

# The Estimate of Coefficients Related to the Water Demand in the Urban Section of Arsanjan and Welfare Effects of a Change in Water Price (A Dynamic Model) "A Comparison between the Regression Method and the Artificial Neural Network"

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## ABSTRACT

This article investigates the prediction for water demand with two methods of two-phase regression and the neural network. The data used here is related to two periods of time between 1982-1993 and 1998-2005. According to the findings of the study, there is a negative relation between the water price and water demand and income strength more than one for water and the reciprocal impression of the variables affecting the water demand. One more finding is that a decrease in the water price by one percent, a 0.1-percent increase in the welfare level of the society was observed. It was also found that prediction through the artificial neural network with a 4.1 percent error was closer to reality comparing with the regression method with an error value of 10.75 percent.

**KEY WORDS:** Urban Water Demand, two-phase regression, neural network, Klein-Rubin, Box-Cox, Arsanjan

## INTRODUCTION

Concurrent with the technological advance in health, industry, etc, more water has been consumed consequently, while water supply has been remained constant. So, as time passes, the human beings have been more aware of the critical value of this material and have been trying to deal with the problem of water shortage through decreasing demand and increasing supply of water.

The demanded water is spent in different sections as in agriculture, industry, commerce, and at homes and also the potable water and the water used for recreational purposes. This paper reviews the factors influencing the potable water demand. The research done regarding the urban water demand based on the models adopted from the microeconomic theories are predominantly seen in the economic literature of the last six decades.

### Materials and methods

In this section, the theoretical bases of the utilized models are studied.

### 1. Klein-Rubin Model

In this model, the function of water demand is obtained through maximizing the utility with regards to the limited budget of the consumer:

$$(1) \text{Max} : u = \prod_{i=1}^n (Q_i - \delta_i)^{\beta_i} \quad \text{s.t.} \quad \sum \beta_i = 1 \Rightarrow 0 < \beta_i < 1, Q_i \geq \delta_i$$

$I = \sum P_i Q_i$   $U$  stands for utility,  $Q_i$  is the consumed amount of {a formula} for the  $i$  object and  $\delta_i$  as the minimum required amount of  $i$  object for survival and  $I$  indicating the consumer's income and  $P_i$  used to show the price of  $i$  object. Model (1) is dependant to the maximum family budget.

In this study, we hypothesize {a formula} for two objects ( $Q_1$  for water and  $Q_2$  for other materials and services). With this theory, model (1) is modified as:

$$(2) \text{max} : u = (Q_1 - \delta_1)^{\beta_1} (Q_2 - \delta_2)^{\beta_2} \quad \text{s.t.} : I = P_1 Q_1 + P_2 Q_2$$

By solving Model (2), then we will reach: {a formula}

In which,  $Q_1$  is indicative of the water demand per capita,  $P_1$  is the water price and  $P_2$  shows the price of other materials and services. Considering the specifications of utility and demand functions, then  $\beta_1, \beta_2 \delta_1$  should be positive and  $\beta_1 \delta_2$  negative. Since the water demand is a function dependant to the atmospheric factors, so to consider the effect of these

variables in the model and to modify it into a dynamic model, it is enough to consider the minimum amount of needed water as a function of not only the temperature (T) and rainfall (R), but as a function dependent on the water consumed in the previous term ( $Q_{it-1}$ ). Indicated as:

$$\bar{\delta}_1 = \delta_1 + K_1 T + K_1 R + K_3 Q_{it-1}$$

Consequently, model (3) will be changed as follows:

$$(4) Q_{it} = \delta_1(1 - \beta_1) - \beta_1 \delta_2 \left( \frac{P_{2t}}{P_{1t}} \right) + \beta_1 \left( \frac{I_t}{P_{1t}} \right) + k_1(1 - \beta_1) T_t + k_2(1 - \beta_1) R_t + (1 - \beta_1) Q_{it-1} + u_t$$

## 2. Box-Cox Model

This model is used to measure the reciprocal effects of variables involved in the water price and the price of other materials and services, the consumer's income, rainfall and the weather temperature. The total form of the above-mentioned model is as follows:

$$(5) \frac{P_{(Q_i)}^\theta - 1}{\theta} = \eta_o + \eta \sum \eta_i \left( \frac{Q_i^\lambda - 1}{\lambda} \right) + \frac{1}{2} \sum \sum \gamma_{ij} \left( \frac{Q_i^\lambda - 1}{\lambda} \right) \left( \frac{Q_j^\lambda - 1}{\lambda} \right)$$

