

## Synchronization of Two Different Chaotic Neural Networks with Unknown Time Delay Using Time-Scale Separation Technique

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### ABSTRACT

This paper addresses a novel synchronization scheme between two different kinds of delayed chaotic neural networks (NNs) with unknown parameters. A time-scale separation based technique is used to design the control law which guarantees the semi-global ultimately boundedness of the error synchronization between two different chaotic NNs with unknown time delay. A simulation example is exploited to illustrate the effectiveness of the proposed approach.

**KEYWORDS:** Chaotic Neural Networks, Non-identical Synchronization, Unknown Time Delay, Time-scale separation

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### INTRODUCTION

The possibility of synchronization of two coupled chaotic systems was first shown by Pecora and Carrol [1]. Thereafter this topic aroused great interest as a potential mean for communications [2], [3]. In recent years a great deal of effort has been devoted to extend the chaotic communication applications to the field of secure communications. A detailed survey of chaotic secure communication systems is presented by Yang [4].

Chaotic neural networks (NNs) have a rich range and flexible dynamics. These properties give them the ability to solve optimization problems [5], [6]. Chaotic NNs synchronization and its applications presented by Milanovic et al. and Tan et al. [7], [8]. The studies on synchronization of chaotic neural networks have attracted considerable attention, see [9]-[13].

The finite switching speed of the amplifiers and communication time may cause time delay in electronic implementations of neural networks, so delayed neural networks (or in fact neural networks with time delay) were proposed. Such neural networks have been applied in signal and image processing, associative memories, combinatorial optimization, automatic control, and other areas. In addition there have been many efforts for the study of dynamical properties of delayed neural networks (DNNs) [14], [15]. It has been shown that such DNNs can exhibit complicated dynamics and even chaotic behavior if the parameters and time delays are appropriately chosen for the DNNs [16]. This class of delayed chaotic NNs can include several well known delayed chaotic NNs, such as delayed chaotic Hopfield NNs and delayed chaotic cellular NNs. The synchronization problem for neural networks with time delay was investigated for identical delayed neural networks by Huang and Cao, Cao and Lu and Yan et al. [17]-[19].

In general, the parameters of the drive and response chaotic NNs are not identical in real-world applications. In other hand, time delays always exist and parameters of the system are inevitably perturbed by external factors. Consequently synchronizing two different time-delayed chaotic NNs with partially unknown parameters is an essential and useful topic in practice. Zhang et al. proposed an adaptive control approach to synchronize two time-delayed chaotic NNs in the presence of partly unknown parameters [20]. Huang and Feng proposed an integral SMC approach to address the synchronization problem of chaotic neural networks with time delays [21]. The main contributions of their study are to investigate the effect of the mismatched parameters on the synchronization of drive-response systems and to propose an integral SMC approach to solving it. To the best of our knowledge, this problem for a fully unknown time delay parameters has not yet been studied and remains a challenge.

Control designs based on time-scale separation have been recently presented by Hovakimyan et al. [22]. They proposed a new method for approximation of dynamic inversion of non-affine-in-control systems via time-scale separation. In [23] the authors extended the methodology to uncertain systems and developed a direct adaptive counterpart of the method. Chakraborty and Arcak proposed two different time-scales separation based robust redesign techniques which recover the trajectories of a nominal control design in the presence of uncertain nonlinearities [24]. They extended their work to nonlinear systems with un-modelled dynamics presented in [25].

Most of the work cited before assumed either system parameters are known or only a few parameters are not given. In this paper a nonlinear controller is proposed for synchronization of two different chaotic neural networks in the presence of unknown time delay. The effects of uncertainty and unknown parameters in the synchronization error dynamic are handled based on a time-scale separation technique. Using the Lyapunov theory, the stability of the closed-loop system is proved. Simulation results illustrate the effectiveness of the proposed method.

The rest of this article is organized as follows. In Section 2, the class of systems under study and the chaos synchronization problem are described. Section 3 presents the main contribution of this paper which consists of a time-scale separation technique based nonlinear controller to synchronize two uncertain delayed chaotic NNs. In order to demonstrate

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the validity of the approach a numerical example including a delayed chaotic cellular NN as a drive system and a delayed chaotic Hopfield NN as a response system is presented in Section 4. Section 5 provides the concluding remarks.

