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New Approach to Generalize Uncertainty Principle

Mahgoub. A. Salih, Taysir. M. Almahdi

Department of Physics, Faculty of Science and Art at al-Muznab, Al Qassim University, KSA

ABSTRACT

We have presented a model based on a combination of Schwarzschild's radius and Compton length to measure the gravitational length of an object. The model laid a new approach to generalize uncertainty principle. The model explained the physical meaning of the deforming linear parameter β_{0i} and it's variation from unity to maximum value.

KEYWORDS: generalized uncertainty principle; Deformed Heisenberg algebra; minimal length.

1. INTRODUCTION

Quantum gravity expresses as the regime where the Planck's quantities, Plank's length (l_p) and Planck mass(m_p) become relevant. On the other hand, general relativity implies a maximal mass of black hole (maximal mass that can be contained in limited system r) which occurred at Schwarzschild's radius. In addition, quantum effect entails mass fluctuation proportional to Planck constant. Therefore, quantum effect and the minimum measurable length that proportional to Planck length [1-7] rule the lower boundary of mass. The upper boundary is ruled by general relativity.

In this paper we will give a simple derivation of generalize uncertainty principle (GUP) and the physical meaning of the linear deformation parameters.

2. Generalized uncertainty principle

The threshold mass can be defined as the mass that satisfies

$$\frac{\hbar}{cl} = \frac{c^2 r}{2G} \qquad (1)$$

The solution depends on the length, $l = r = l_P$,where the threshold mass becomes Planck's mass. Now, if one tries to measure the gravitational radius of an object with mass $\bar{m}$ using both relations $r_1 = \frac{2G\bar{m}}{c^2}$ and $r_2 = \frac{\hbar}{\bar{m}c}$ the resultant measurement is $r = \bar{r} \pm \overline{\Delta r}$ which can be written as

$$r = \frac{1}{2}\left(\frac{2G\bar{m}}{c^2} + \frac{\hbar}{\bar{m}c}\right) \pm \frac{1}{2}\left(\frac{2G\bar{m}}{c^2} - \frac{\hbar}{\bar{m}c}\right) \qquad (2)$$

When $\bar{m} = m_p$,equation(2) gives

$$r = \bar{r} = \frac{1}{2}\left(\frac{2Gm_p}{c^2} + \frac{\hbar}{m_p c}\right) = \frac{2Gm_p}{c^2} = \frac{\hbar}{m_p c} = \sqrt{\frac{2\hbar G}{c^3}} = l_P \quad \text{, With } \overline{\Delta r} = 0 \qquad (3)$$

Where l_P is Planck length. But in general $r_1 \neq r_2$ and $\overline{\Delta r} \neq 0$.

We remark here, in microscopic measurement ($m < m_p$), the Compton length is greater than Schwarzschild's radius. Thus, the term $\frac{2Gm}{c^2}$ is neglected and the measuring length is equal to Compton length $\lambda_c = \frac{\hbar}{mc}$ .in contrary, in macroscopic measurement ($m > m_p$) the omitting term is $\frac{\hbar}{mc}$. Therefore, there is no appearance of quantum effects in macroscopic scale. Hence, one can says explicitly, the mass selects the equation that can be measured with, either Schwarzschild's radius or Compton length. This represents a transformation between macroscopic scale and microscopic scale. To this end one has

$$\frac{\hbar c}{m} \to 2GM \quad ; \quad M \to \frac{\alpha}{m} \quad ,\alpha = \frac{\hbar c}{2G} \qquad (4)$$

Where, m refers to microscopic mass ($m < m_p$) and M refers to macroscopic mass ($M > m_p$). This transformation represents mass duality [8].

Hence, the approach sign in the transformation (4) becomes exactly an equal sign with Planck's mass that what motivated Arbab[9] to builds on his characteristic Planck's constant $\hbar_c$ which depends on the size of the system.

