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Some Fixed Point Results in Metric Type Space

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ABSTRACT

In this paper we consider metric type spaces which are introduced as a generalization of symmetric and metric spaces by M.A. Khamsi and N. Hussain [M.A. Khamsi, N. Hussain, KKM mappings in metric type spaces, Nonlinear Anal. 73 (2010) 3123-3129.] and [M. Jovanovic, Z. Kadelburg, S. Radenovic, Common Fixed Point Results in Metric-Type Spaces, Fixed Point Theory and Applications, Volume 2010, Article ID 978121, 15 pages.]. Then we develop the existence of coincidence points and common fixed points for mappings satisfying generalized contractive conditions, without appealing to continuity of any map involved therein, in a metric type space. The results extend and improve recent related results.

KEYWORDS: Metric type space; Common fixed point; Coincidence point; Weakly compatible maps; Cone metric space

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1. INTRODUCTION

The following famous fixed point theorem was proved by Banach in 1922 [4], "Let (X,d) be a complete metric space and T be a contractive mapping, that is, there exist $\lambda \in [0,1)$ satisfying $\mathrm{d}(\mathrm{Tx},\mathrm{Ty}) \le \lambda \mathrm{d}(\mathrm{x},\mathrm{y})$ for all $x, y \in X$, then T has a unique fixed point". Afterward, some various definitions of contractive mappings were proved by other researchers and several fixed and common fixed point theorems were considered in [5, 10, 15, 20]. Common fixed point theorem for commuting maps was considered by Jungck [13]. This theorem has many applications but their results require the continuity of one of the two maps involved. In 1996, Jungck defined a pair of self-mappings to be weakly compatible if they commute at their coincidence points [12].

The cone metric space was initiated by Huang and Zhang [9]. Then several fixed and common fixed point results in cone metric spaces were introduced in [6, 16, 18, 19, 21]. Also, Jungck and Rhoades [14], Abbas and Jungck [1] proved some fixed and common fixed point theorems for noncommuting and compatible maps in metric and cone metric spaces.

Symmetric space, as metric-like spaces lacking the triangle inequality was introduced by Wilson in 1931 [22]. Recently, a new type of spaces was defined by Khamsi and Hussain which they called metric type spaces [16, 17]. Metric type spaces are introduced as a generalization of symmetric and metric spaces. Afterward, other researchers proved several fixed point theorems in metric type space [7, 11].

The purpose of this paper is to generalize and unify common fixed point theorems for two pairs of weakly compatible maps of Abbas and Jungck [1], Abbas and Rhoades [14], Arshad et al. [3], Huang and Zhang [9], on metric type spaces. Our main theorem extend Theorem 2 of [3] and Theorem 2.8 of [2] in the metric type space case. In Theorem 3.1.and any one of Corollaries from 3.2 to 3.4, set $K = 1$. Then we obtain analogues theorem and analogues corollaries for metric space. Thus, our results are new and extend old papers on metric type space.

This paper is organized to four sections. In the following, we first provide necessary background of old definitions in section 2. In section 3, we prove our main theorem and obtain some results of our theorem. Finally, the conclusions are found in section 4.

Preliminaries

Let us start by defining some important definitions.

In 1931, Wilson introduced symmetric space, as metric-like spaces lacking the triangle inequality.

Definition 2.1.(See [22]). Let X be a nonempty set. Suppose the mapping $D: X \times X \to [0,\infty)$ satisfies

(S1) $D(x,y) = 0 \Leftrightarrow x = y$;

(S2) $D(x,y) = D(y,x)$,

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for all $x, y \in X$. Then D is called a symmetric on X and (X, D) is called a symmetric space.

Definition 2.2. (See [8, 9]). Let E be a real Banach space and P be a subset of E. Then P is called a cone if and only if

(a) P is closed, non-empty and $P \neq \{0\}$;

(b) $a, b \in R, a, b \geq 0, x, y \in P$ imply that $ax + by \in P$;

(c) if $x \in P$ and $-x \in P$, then $\mathrm{x} = 0$.

