This value can be used to make limitations in the new observations estimations. The mean absolute error is the average 0.000653426 of residual value. In Durbin – Watson test the mean absolute error is less than the average value. As the Durbin – Watson test is higher than 1.4, it shows the independence of the research results.

Hypothesis Test:
Sample mean = 0.0
Sample standard deviation = 1.0
Sample size = 100
95% confidence interval for mean: 0.0 + / - 0.198422
Null Hypothesis: mean = 0.5
Computed T statistic = -5.0

P-value = 0.00000265129
Rejected the null hypothesis for alpha = 0.05.

Test results are as follows:

H0: mu = 0.5
H1: mu <> 0.5

A sample of 100% observations with mean of zero and standard deviation of one, obtained the statistical T test equal with -0.5. P-value of the test is less than 0.05 and the zero assumption is rejected at the 95% of confidence level. Confidence interval shows that the average value of data is -0.198422 to 0.198422.

## 6 - Conclusion and Summary

Hypothesis of a direct linear correlation between corporate profitability and dynamic pricing is established and the existence of this relationship was demonstrated in this paper. This relationship shows that the linear relationship between dynamic pricing and direct costs shall be considered in the annual planning to increase profits. The researchers suggest that future studies use the mathematical modeling in optimization of dynamic pricing and import the dynamic pricing optimization in the model restrictions. [14]

## REFERENCES

1- P. Dasgupta and R. Das, ”Dynamic Service Pricing for Brokers in a Multi-Agent Economy”, Proc. of the Third International Conference for Multi-Agent Systems(ICMAS), Boston, MA, July 4-9, 2000, pp. 375-376.

2 – Mohammad Modarres, Ehsan Bolandi Far, the fourth international management conference, dynamic pricing with the periodic review and discrete prices.

3- Araman, V. F., R. Caldentey. 2009. Dynamic pricing for nonperishable products with demand learning. Oper. Res. 57(5) 1169–1188.

4- Elmaghraby, W., P. Keskinocak. 2003. Dynamic pricing in the presence of inventory considerations: Research overview, current practices and future directions. Management Science 49 1287{1309.

5- Gallego, G., G. van Ryzin. 1997. A multiproduct dynamic pricing problem and its applications to network yield management. Oper. Res. 45(1) 24–41.

6- Maglaras, C., J. Meissner. 2006. Dynamic pricing strategy for multi-product revenue management problems. Manufacturing and Service Operations Management 136-148.

7- Brumelle, S., J. McGill. 1993. Airline seat allocation with multiple nested fare classes. Oper. Res. 41(1) 127–137.

8- Lin, K. Y. 2006. Dynamic pricing with real-time demand learning. Eur. J. Oper. Res. 174(1) 522–538.

9- Cope, E. 2006. Bayesian strategies for dynamic pricing in e-commerce. Navtal Res. Logist. 54(3) 265–281.

10 - Mohammad Modarres, Ehsan Bolandi Far, dynamic pricing with the periodic review and discrete prices, Fourth International Conference on Industrial Engineering, 2007.

11- Gallego,G.,G.Van Ryzin. 1994. Optimal dynamic pricing of inventories with stochastic demand over finite horizons. Management Sci. 40 999-1020

12- Bitran,G.,S.Monschein. 1997. Periodic pricing of seasonal product in retailing. Management Sci.43 427-443.

13- N. Ravishankar and D. K. Dey (2002), "A First Course in Linear Model Theory", Chapman and Hall/CRC, Boca Raton (p. 101)

14- Gallego, G., G. van Ryzin. 1994. Optimal dynamic pricing of inventories with stochastic demand over finite horizons. Management Sci. 40(8) 999–1020.