***Corresponding Author:** Mahgoub. A. Salih, Department of Physics, Faculty of Science and Art at al-Muznab, Al Qassim University, KSA, mahgoubasalih@hotmail.com; taysirelmahdi@yahoo.com

On the other hand, T duality also suggests a generalization of the Heisenberg uncertainty principle according to which the best possible spatial resolution Δl is bounded below not only by the reciprocal of the momentum spread Δp but also by the string scale L_s.[10]
Return back to the measurement, the uncertainty in measuring the length l is Δl.The standard deviations is defined by

$$\Delta l = \sqrt{\langle l^2\rangle - \langle l\rangle^2} \qquad (5)$$

By substituting $l = l_P{}^2 \frac{p}{2\hbar} + \frac{\hbar}{2p}$ .the uncertainty in measuring l is given by

$$\Delta l \geq l_P{}^2 \frac{\Delta p}{2\hbar} \sqrt{1 + \frac{\hbar^4}{l_P{}^4} \frac{\langle \frac{1}{p^2}\rangle - \langle \frac{1}{p}\rangle^2}{\langle p^2\rangle - \langle p\rangle^2} + \frac{2\hbar^2}{l_P{}^2 (\Delta p)^2}\left(1 - \langle p\rangle \langle \tfrac{1}{p}\rangle\right)} \qquad (6)$$

Where p is the momentum ,$\frac{G\bar{m}}{c^2} = l_P{}^2 \frac{p}{2\hbar}$, $\Delta\left(\frac{1}{p}\right) = \sqrt{\langle \frac{1}{p^2}\rangle - \langle \frac{1}{p}\rangle^2}$ and $\Delta P = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$.

If the term $\frac{\hbar^4}{l_P{}^4} \frac{\langle \frac{1}{p^2}\rangle - \langle \frac{1}{p}\rangle^2}{\langle p^2\rangle - \langle p\rangle^2} + \frac{2\hbar^2}{l_P{}^2 (\Delta p)^2}\left(1 - \langle p\rangle \langle \frac{1}{p}\rangle\right) \ll 1$, one has

$$\Delta l \geq l_P{}^2 \frac{\Delta p}{2\hbar} + \frac{\hbar^3 \left(\Delta\left(\frac{1}{p}\right)\right)^2}{4 l_P{}^2 \Delta p} + \frac{\hbar}{2\Delta p}\left(1 - \langle p\rangle \langle \tfrac{1}{p}\rangle\right) \qquad (7)$$

Therefore, the generalized uncertainty takes the form

$$\Delta l \Delta P \geq l_P{}^2 \frac{(\Delta p)^2}{2\hbar} + \frac{\hbar^3 \left(\Delta\left(\frac{1}{p}\right)\right)^2}{4 l_P{}^2} + \frac{\hbar}{2}\left(1 - \langle p\rangle \langle \tfrac{1}{p}\rangle\right) \qquad (8)$$

Moreover, the commutation relation for position and momentum is given by

$$[X, P] = i\hbar \left(1 + l_P{}^2 \frac{(\Delta p)^2}{\hbar^2} + \frac{\hbar^2 \left(\Delta\left(\frac{1}{p}\right)\right)^2}{2 l_P{}^2} - \langle p\rangle \langle \tfrac{1}{p}\rangle\right) \qquad (9)$$

By comparing (8) with the deformed Heisenberg uncertainty [11-13]

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + \sigma(\Delta p)^2 + \sigma\langle P\rangle^2\right) \qquad (10)$$

Where σ is the deformation parameter. One finds

$$\sigma = \frac{l_P{}^2}{\hbar^2} \qquad (11)$$

Moreover, the minimum measuring length is

$$\Delta x_{min}(\langle P\rangle) = \hbar\sqrt{\sigma}\sqrt{1 + \sigma\langle P\rangle^2} \qquad (12)$$

The smallest measuring length according to equations (11) and (12) when $\langle P\rangle = 0$ is [14,15]

$$\Delta x_{min} = \hbar\sqrt{\sigma} = l_P \qquad (13)$$

According to the model, equation (7) gives the smallest measuring length,$\Delta l = l_P$, in Plank's scale which implies

$$\Delta\left(\tfrac{1}{P}\right) = \frac{l_P{}^2}{\hbar^2}\sqrt{2}\Delta P = \sqrt{2}\,\sigma \Delta P \text{and } \langle p\rangle \langle \tfrac{1}{p}\rangle = 1 \qquad (14)$$

The above result consents with the result in (3) that, the measurement $r = l_P$, when $m = m_p$.
By rewriting equation (7) in linear combination of the form

$$\Delta l \Delta P \geq \frac{1}{2}\left[\beta_1\left(1 - \langle p\rangle \langle \tfrac{1}{p}\rangle\right) + \beta_2 \left(\Delta\left(\tfrac{1}{p}\right)\right)^2 + \beta_3 (\Delta p)^2\right] \qquad (15)$$