## I. PROBLEM DESCRIPTION

A class of delayed chaotic NNs is described by the following time delay differential equations:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^n a_{ij} g_j(x_j(t)) + \sum_{j=1}^n b_{ij} g_j(x_j(t - \tau_j)) + H_i(t) \quad (1)$$

where  $n > 2$  denotes the number of neurons in the network,  $x_i(t)$  denotes the state variable associated with the  $i$ th neuron, the bounded delay  $\tau_j$  is a positive constant,  $c_i > 0$  denotes the self connection of neurons,  $a_{ij}$  indicates the strength of the neuron interconnections within the network,  $b_{ij}$  indicates the strength of the neuron interconnections within the network with constant delay  $\tau_j \geq 0$ , and  $H_i(t)$  is an unknown external input,  $i, j = 1, 2, \dots, n$ . Suppose that the drive system (1) is partially known

$$\dot{x}_i(t) = \bar{g}_i(x_i) + \Delta \bar{g}_i(x_i, \tau, t) \quad (2)$$

where  $\bar{g}_i(x_i)$  is the known part of the drive system (1) and  $\Delta \bar{g}_i(x_i, \tau, t)$  denotes the unknown part of system (1). The response system is described as follows:

$$\begin{aligned} \dot{z}_i(t) = & -d_i z_i(t) + \sum_{j=1}^n p_{ij} f_j(z_j(t)) + \sum_{j=1}^n q_{ij} f_j(z_j(t - \tau_j)) \\ & + W_i(t) + u_i(t) \end{aligned} \quad (3)$$

where  $n > 2$  denotes the number of neurons in the network,  $z_i(t)$  denotes the state variable associated with the  $i$ th neuron,  $d_i > 0$  denotes the self connection of neurons,  $p_{ij}$  and  $q_{ij}$  indicate the interconnection strength among neurons within the network with the bounded delay  $\tau_j \geq 0$ ,  $W_i(t)$  is an unknown external input, and  $u_i(t)$  denotes the external control input and will be appropriately designed to obtain a certain control objective,  $i, j = 1, 2, \dots, n$ . In the similar way, the response system (3) can be rewritten as

$$\dot{z}_i(t) = \bar{f}_i(z_i) + \Delta \bar{f}_i(z_i, \tau, t) + u_i(t) \quad (4)$$

where  $\bar{f}_i(z_i)$  is the known part of the response system (3) and  $\Delta \bar{f}_i(z_i, \tau, t)$  denotes the unknown part of system (3).

*Remark 1:* The external input terms in the model of the drive and response system in (1) and (3) are assumed to be the time-varying unknown parameters contrary to known constant one in the similar works [20,21].

*Remark 2:* The models in (2) and (4) are proposed to generalize the problem formulation for different cases of known and unknown parts.

Define the synchronization error signal  $e_i(t) = z_i(t) - x_i(t)$ . The objective of the paper is to design controller  $u(t) = [u_1(t), u_2(t), \dots, u_n(t)]^T$  such that the trajectory of the response DNN (2) can be synchronized with the trajectory of the drive DNN (1) in the presence of uncertainties  $\Delta \bar{g}_i(x_i, \tau_j, t)$  and  $\Delta \bar{f}_i(z_i, \tau_j, t)$ .

## II. MAIN RESULTS

This section proposes a semi-global nonlinear controller to synchronize two uncertain delayed chaotic NNs based on the time-scale separation technique. The synchronization error dynamics is rewritten as:

$$\begin{aligned} \dot{e}_i(t) = & \dot{z}_i(t) - \dot{x}_i(t) = \bar{f}_i(z_i) + \Delta \bar{f}_i(z_i, \tau, t) \\ & - \bar{g}_i(x_i) - \Delta \bar{g}_i(x_i, \tau, t) + u_i(t) \\ = & h_i(x_i, z_i) + \Delta h_i(x_i, z_i, \tau, t) + u_i(t) \end{aligned} \quad (5)$$

*Assumption 1:* Suppose that  $\Delta h_i(x_i, z_i, \tau, t)$  is a smooth function with respect to  $t$  such that

$$\left| \frac{\partial \Delta h_i(x_i, z_i, \tau, t)}{\partial t} \right| \leq A_i \quad \forall t \in \mathbb{R}^+$$

where  $A_i$  is an unknown positive constant.

