

Time-Space Noise

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ABSTRACT

In this paper, has been introduced the Time-Space noise. Since the noise dependence on both space and time must be considered in the calculation of this dependence that it is not in most cases. Therefore, in this paper has been introduced the time-space noise, obtained by some of the parameters and its application are described.

KEYWORDS: Time-Space Noise; Variance; Mean; Covariance; Spectrum.

1. INTRODUCTION

Time-Space processes are processes that output of a random process joins a waveform relative that is dependent time and place[1]. in these processes, the variable parameters are x (space) and t (time) that should be considered simultaneously.

This is important that review time-space processes, because for example in channel estimation some of the factors are simultaneously dependence time and space and its dependence should be considered in the calculations. In most cases, only one dependency (time or place) is considered and other parameters are assumed constant.

The noise is the kind of process that dependence synchronized time and space. Noise is the most influential factors on the information that is strong dependence on the environment (weather, transmission's route, etc.) and time information. Based on these two factors (time and place), can be variable the effects of noise on the signal.

In different sources has been mentioned dependence time and place of noise and Its applications have been studied in image and image processing [2]. Only in reference [3] has been introduced the Time-Space white noise and defined as the mean and variance.

None of the reviewed papers and references, the noise has not been studied process. In this paper, has been review the time-space noise process and Some parameters has been obtained such as covariance, correlation coefficient and the spectrum. In order, to better understand the time-space noise, has been introduced time-space process and Based on it defined the time-space noise and have been calculated desired parameters.

2. Time-space noise

As in reference [1], can be defined time-space process:

$$\{g(x, t) : x \in D \subset \mathfrak{R}^n, t \in [0, \infty)\}. \quad (1)$$

Where, g is the function depending on x and t, \mathfrak{R}^n N-dimensional space and t is the process time. Accordingly, the mean and covariance of time-space random process order is defined as follows [4]:

$$\mu(x, t) \triangleq E(g(x, t)). \quad (2)$$

$$K(x_1, x_2, t_1, t_2) \triangleq Cov(g(x_1, t_1), g(x_2, t_2)) \quad (3)$$

Where, expected value is defined the following for time-space process:

$$E(g(x, t)) = \iint_{x, t} g(x_i, t_i) f(x_i, t_i) dx_i dt_i. \quad (4)$$

Where, $f(x_i, t_i)$ is the probability density function at point (x_i, t_i) . Using the above definition and the definition of noise, time-space noise is defined as follows:

$$W(x, t) \sim N(\mu(x, t), \sigma(x, t)). \quad (5)$$

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Where, Noise has considered a normal distribution with mean $\mu(x, t)$ and variance $\sigma(x, t)$, that Mean and Variance of noise are related to time and place.

In Figure (1) is shown the time-space noise. Unlike simple noise (time or place) that is shown in Figure (2), This is a three-dimensional noise.

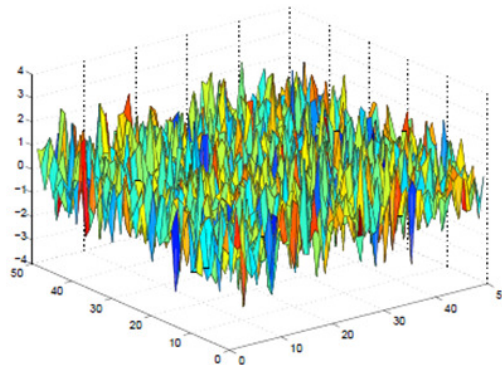


Figure 1. Time-Space Noise

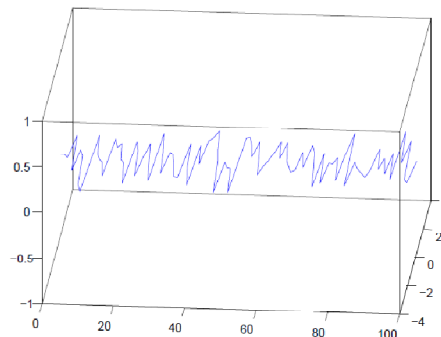


Figure 2. Sample Noise

Now, Been used a defined time-space noise, are obtained the desired values. $R(W(x_1, t_1), W(x_2, t_2))$ Shown the correlation between two functions $W(x_1, t_1)$ and $W(x_2, t_2)$, and is defined as follows:

$$\begin{aligned}
 &R(W(x_1, t_1), W(x_2, t_2)) \\
 &= E\{W(x_1, t_1), W^*(x_2, t_2)\} \\
 &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[W(x_1, t_1) W^*(x_2, t_2) \cdot f(x_1, t_1) f(x_2, t_2) dx_1 dt_1 dx_2 dt_2 \right].
 \end{aligned}
 \tag{6}$$

For the time-space noise can be considered correlation function as:

$$\begin{aligned}
 &R(W(x_1, t_1), W(x_2, t_2)) \\
 &= \begin{cases} \alpha & x_1 = x_2, t_1 = t_2 \\ 0 & else \end{cases}.
 \end{aligned}
 \tag{7}$$