Given a cone $P \subset E$, we define a partial ordering $\leq$ with respect to P by $x \leq y \Leftrightarrow y - x \in P$. We shall write $x < y$ if $x \leq y$ and $x \neq y$. Also, we write $x \ll y$ if and only if $y - x \in intP$ (where $intP$ is interior of P). The cone P is named normal if there is a number $k > 0$ such that for all $x, y \in E$, we have $0 \leq x \leq y \Rightarrow \| x \| \leq k \| y \|$.

The least positive number satisfying the above is called the normal constant of P.

Example 2.3. (See [19]).

(i) Let $E = C_{\mathbf{R}}[0,1]$ with the supremum norm and $P = \{ f \in E : f \geq 0 \}$. Then, P is a normal cone with normal constant $\mathrm{k} = 1$.

(ii) Let $E = C^2_{\mathbf{R}}[0,1]$ with the norm $\| f \| = \| f \|_\infty + \| f' \|_\infty$ and consider the cone $P = \{ f \in E : f \geq 0 \}$ for every $k \geq 1$. Then P is a non-normal cone.

In the following, Huang and Zhang [9] replaced the real numbers by ordering Banach space and defined cone metric spaces (X, d).

Definition 2.4.(See [9]). Let X be a nonempty set. Suppose the mapping $d : X \times X \to E$ satisfies

(d1) $0 \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;

(d2) $d(\mathrm{x}, \mathrm{y}) = d(\mathrm{y}, \mathrm{x})$ for all $x, y \in X$;

(d3) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

Then, d is called a cone metric on X and (X, d) is called a cone metric space. It is obvious that the cone metric spaces generalize metric spaces.

Example 2.5. (See [9]). Let $E = R^2$, $P = \{(x, y) \in E \mid x, y \geq 0\} \subset R^2$, $X = R$ and $d : X \times X \to E$ such that $d(x, y) = (|x - y|, \alpha |x - y|)$, where $\alpha \geq 0$ is a constant. Then (X, d) is a cone metric space.

Definition 2.6.(See [16, 17]). Let X be a nonempty set and $K \geq 1$ be a real number. Suppose the mapping $D : X \times X \to [0, \infty)$ satisfies

(D1) $D(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;

(D2) $D(x, y) = D(y, x)$ for all $x, y \in X$;

(D3) $D(x, z) \leq K(D(x, y) + D(y, z))$ for all $x, y, z \in X$.

(X, D, K) is called metric type space. Obviously, for $\mathrm{K} = 1$, metric type space is a metric space.

Note that property (D3) is discouraging since it does not give the classical triangle inequality satisfied by a distance. But there are many examples where the triangle inequality fails (See[17]).

Example 2.7. (See [17]). Let X be the set of Lebesgue measurable functions on $[0,1]$ such that

$\int_0^1 |f(x)|^2\, dx < \infty$. Suppose $D: X \times X \to [0,\infty)$ is defined by

$$D(f,g) = \int_0^1 |f(x) - g(x)|^2\, dx$$

for all $f,g \in X$. Then (X,D) is a metric type space with $\mathrm{K} = 2$.

Similarly, we can define convergence in metric type spaces.

Definition 2.8.(See [17]). Let (X,D) be a metric type space and $\{x_n\}$ be a sequence in X and $x \in X$. we say $\{x_n\}$ is

(i) a Convergent sequence if and only if $\lim\limits_{n\to\infty} D(x_n, x) = 0$,

(ii) a Cauchy sequence if and only if $\lim\limits_{n,m\to\infty} D(x_n, x_m) = 0$.

In the sequel, we consider some old definitions such as coincidence points and weakly compatibility conditions.

Definition 2.9.(See [14]). Let $f,g: X \to X$ be mappings of a set X. If $fw = gw = z$ for some $z \in X$, then w is called a coincidence point of f and g, and z is called a point of coincidence of f and g.

Definition 2.10.(See [14]). Let f and g be two self-maps defined on a set X. Then f and g are said to be weakly compatible if they commute at every coincidence point, that is, if $fgw = gfw$ for all coincidence points w.