*Remark 3:* Assumption 1 is less conservative than that of the similar works [20,21].

*Remark 4:* The parameter indexes are dropped for notation simplicity in the rest of this section.

To design a control input  $u$  which stabilizes the origin  $e = 0$  of (5) despite the presence of  $\Delta h_i$ , we define the following nonlinear system

$$\dot{\hat{e}}_i = h_i + u_i - \frac{\hat{e}_i - e_i}{\varepsilon_i} \quad (6)$$

where  $\varepsilon_i > 0$  is a small constant design parameter. Let

$$l_{e_i} \equiv \frac{\hat{e}_i - e_i}{\varepsilon_i} \quad (7)$$

The time derivative of  $l_{e_i}$  is

$$\varepsilon_i \dot{l}_{e_i} = \dot{\hat{e}}_i - \dot{e}_i = -l_{e_i} - \Delta h_i \quad (8)$$

It is obvious that  $\varepsilon_i \rightarrow 0$ ,  $l_{e_i} \rightarrow -\Delta h_i$ . When  $\varepsilon_i$  is small,  $l_{e_i}$  grows in a faster time-scale than  $e_i$ , and reaches a small neighbourhood of the manifold  $l_{e_i} = -\Delta h_i$ .

To compensate the effect of  $\Delta h_i$ , one can choose  $i$ th component of the control input  $u$  as:

$$u_i = -\alpha_i e_i - h_i + l_{e_i} \quad (9)$$

where  $\alpha_i$  is a positive constant design parameter. Now, we summarize the above control scheme in the following theorem.

**Theorem 1.** Consider the error system (5). Given a compact set  $\Omega_e \subset \mathbb{R}^n$  of initial conditions there exists  $\varepsilon_i^* > 0$  such that for all  $0 < \varepsilon_i < \varepsilon_i^*$  and for all  $e(0) \in \Omega_e$ , the controller (8) and (9) guarantees semi-globally ultimately boundedness of  $e(t)$ ,  $\hat{e}(t)$ , and  $z(t)$ . Furthermore, the synchronization error converges to an arbitrary small neighbourhood of the origin.

*Proof.* Define the off-manifold  $\eta_i$  as:

$$\eta_i = l_{e_i} + \Delta h_i. \quad (10)$$

The time derivative of  $\eta_i$  is:

$$\dot{\eta}_i = \dot{l}_{e_i} + \dot{\Delta h}_i = -\frac{\eta_i}{\varepsilon_i} + \frac{\partial \Delta h_i}{\partial x_i} \dot{x}_i + \frac{\partial \Delta h_i}{\partial z_i} \dot{z}_i + \frac{\partial \Delta h_i}{\partial t} \quad (11)$$

Consider the Lyapunov function  $V = \frac{1}{2} \sum_{i=1}^n (e_i^2 + \eta_i^2)$ .

The time derivative of  $V$  along the trajectories (5) and (11) is:

$$\dot{V} = \sum_{i=1}^n e_i [h_i + \Delta h_i + u_i] + \eta_i \left[ -\frac{\eta_i}{\varepsilon_i} + \frac{\partial \Delta h_i}{\partial x_i} \dot{x}_i + \frac{\partial \Delta h_i}{\partial z_i} \dot{z}_i + \frac{\partial \Delta h_i}{\partial t} \right] \quad (12)$$

Substituting the proposed control law (9) into (12), one has:

$$\dot{V} = \sum_{i=1}^n -\alpha_i e_i^2 + e_i \eta_i - \frac{\eta_i^2}{\varepsilon_i} + \eta_i \left[ \frac{\partial \Delta h_i}{\partial e_i} \dot{e}_i + \frac{\partial \Delta h_i}{\partial x_i} \dot{x}_i + \frac{\partial \Delta h_i}{\partial t} \right]$$

Suppose that a compact set  $\Omega_e$  of initial conditions is given. From (6) and (7) one concludes that  $l_i(0) = 0$ ,  $i = 1, \dots, n$ .