In which  $P_{(Q_i)}$  is the price of i material.  $\theta$  and  $\eta_j, \lambda$  are also the variables to be estimated. Model (5) in the present study is modified as follows:

$$(6) \frac{Q_{it}^\theta - 1}{\theta} = \eta_o + \eta_1 \left( \frac{P_{1t}^\lambda - 1}{\lambda} \right) + \eta_2 \left( \frac{P_{2t}^\lambda - 1}{\lambda} \right) + \eta_3 \left( \frac{I_t^\lambda - 1}{\lambda} \right) + \eta_4 \left( \frac{P_{1t}^\lambda - 1}{\lambda} \right) \left( \frac{P_{2t}^\lambda - 1}{\lambda} \right) + \eta_5 \left( \frac{P_{1t}^\lambda - 1}{\lambda} \right) \left( \frac{I_t^\lambda - 1}{\lambda} \right) + \eta_6 \left( \frac{P_{1t}^\lambda - 1}{\lambda} \right) \left( \frac{T_t^\lambda - 1}{\lambda} \right) + \eta_7 \left( \frac{P_{1t}^\lambda - 1}{\lambda} \right) \left( \frac{R_t^\lambda - 1}{\lambda} \right) + u_t$$

## The Artificial Neural Network (ANN)

In today world, more advanced methods are invented to predict the variables process as the artificial neural networks, which operate like the human brain to process the experimental data, and transit the latent rule beyond the data to the network structure. A neural network is formed of artificial neurons. Each one of the neurons receives the input and after processing them, an output signal is produced. The output equation of neuron is as follows:

$$\mathbf{a} = \mathbf{f}(\mathbf{W}\bar{\mathbf{P}} + \mathbf{b})$$

The moving  $f$  function is selected by the designer. According to selection of  $f$  and the training algorithm method, the  $w$  and  $b$  parameters are formulated. In fact, training is meant that to modify  $w$  and  $b$  in a way that the neuron input-output relation concords with a specific purpose. In general, each neuron has more than one input. In figure (1), a neuron model with R input is indicated.

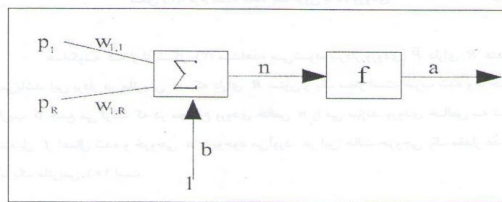


Figure (1): the multi-input model of a neuron

In figure (1), the  $P_i$  figures are the input vector ( $\bar{\mathbf{P}}$ ) elements and with the weight matrix of  $w$  and the diagonal term of  $b$ , the net input is made as the following relation:

$$(7) \mathbf{n} = \sum_{i=1}^R P_i W_{1,i} + \mathbf{b} = \mathbf{W}\bar{\mathbf{P}} + \mathbf{b}$$

$$\bar{\mathbf{P}} = [P_1, P_2, \dots, P_R]^T \quad \mathbf{W} = [W_{1,1}, W_{1,2}, \dots, W_{1,R}]$$

And finally, the neuron output will be as the following relation:

$$\mathbf{a} = \mathbf{f}(\mathbf{W}\bar{\mathbf{P}} + \mathbf{b})$$

Generally, a single neuron even with a multitude of inputs is inadequate for solving the problems. Therefore, in most cases, a mass of neurons are used as a layer. The artificial neural networks just like the biological neural networks are

capable to be organized in different ways. It means that neurons are able to attach to each other in a number of ways and produce neural networks with diverse structures.

### Data sources

The periods under study as of 1982-1993 and 1998-2005 and all the data are arranged on a monthly basis. The information related to  $I, P_2$  is extracted from the magazines of Central Bank of I.R. of Iran, the State Management and Planning Organization and the data related to the water demand are obtained from a 400-family sample population.

### Assessment method

To assess the relation between the water demand and the factors influencing the demand, the regression and neural network methods are utilized. With regard to the fact that it is expected to be a relation between  $ut$  and  $P1t$  in equation 4, thus, the two-phase minimum squares method and  $\hat{P}_{it}$  as the instrumentation variable are used in this case. At first,  $P1$  is regarded as a function of other exogenous variables and the following equation is generated.

$$(8) \hat{P}_{1t} = \gamma_o + \gamma_1 P_{1t-1} + \gamma_2 I_t + \gamma_3 P_{2t} + \gamma_4 T_t + e \quad \Rightarrow P_{1t} = \hat{P}_{1t} + e$$

Then, equation No. 8 is replaced with no 4 and 5. It should be noted that the first period data is of series time-cross section type and the second period data is series-time model. So, the test of single root existence (stagnation) was implemented and all their stagnations  $\{I(0)\}$  are approved (Dickey-Fuller test).