Lemma 2.11.(See [1]). Let f and g be weakly compatible self-maps of a set X. If f and g have a unique point of coincidence $fw = gw = z$, then z is the unique common fixed point of f and g.

MAIN RESULTS

The following theorem is the extension of Theorem 2 of [3] and Theorem 2.8 of [2] in the metric type space case.

Theorem 3.1. Let (X,D,K) be a metric type space with constant $K \geq 1$.Suppose the mappings f, g, S and T are four self-maps on X, satisfying
$f(X) \subset T(X)$, $g(X) \subset S(X)$ and

$$D(fx,gy) \leq \alpha D(Sx,Ty) + \beta D(fx,Sx) + \gamma D(gy,Ty) + \delta[D(fx,Ty) + D(gy,Sx)], \quad (1)$$

for all $x,y \in X$, where $\alpha,\beta,\gamma,\delta \in [0,\frac{1}{K})$ satisfying

$$K\alpha + (K+1)\max\{\beta,\gamma\} + (K^2 + K)\delta < 1. \quad (2)$$

If one of $f(X)$, $g(X)$, $\mathrm{S(X)}$ or $T(X)$ is a complete subspace of X, then $\{f,S\}$ and $\{g,T\}$ have a unique point of coincidence in X. Moreover, if $\{f,S\}$ and $\{g,T\}$ are weakly compatible, then f, g, S and T have a unique common fixed point.

Proof. Suppose x_0 is an arbitrary point of X. Since $f(X) \subset T(X)$, so there exists $x_1 \in X$ such that $\mathrm{f(x_0) = T(x_1) = y_1}$. Since $g(X) \subset S(X)$, then there exists $x_2 \in X$ such that $\mathrm{g(x_1) = S(x_2) = y_2}$.
If we continue in this manner, then

$$\exists x_{2n+1} \in X \quad \text{s.t} \quad y_{2n+1} = fx_{2n} = Tx_{2n+1},$$
$$\exists x_{2n+2} \in X \quad \text{s.t} \quad y_{2n+2} = gx_{2n+1} = Sx_{2n+2}$$

for all $n = 0, 1, \cdots$. Now,

$$\begin{aligned}
&D(y_{2n+1}, y_{2n+2}) = D(fx_{2n}, gx_{2n+1})\\
&\leq \alpha D(Sx_{2n}, Tx_{2n+1}) + \beta D(fx_{2n}, Sx_{2n}) + \gamma D(gx_{2n+1}, Tx_{2n+1})\\
&+\delta[D(fx_{2n}, Tx_{2n+1}) + D(gx_{2n+1}, Sx_{2n})]\\
&= \alpha D(y_{2n}, y_{2n+1}) + \beta D(y_{2n+1}, y_{2n}) + \gamma D(y_{2n+2}, y_{2n+1})\\
&+\delta[D(y_{2n+1}, y_{2n+1}) + D(y_{2n+2}, y_{2n})]\\
&\leq (\alpha + \beta + \delta K) D(y_{2n}, y_{2n+1}) + (\gamma + \delta K) D(y_{2n+1}, y_{2n+2}), \qquad (3)
\end{aligned}$$

which implies that $D(y_{2n+1}, y_{2n+2}) \leq \frac{\alpha + \beta + \delta K}{1 - \gamma - \delta K} D(y_{2n}, y_{2n+1})$. Similarly,

$$\begin{aligned}
&D(y_{2n+3}, y_{2n+2}) = D(fx_{2n+2}, gx_{2n+1})\\
&\leq \alpha D(Sx_{2n+2}, Tx_{2n+1}) + \beta D(fx_{2n+2}, Sx_{2n+2}) + \gamma D(gx_{2n+1}, Tx_{2n+1})\\
&+\delta[D(fx_{2n+2}, Tx_{2n+1}) + D(gx_{2n+1}, Sx_{2n+2})]\\
&= \alpha D(y_{2n+2}, y_{2n+1}) + \beta D(y_{2n+3}, y_{2n+2}) + \gamma D(y_{2n+2}, y_{2n+1})\\
&+\delta[D(y_{2n+3}, y_{2n+1}) + D(y_{2n+2}, y_{2n+2})]\\
&\leq (\alpha + \gamma + \delta K) D(y_{2n+2}, y_{2n+1}) + (\beta + \delta K) D(y_{2n+3}, y_{2n+2}), \qquad (4)
\end{aligned}$$