Therefore, for any compact set  $\Omega_e$  of initial conditions  $e(0)$ , we can find a corresponding compact set  $\Omega_\eta$  of initial conditions  $\eta(0) = [\eta_1(0), \dots, \eta_n(0)]$ . Thus, one finds a level set  $\Omega_c$  of  $V$  such that  $\Omega_e \times \Omega_\eta \subseteq \Omega_c$  and a positive number  $L_i$  such that on this level set:

$$\left| \frac{\partial \Delta h_i}{\partial e_i} \right| \leq L_i. \quad (13)$$

Moreover, from Assumption 1 and the boundedness of the trajectories of the driving system, one concludes that:

$$\left| \frac{\partial \Delta h_i}{\partial x_i} \dot{x}_i \right| + \left| \frac{\partial \Delta h_i}{\partial t} \right| \leq F_i \quad (14)$$

Using the inequalities (13) and (14), we have:

$$\dot{V} \leq \sum_{i=1}^n -\alpha_i e_i^2 + |e_i| |\eta_i| - \frac{\eta_i^2}{\varepsilon_i} + L_i |\eta_i| |\dot{e}_i| + F_i |\eta_i|$$

From Young's inequality, we have:

$$\dot{V} \leq \sum_{i=1}^n \left[ (-\alpha_i + \lambda_i) e_i^2 + \left( -\frac{1}{\varepsilon_i} + \frac{1}{4\lambda_i} + \frac{1}{4\beta_i} \right) \eta_i^2 + L_i |\eta_i| |-\alpha_i e_i + \eta_i| + F_i^2 \beta_i \right] \quad (15)$$

where  $\beta_i$  and  $\lambda_i$  are arbitrary small positive values. After some straightforward manipulations, one has

$$\begin{aligned} \dot{V} \leq \sum_{i=1}^n & \left[ (-\alpha_i + \lambda_i + \alpha_i^2 \varpi_i) e_i^2 + \left( -\frac{1}{\varepsilon_i} + \frac{1}{4\lambda_i} + \frac{1}{4\beta_i} \right. \right. \\ & \left. \left. + \frac{1}{4\varpi_i} + L_i \right) \eta_i^2 + F_i^2 \beta_i \right] = \sum_{i=1}^n \left[ (-\alpha_i + \kappa_i) e_i^2 \right. \\ & \left. + \left( -\frac{1}{\varepsilon_i} + \bar{L}_i \right) \eta_i^2 + F_i^2 \beta_i \right] \end{aligned}$$

where  $\varpi_i$  is also an arbitrary small positive value. It should be noted that the parameters  $\lambda_i$  and  $\varpi_i$  have to be chosen such  $-\alpha_i + \lambda_i + \alpha_i^2 \varpi_i < 0$ . Moreover, for any  $0 < \varepsilon_i < \varepsilon_i^* = \frac{1}{\bar{L}_i}$ , we have  $\left( -\frac{1}{\varepsilon_i} + \bar{L}_i \right) < 0$ . Then

$$\dot{V} \leq -cV + \rho \quad (16)$$

where

$$c = \min \left\{ (-\alpha_i + \kappa_i), \left( -\frac{1}{\varepsilon_i} + \bar{L}_i \right), \quad i = 1, \dots, n \right\},$$

$$\rho = \sum_{i=1}^n F_i^2 \beta_i$$

From (16), we have

$$V(t) \leq V(t_0) e^{-c(t-t_0)} + \frac{\rho}{c}. \quad (17)$$

From (17), it can be shown the all closed-loop state variables  $e_i(t)$ ,  $\hat{e}_i(t)$ , and  $z_i(t)$ ,  $i=1, \dots, n$ , are semi-globally uniformly ultimately bounded. In order to achieve the error convergences to an arbitrary small neighbourhood around zero, the parameter  $\rho$  is sufficiently chosen small. To do this, the parameters  $\beta_i$ ,  $i=1, \dots, n$ , must be selected as small as desired.