With the linear parameters $\beta_1 = \hbar$,$\beta_2 = \frac{\hbar^3}{2 l_P{}^2}$ and $\beta_3 = \frac{l_P{}^2}{\hbar}$.
The general form of the linear parameter β can be formed and related to Plank's length as

$$\beta_i = \frac{\hbar}{(n+1)!} \cdot \left(\frac{\hbar}{l_p}\right)^{2n} = \beta_{0i} l_P{}^2 \qquad (16)$$

Where $n = 0, \pm 1, \pm 2, \ldots$,which represents the state of the scale e.g. $n = 0$ is the Plank's scale where $m = m_p$ .we find the coupling of macroscopic and microscopic state relation (15) shows this state in term of $(\langle p\rangle \langle \frac{1}{p}\rangle)$.also the state $n = +1$ represents microscopic scale where $m < m_p$ and the state $n = -1$ represents macroscopic scale with $m > m_p$.
$i = 1,2,3, \ldots$, is the β parameter index, which indicates scales order.
Finally, the general form of uncertainty and the commutation relation for position and momentum according to relation (15) and equation (16) is given by

$$\Delta l \Delta P \geq \frac{\hbar}{2}\left[\sum_{i,n} \frac{C_i}{(n+1)!} \cdot \left(\frac{\hbar}{l_p}\right)^{2n}\right] \qquad (17)$$

$$[X, P] = i\hbar \left[\sum_{i,n} \frac{C_i}{(n+1)!} \cdot \left(\frac{\hbar}{l_p}\right)^{2n}\right] \qquad (18)$$

Where C_i is the relation of the standard deviation of momentum and the fundamental length of the scale that conserve the dimension of the term $\Delta l \Delta P$.Therefore,$\Delta l \Delta P = \beta_i C_i \equiv \hbar$.

e.g. when $n = +2$, $i = 4$ and $\beta_4 = \frac{\hbar^5}{6{l_P}^4}$. The dimension of $\Delta l \Delta P$ should be equal to $\hbar$ dimension. Hence, $C_i = (\Delta p)^4$.
According to equation (16), one finds the dimension of β_{0i} as

$$\beta_{01} = \frac{\hbar}{{l_P}^2}([M][T]^{-1}), \beta_{02} = \frac{\hbar^3}{2{l_P}^4} = \frac{1}{2}\hbar(\beta_{01})^2([M]^3[L]^2[T]^{-3}), \beta_{03} = \frac{1}{\hbar} j^{-1}.s^{-1} \quad (19)$$

Equation (13) gives in Planck scale, $\Delta x_{min} = l_P = \sqrt{\beta_0} l_P$,therefore,$\beta_0 = 1$.
In natural units, where $\hbar = c = 1$,one finds , $\beta_1 = 1$,$\beta_3 = {l_P}^2$ and $\beta_2 = \frac{1}{2{l_P}^2}$.similarly ,$\beta_{01} = \frac{1}{{l_P}^2}$,$\beta_{03} = 1$ and $\beta_{02} = \frac{1}{2{l_P}^4}$.
Most of quantum theories of gravity indicated that, β_0 in order of unity in Planck scale. On the other hand, β_{0i} has believed to be energy scale constant and vary with the scale [16].
From above there are numerous values of β_{0i} related to the various scales or energies.
According to equation(18) the hidden physical meaning of β_{0i} became cleared as the following:

- β_{01} is the mass (creation-annihilation) rate. Therefore, space-time accelerate to require region, space quanta, to allow matter to be placed in it and vice versa.
- β_{02} is the cosmic string energy loss (cosmic string had formed at a phase transition in the early universe, which is responsible for the large-scale structure of the universe).β_{02} demands that, the energy loss mechanism is sufficient so that the energy density of strings will scale as l^{-4} as is necessary for the consistency of the string scenario. [16-19]