which implies that $D(y_{2n+3}, y_{2n+2}) \leq \frac{\alpha + \gamma + \delta K}{1 - \beta - \delta K} D(y_{2n+2}, y_{2n+1})$. From (3) and (4), we have

$$D(y_n, y_{n+1}) \leq \lambda D(y_{n-1}, y_n) \leq \cdots \leq \lambda^n D(y_0, y_1) \; (5)$$

where $\lambda = \max\{\frac{\alpha + \beta + \delta K}{1 - \gamma - \delta K}, \frac{\alpha + \gamma + \delta K}{1 - \beta - \delta K}\} < \frac{1}{K}$ for $K \geq 1$ (by (2)). For $m > n$, we have

$$\begin{aligned}
&D(y_n, y_m) \leq K[D(y_n, y_{n+1}) + D(y_{n+1}, y_m)]\\
&\leq KD(y_n, y_{n+1}) + K^2[D(y_{n+1}, y_{n+2}) + D(y_{n+2}, y_m)]\\
&\leq \cdots \leq KD(y_n, y_{n+1}) + K^2 D(y_{n+1}, y_{n+2}) + \cdots + K^{m-n-1} D(y_{m-2}, y_{m-1}) + K^{m-n} D(y_{m-1}, y_m).
\end{aligned}$$

Now (5) and $\lambda K < 1$ imply that

$$\begin{aligned}
&D(y_n, y_m) \leq (K\lambda^n + K^2\lambda^{n+1} + \cdots + K^{m-n-1}\lambda^{m-2} + K^{m-n}\lambda^{m-1}) D(y_0, y_1)\\
&= K\lambda^n (1 + K\lambda + \cdots + (K\lambda)^{m-n-1}) D(y_0, y_1)\\
&\leq \frac{K\lambda^n}{1 - K\lambda} D(y_0, y_1) \to 0 \text{ as } n \to \infty.
\end{aligned}$$

It follows that $\{y_n\}$ is a Cauchy sequence. Suppose $S(X)$ is a complete subspace of X. Then $\{y_n\}$ is convergent in $S(X)$ and so there exists $z \in X$ such that $\lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} y_{2n} = z$. Consequently, there is a $w \in X$ such that $Sw = z$. Now, we show $\mathrm{fw} = \mathrm{z}$.

$$\begin{aligned}
&D(fw,z) \le K[D(fw,gx_{2n+1}) + D(gx_{2n+1},z)] \\
&\le K\big[\alpha D(Sw,Tx_{2n+1}) + \beta D(fw,Sw) + \gamma D(gx_{2n+1},Tx_{2n+1}) \\
&+\delta[D(fw,Tx_{2n+1}) + D(gx_{2n+1},Sw)]\big] + KD(gx_{2n+1},z) \\
&= K\big[\alpha D(z,y_{2n+1}) + \beta D(fw,z) + \gamma D(y_{2n+2},y_{2n+1}) \\
&+\delta[D(fw,y_{2n+1}) + D(y_{2n+2},z)]\big] + KD(y_{2n+2},z) \\
&\le (K\alpha + K^2\delta)D(z,y_{2n+1}) + K\gamma D(y_{2n+2},y_{2n+1}) + K(\delta+1)D(y_{2n+2},z) \\
&+(K\beta + K^2\delta)D(fw,z)
\end{aligned}$$

which implies that

$$(1-K\beta-K^2\delta)D(fw,z) \le (K\alpha+K^2\delta)D(z,y_{2n+1}) + K\gamma D(y_{2n+2},y_{2n+1}) + K(\delta+1)D(y_{2n+2},z). \quad (6)$$

Since $\{y_n\}$ is convergent, and by (2) and (6), we have

$$\begin{aligned}
&D(fw,z) \le \frac{1}{1-K\beta-K^2\delta}[(K\alpha+K^2\delta)D(z,y_{2n+1}) + K\gamma D(y_{2n+2},y_{2n+1}) \\
&+K(\delta+1)D(y_{2n+2},z)] \to 0 \text{ as } n \to \infty.
\end{aligned}$$