*Remark 5:* According to inequality (15), in order to compensate the decreasing effect of  $\beta_i$ , the parameters  $\varepsilon_i$  has to be chosen appropriately.

*Remark 6:* The proposed controller in theorem 1 is simpler than the ones presented in [20] and [21]. Furthermore, the assumptions which are considered in this paper are less conservative.

### III. ILLUSTRATIVE EXAMPLE

Assume that delayed chaotic cellular NN (18) is the drive system [26] :

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = -C \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + A \begin{bmatrix} g_1(x_1(t)) \\ g_2(x_2(t)) \end{bmatrix} + B \begin{bmatrix} g_1(x_1(t-\tau_1)) \\ g_2(x_2(t-\tau_2)) \end{bmatrix} \quad (18)$$

where  $g_j(x_j(t)) = 5(|x_j(t)+1| - |x_j(t)-1|)$ ,  $j=1,2$  and the delayed chaotic Hopfield NN (16) is the response system [27] :

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = -D \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + P \begin{bmatrix} f_1(z_1(t)) \\ f_2(z_2(t)) \end{bmatrix} + Q \begin{bmatrix} f_1(z_1(t-\tau_3)) \\ f_2(z_2(t-\tau_4)) \end{bmatrix} + u(t) \quad (19)$$

where  $f_j(z_j(t)) = \tanh(z_j(t))$ ,  $j=1,2$ .

We also selected the following values for the systems (18) and (19) parameters:

$$C = \text{diag}[1,1], \quad D = \text{diag}[1,1]$$

$$A = \begin{bmatrix} 1+\pi/4 & 20 \\ 0.1 & 1+\pi/4 \end{bmatrix}, \quad B = \begin{bmatrix} -1.3\sqrt{2}\pi/4 & 0.1 \\ 0.1 & -1.3\sqrt{2}\pi/4 \end{bmatrix}$$

$$P = \begin{bmatrix} 2 & -0.1 \\ -5 & 4.5 \end{bmatrix}, Q = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -4 \end{bmatrix}$$

$$\tau_1 = \tau_2 = \tau_3 = \tau_4 = 1$$

$$[x_1(s), x_2(s)] = [0.01, 0.1]$$

$$[z_1(s), z_2(s)] = [0.4, 0.6] \quad \text{for } -1 \leq s \leq 0$$

The proposed controller is

$$u(t) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -e_1 + l_{e_1} \\ -e_2 + l_{e_2} \end{bmatrix}. \quad (20)$$

It should be noted that we suppose that is

$$\begin{aligned} \Delta h = & -D \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + P \begin{bmatrix} f_1(z_1(t)) \\ f_2(z_2(t)) \end{bmatrix} + Q \begin{bmatrix} f_1(z_1(t - \tau_3)) \\ f_2(z_2(t - \tau_4)) \end{bmatrix} \\ & + C \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} - A \begin{bmatrix} g_1(x_1(t)) \\ g_2(x_2(t)) \end{bmatrix} - B \begin{bmatrix} g_1(x_1(t - \tau_1)) \\ g_2(x_2(t - \tau_2)) \end{bmatrix} \end{aligned}$$

It is worth mentioning that all functions in both the drive and response systems are assumed unknown (worst case).

The chaotic behaviours of the drive system (18) and the response system (19) without the controller  $u(t)$  are shown in Figs. 1 and 2, respectively. The proposed control law is depicted in Fig. 3. The simulation results for the synchronization error are shown in Fig. 4. It is obvious that drive and response systems are synchronized and the synchronization errors converge to zero asymptotically.

Meanwhile, to compare the results with the one presented by Zhang et al. [21], their method is implemented to determine the adaptive controller. Figs. 5 and 6 illustrate the simulation results when the initial adaptive parameters choose properly. One can see that the control laws in Figs. 3 and 5 act in the same manner with small differences.