Hence,β_{0i} varies through the range $1 \leq \beta_{0i} \leq \frac{\hbar^3}{2{l_P}^4}$ .this result in a good agreement with the values predicted in [20] and references there in.
Moreover, Das S and Vagenas ,[21]Showed that the GUP effect is unobservable with$\beta_0 = 1$.this consists with the arguments given above. That is, in Plank's scale, from equation (14) ,$\langle p \rangle \langle \frac{1}{p} \rangle = 1$ and $\Delta\left(\frac{1}{P}\right) = \frac{{l_P}^2}{\hbar^2}\sqrt{2}\Delta P$. By substituting these in equation (15) the first term in RHS vanished in Plank's scale and the term $\frac{\hbar^2}{2{l_P}^2}\left(\Delta\left(\frac{1}{p}\right)\right)^2 + \frac{{l_P}^2}{\hbar^2}(\Delta p)^2 = 2$ .Therefore, equation (15) construes to the well-known form of uncertainty relation $\Delta l \Delta P \geq \hbar$ and GUP effect disappeared.

3. Conclusion

Instructively, in this work we generalized uncertainty principle by combining microscopic and macroscopic measurement. Equations (17) and (18) are the complete generalization of uncertainty and commutation relation formulation respectively. The physical meaning of the linear parameters, β_{0i}, can be used to explain the mass (creation-annihilation) rate and cosmic string energy loss.

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REFERENCES

[1] G. Veneziano,1986.A Stringy Nature Needs Just Two Constants.Europhys. Lett.2, 199.

[2] D. J. Gross and P. F. Mende, 1988.String theory beyond the Planck scale.Nucl. Phys. B 303, 407.

[3] D. Amati, M. Ciafaloni and G. Veneziano,1989.Can spacetimebe probed below the string siz

. Phys. Lett. B 216, 41.

[4] K. Konishi, G. Paffuti and P. Provero,1990.Minimum physical length and the generalized uncertainty principle in string theory.Phys. Lett.B 234, 276.

[5] Pouria Pedram,2012.New Approach to Nonperturbative Quantum Mechanics with Minimal Length Uncertainty.arXiv:1112.2327v2 [hep-th].

[6] M. Maggiore,1993.Anti- de Sitter black hole thermodynamics and the generalized uncertainty principle.Phys.Lett. B 304, 65.[arXiv: hep-th/9301067].

[7] L. J. Garay,1995.Quantum gravity and minimum length. Int. J. Mod. Phys. A 10, 145[arXiv:gr-qc/9403008].

[8] E. Witten,1996. Duality, spacetime and quantum mechanics.p. 24 in April 1996 Physics Today.

[9] Arbab.I.Arbab,2004.Quantization of Gravitational System and its Cosmological Consequences.arXiv:gr-qc/0309040v2.

[10] JOHN H. SCHWARZ,1996.The second superstring revolution.arXiv:hep-th/9607067v1.

[11] G. Amelino-Camelia,2003. Phenomenology of Doubly Special Relativity.arXiv:gr-qc/0312124v1.

[12] T Masłowski, A Nowicki and V M Tkachuk,2012.Deformed Heisenberg algebra and minimal length. J. Phys. A: Math.Theor.45 075309 doi:10.1088/1751-8113/45/7/075309.

[13] F. Girelli, E.R. Livine, and D. Oriti,2005.Deformed Special Relativity as an effective flat limit of quantum gravity.Nucl.Phys. B 708, 411, gr-qc/0406100.

[14] AchimKempf, GianpieroMangano, Robert B. Mann,1995.Hilbert space representation of the minimal length uncertainty relation. Phys.Rev.D52:1108.

[15] Achim Kempf,1996.Nonpointlike Particles in Harmonic Oscillators.arXiv:hep-th/9604045v1 9.

[16] T. W. B. Kibble, J. Phys.. A 9 (1976) 1387;Phys. Rep. 67 (1980) 183

[17] A. Vilenkin,1985.Cosmic strings and domain walls. Phys. Rep. Volume 121, Issue 5,Pages 263–315

[18] E. Witten,1985.Cosmic Superstrings. Phys. Lett. 153B . 243.

[19] A. Vilenkin,1981.Cosmological Density Fluctuations Produced by Vacuum Strings.Phys. Rev. Lett. 46, 1169–1172

[20] Benrong Mu, Houwen Wu and Haitang Yang,2009.The generalized uncertainty principle in the presence of extra dimensions.arXiv:0909.3635v2 [hep-th].

[21]Saurya Das,Elias C. Vagenas,2008.Universality of Quantum Gravity Corrections.Phys. Rev. Lett. 101, 221301.