It follows that $\mathrm{D(fw,z)} = 0$ that is, $fw = z$. Thus, we have $\mathrm{fw} = \mathrm{Sw} = \mathrm{z}$, that is, z is a point of coincidence of mappings f and S and w is a coincidence point of mappings f and S. Since $z \in f(X) \subset T(X)$ then there exists $u \in X$ such that $Tu = z$. Now, we prove $\mathrm{gu} = \mathrm{z}$.

$$\begin{aligned}
&D(z,gu) \le K[D(z,fx_{2n}) + D(fx_{2n},gu)] \\
&\le KD(fx_{2n},z) + K\big[\alpha D(Sx_{2n},Tu) + \beta D(fx_{2n},Sx_{2n}) + \gamma D(gu,Tu) \\
&+\delta[D(fx_{2n},Tu) + D(gu,Sx_{2n})]\big] \\
&= KD(y_{2n+1},z) + K\big[\alpha D(y_{2n},z) + \beta D(y_{2n+1},y_{2n}) + \gamma D(gu,z) \\
&+\delta[D(y_{2n+1},z) + D(gu,y_{2n})]\big] \\
&\le (K\alpha+K^2\delta)D(z,y_{2n}) + K\beta D(y_{2n+1},y_{2n}) + K(\delta+1)D(y_{2n+1},z) \\
&+(K\gamma+K^2\delta)D(gu,z),
\end{aligned}$$

which implies that

$$(1-K\gamma-K^2\delta)D(z,gu) \le (K\alpha+K^2\delta)D(z,y_{2n}) + K\beta D(y_{2n+1},y_{2n}) + K(\delta+1)D(y_{2n+1},z). \quad (7)$$

Since $\{y_n\}$ is convergent, and by (2) and (7), we have

$$\begin{aligned}
&D(z,gu) \le \frac{1}{1-K\gamma-K^2\delta}[(K\alpha+K^2\delta)D(z,y_{2n}) \\
&+K\beta D(y_{2n+1},y_{2n}) + K(\delta+1)D(y_{2n+1},z)] \to 0 \text{ as } n \to \infty
\end{aligned}$$

It follows that $D(z,gu)=0$, that is, $gu=z$. So, we have $gu=Tu=z$, that is, z is a point of coincidence of mappings g and T, and u is a coincidence point of mappings g and T. Hence $fw=gu=Sw=Tu=z$.

Now we shall show that z is unique point of coincidence of pairs $\{f,S\}$ and $\{g,T\}$. Let z' be also a point of coincidence of these four mappings, then $fw'=gu'=Sw'=Tu'=z'$ for $w',u'\in X$. From (1), we have

$$D(z,z')=D(fz,gz')$$
$$\le \alpha D(Sz,Tz')+\beta D(fz,Sz)+\gamma D(gz',Tz')+\delta[D(fz,Tz')+D(gz',Sz)]$$
$$=\alpha D(z,z')+\beta D(z,z)+\gamma D(z',z')+\delta[D(z,z')+D(z',z)]$$
$$=(\alpha+2\delta)D(z,z').$$

Therefore $z=z'$. By Lemma 2.11, if the pairs $\{f,S\}$ and $\{g,T\}$ are weakly compatible, then z is the unique common fixed point of f, S, g, and T.

The proofs for the cases in which $f(X)$, $g(X)$, or $T(X)$ are complete are similar.

The following results is obtained from Theorem 3.1.