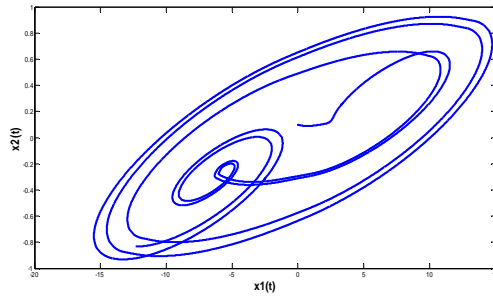


Fig. 1. Chaotic behaviour of the drive system

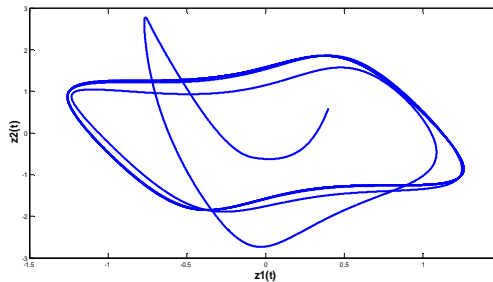


Fig. 2. Chaotic behaviour of the response system without the controller  $u(t)$ .

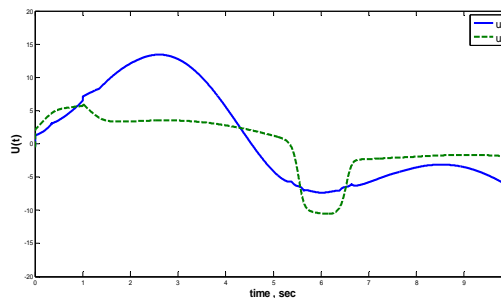


Fig. 3. The proposed control law trajectories.

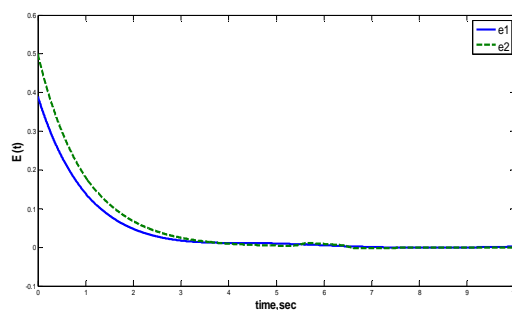


Fig. 4. Synchronization error with the proposed control law.

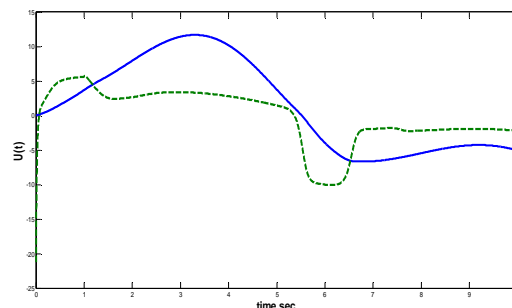


Fig. 5. The control law trajectories due to Zhang et al.'s controller.

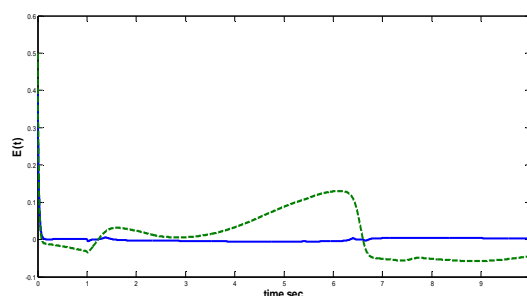


Fig. 6. Synchronization error due to Zhang et al.'s controller.

#### IV. CONCLUSION

This paper presented a novel synchronization scheme between two different kinds of delayed chaotic neural networks (NNs) with unknown parameters. The effects of uncertainty and unknown parameters in the synchronization error dynamic handled based on a time-scale separation technique. Using the Lyapunov theory, the stability of the closed-loop system was proved. Simulation results illustrated the effectiveness of the proposed method.

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