### ANALYSIS

The parameters of demand model in the static and dynamic modes and with the neuron network, Stoon-Grey and Bucks-Cox methods are assessed and the results are provided as follows:

1. The assessment results of Stoon-Grey – static mode – first period

$$Q_{1t} = 2 - 0.39 \frac{P_{2t}}{\hat{P}_{1t}} + 0.0042 \frac{I_t}{\hat{P}_{1t}} + 0.15 T_t, R^2 = 0.95$$

( t ) ( 4.5 ) ( 0018.6 ) ( 10 ) ( 8 )

2. The assessment results of Stoon-Grey – dynamic mode – second period

$$\hat{Q}_{it} = 1 - 6.5 \frac{P_t}{P_{it}} + 0.00067 \frac{I_t}{P_{it}} + 0.041 T_t + 0.3 D_1 + 0.48 Q_{it-2}, R^2 = 0.72$$

( 8.5 ) ( 8.3 ) ( 14 ) ( 7 ) ( .85 )

(The *camera h* test reveals lack of correlation)

The reason behind entering  $Q_{it-2}$  instead of  $Q_{it-1}$  is that the water meters indicators were read every two months. According to the above relation, the water consumption of the previous period has a positive impact on the demand of the current period.

Some .00075 percent of the citizens' expenditures is spent on water and each citizen needs at least 4.25 cubic meter of water per month. This amount had been 2 cubic meters in the first period. The two-fold increase in the minimum required water is due to observing sanitation rules and making inner-house bathroom and the increase in the number of automobiles per capita.

3. The assessment results of Box-Cox model – second period – dynamic mode

$$\frac{Q_{1t}^{0.5}}{0.5} = 8.1 - 0.043 \left( \frac{\hat{P}_{1t}^{0.64}}{0.64} \right) - 0.44 \left( \frac{P_{2t}^{0.64}}{0.64} \right) + 0.0007 \left( \frac{I_t^{0.64}}{0.64} \right) + 0.004 \left( \frac{\hat{P}_{it}^{0.64}}{0.64} \cdot \frac{P_{it}^{0.64}}{0.64} \right)$$

( 7 )      7.05 )      ( 8 )      ( 8 )      ( 3.8 )

$$- .00000062 \left( \frac{\hat{P}_{1t}^{0.64}}{0.64} \cdot \frac{I_t^{0.64}}{0.64} \right) + 0.0003 \left( \frac{\hat{P}_{1t}^{0.64}}{0.64} \right) + 0.004 \frac{T_t^{0.64}}{0.64}$$

( .8 )      ( .86 )      ( 5 )

( t )

$$+ 0.12D_1 + 0.39 \frac{Q_{11-2} 0.64}{0.64}, R^2 = 0.77$$

( 3.7 ) ( 4 )

All the assessed coefficients are meaningful. The interesting thing in the above model is the reciprocal and positive effect of water price and the price indexes. It means that when the price indexes increase, a decrease is witnessed in the ratio of water demand in relation to the water price, but if the price indexes show a decrease and water price increases, the sensitivity for water demand increases accordingly.

The assessment result of the Neural Network Method

In this method, 180 out of the total 240 data items were selected to be used for designing and training of the neural network and the rest were utilized to evaluate the prediction ability of the model. In order to predict all the data available, the cross validation method was used. In this method, all sets of data were divided to two sub-groups of training and testing. Then, using the training data, the said model were set to be trained and with the testing data, the results were validated. This procedure is repeated for several times and the average of the obtained results is considered as the final assessment. In this study, the data was used as experimental data in torque form for 8 times. The prediction results are presented in the following table:

Table (1) the model prediction results (Regression Method)

Error percentage	Correct prediction percentage	Number of Correct prediction	samples	No.
13	87	26	30-1	1
10	90	27	60-30	2
17	83	25	90-60	3
20	80	24	120-90	4
3	97	29	150-120	5
10	90	27	180-150	6
10	90	27	210-180	7
3	97	29	240-210	8

In the artificial neural network for network training, the data are divided into two parts like the normal prediction methods. The number of neurons in the latent layer of this network is regarded 1-5 and the results are provided in the following table.

Table (2) the prediction results of the Artificial Neural Network Model

Error percentage	Correct prediction percentage	Number of Correct prediction	Number of optimal repetition	Number of optimal neurons	samples	No.
10	90	27	11	3	30-1	1
3	97	29	6	4	60-30	2
7	93	28	18	5	90-60	3
7	93	28	4	4	120-90	4
3	97	29	12	5	150-120	5
0	100	30	19	5	180-150	6
0	100	30	15	5	210-180	7
3	97	29	14	4	240-210	8

Briefly speaking, prediction based on artificial neural network model with 4.1 error percent is more efficient comparing to the regression model with 10.75 error percent. Moreover, in the torques No. 6 and 7 with five neurons in the latent layer, in this study it was chosen as the superior network with 100 percent precision.

The effect of water price changes in the consumers' welfare

With calculation of extra consumers (through integration from the function No. 10 and then calculating the changing percentage in it by a 10-percent decrease in water price, we find out that the number of extra consumers were increased by 1 percent. In other words, a ten percent decrease in the water price caused an increase in the society welfare by 1 percent.

## Conclusion and discussion

In all the functions, the marked resulting coefficients were expected and with respect to the income strength, water is known as a luxurious merchandise. It was also found out that the increase in the water consumption in one period of time is the cause of more consumption in the next time cycle.

Briefly, in whole the artificial neural network model with less error percent in comparison to the regression model had a higher prediction strength.

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