Corollary 3.2. Let (X,D,K) be a metric type space with constant $K\ge 1$. Suppose the mappings f, g, S and T are four self-maps on X, satisfying

$f(X)\subset T(X)$, $g(X)\subset S(X)$ and for some $m,n\in\mathbf{N}$

$$D(f^m x,g^n y)\le \alpha D(S^m x,T^n y)+\beta D(f^m x,S^m x)+\gamma D(g^n y,T^n y)$$
$$+\delta[D(f^m x,T^n y)+D(g^n y,S^m x)] \quad (8)$$

for all $x,y\in X$, where $\alpha,\beta,\gamma,\delta\in[0,\frac{1}{K})$ satisfying

$$K\alpha+(K+1)\max\{\beta,\gamma\}+(K^2+K)\delta<1. \quad (9)$$

If one of $f(X)$, $g(X)$, $S(X)$ or $T(X)$ is a complete subspace of X, then $\{f,S\}$ and $\{g,T\}$ have a unique point of coincidence in X. Moreover, if $\{f,S\}$ and $\{g,T\}$ are weakly compatible, then f, g, S and T have a unique common fixed point.

Proof. In (1), set $f\equiv f^m$, $g\equiv g^n$, $S\equiv S^m$ and $T\equiv T^n$. Theorem 3.1.implies $\{f,S\}$ and $\{g,T\}$ have a unique common fixed point z.

The following corollaries extend and improve results in [1, 2] on metric type space.

Corollary 3.3. Let (X,D,K) be a metric type space with constant $K\ge 1$. Suppose the mappings f, g, T are three self-maps on X, satisfying

$f(X)\cup g(X)\subset T(X)$ and

$$D(fx,gy)\le \alpha D(Tx,Ty)+\beta D(fx,Tx)+\gamma D(gy,Ty)+\delta[D(fx,Ty)+D(gy,Tx)] \quad (10)$$

for all $x,y\in X$, where $\alpha,\beta,\gamma,\delta\in[0,\frac{1}{K})$ satisfying

$$K\alpha+(K+1)\max\{\beta,\gamma\}+(K^2+K)\delta<1. \quad (11)$$

If one of $f(X)$, $g(X)$ or $T(X)$ is a complete subspace of X, then $\{f,T\}$ and $\{g,T\}$ have a unique point of coincidence in X. Moreover, if $\{f,T\}$ and $\{g,T\}$ are weakly compatible, then f, g and T have a unique common fixed point.

Proof. In Theorem 3.1, set $S = T$ and suppose $f(X) \cup g(X) \subset T(X)$. Theorem 3.1 implies $\{f, T\}$ and $\{g, T\}$ have a unique common fixed point z.

Corollary 3.4. Let (X, D, K) be a metric type space with constant $K \geq 1$. Suppose the mappings f and T are two self-maps on X, satisfying

$f(X) \subset T(X)$ and

$$D(fx, fy) \leq \alpha D(Tx, Ty) + \beta D(fx, Tx) + \gamma D(fy, Ty) + \delta[D(fx, Ty) + D(fy, Tx)] \quad (12)$$

for all $x, y \in X$, where $\alpha, \beta, \gamma, \delta \in [0, \frac{1}{K})$ satisfying

$$K\alpha + (K+1)\max\{\beta, \gamma\} + (K^2 + K)\delta < 1. \quad (13)$$

If one of $f(X)$ or $T(X)$ is a complete subspace of X, then $\{f, T\}$ have a unique point of coincidence in X. Moreover, if $\{f, T\}$ are weakly compatible, then f and T have a unique common fixed point.

Proof. In (10), set $f = g$. Corollary 3.3 implies $\{f, T\}$ have a unique common fixed point z.

Remark 3.5. In Theorem 3.1.and any one of Corollaries from 3.2 to 3.4, set $K = 1$. Then we obtain analogues theorem and analogues corollaries for metric space. Moreover, if (X, D) is a cone metric space. Then we obtain some theorems introduced in [1-3]. Also, several fixed point theorems on cone metric type space for four mapping were proved by Cvetkovic et al. [7], that can obtain them by Theorem 3.1, Corollaries 3.3 and 3.4. Moreover, most of the examples in [7] will easily translate into metric type spaces.

Conclusion

In this paper we consider some old and new definitions on metric spaces, cone metric spaces and metric type spaces with some examples. Then, we prove a common fixed point theorem on metric type spaces, without appealing to continuity of any map involved therein. Also, we obtain some results of our main theorem in section 3. Our results generalize and unify old common fixed theorems of [1-3, 11].

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