This means that only one point has a correlation function's value, and Value (α) is considered as the auto-correlation coefficient. Because of this correlation function takes the value of a point that different parts of the time-space noise have not dependence and each point will be associated only with itself. Using this definition, time-space noise covariance is defined as:

$$\begin{aligned}
 & Cov(W(x_1, t_1), W(x_2, t_2)) \\
 &= \left\{ E(W(x_1, t_1)W(x_2, t_2)) - \right. \\
 & \left. E(W(x_1, t_1)) \cdot E(W(x_2, t_2)) \right\} \\
 &= \{ E(W(x_1, t_1)W(x_2, t_2)) - \mu(x_1, t_1) \cdot \mu(x_2, t_2) \} \\
 &= \{ R(W(x_1, t_1), W(x_2, t_2)) - \mu(x_1, t_1) \cdot \mu(x_2, t_2) \} \\
 &= \alpha - (\mu(x_1, t_1))^2.
 \end{aligned} \tag{8}$$

To obtain the spectral density function should be fixed that time-space noise is the Wide-Sense Stationary (WSS). So will had:

1- the statistically average is independent of time[1]. So:

$$E(W(x_1, t_1)) = \mu(x_1, t_1). \tag{9}$$

In general, the noise considered White time-space noise. In this case, the different points have not affiliation (Unlike the color noise that extend to the points are dependent on the others). Noise's mean is zero. So:

$$E(W(x_1, t_1)) = 0. \tag{10}$$

That shown the statistically average is independent of time.

2- The auto-correlation function is a function of the difference time(t) [2]. So:

$$\begin{aligned}
 & R(W(x_1, t_1), W(x_1, t_2)) \\
 &= R(W(x_1, t_1 - t_2)) \\
 &= R(W(x_1, \tau)).
 \end{aligned} \tag{11}$$

White time-space noise is related to the following:

$$\begin{aligned}
 & C(W(x_1, t_1), W(x_1, t_2)) \\
 &= q(x_1, t_1) \cdot \delta(x_1, t_1 - t_2).
 \end{aligned} \tag{12}$$

Since white time-space noise has zero mean, Auto-correlation function is equal to the covariance function. So:

$$\begin{aligned}
 & R(W(x_1, t_1), W(x_1, t_2)) \\
 &= C(W(x_1, t_1), W(x_1, t_2)) \\
 &= q(x_1, t_1) \cdot \delta(x_1, t_1 - t_2).
 \end{aligned} \tag{13}$$

Which indicates that Auto-correlation function is as a function of the difference t. Because time-space noise is a WSS (Unlike the time-space process that are the time dependent and why not WSS), can be defined the spectral density as follows:

$$\begin{aligned}
 S_{x_j}(\omega) &= FT \{ R(W(x, t_2 - t_1)) \} \\
 &= FT \{ R(W(x, \tau)) \} \\
 &= \int_{-\infty}^{\infty} R(W(x, \tau)) e^{-j\omega\tau} d\tau.
 \end{aligned} \tag{14}$$

From equation (13) and Placement in equation (14) can be obtained that:

$$\begin{aligned}
S_{x,t}(\omega) &= \int_{-\infty}^{\infty} R(W(x,\tau))e^{-j\omega\tau}d\tau \\
&= \int_{-\infty}^{\infty} q(x_1,t_1)\cdot\delta(x_1,t_1-t_2)e^{-j\omega\tau}d\tau.
\end{aligned} \tag{15}$$

Because $q(x_1,t_1)$ is not dependent to τ , So:

$$S_{x,t}(\omega) = q(x_1,t_1) \cdot \int_{-\infty}^{\infty} \delta(x_1,t_1-t_2)e^{-j\omega\tau}d\tau. \tag{16}$$

$$t_1 - t_2 = \tau \Rightarrow$$

$$\begin{aligned}
S_{x,t}(\omega) &= q(x_1,t_1) \cdot \int_{-\infty}^{\infty} \delta(x_1,\tau)e^{-j\omega\tau}d\tau.
\end{aligned} \tag{17}$$

Fourier transform of the impulse function is obtained that:

$$I = \int_{-\infty}^{\infty} \delta(x_1,\tau)e^{-j\omega\tau}d\tau = 1. \tag{18}$$

So:

$$\begin{aligned}
S_{x,t}(\omega) &= q(x_1,t_1) \cdot \int_{-\infty}^{\infty} \delta(x_1,\tau)e^{-j\omega\tau}d\tau \\
&\Rightarrow S_{x,t}(\omega) = q(x_1,t_1).
\end{aligned} \tag{19}$$

That is consistent with the noise (time or place) was obtained. Including those that used to have of the time-space processes, can be pointed out Speech processing [5], Optical communications [6], Biomedical engineering and medicine [7], Array antenna [8] and MIMO communication channel and its estimate [9-10]. Because most of the data to have a strong dependence on the time and place, to obtain their function is important, Especially in references [9-10] of this function is used for safe transport and low error.

3. CONCLUSION

In this paper was introduces time-space noise and was calculated Parameters such as correlation, covariance and the spectrum.

Unlike the other time - space processes, time-space noise is the WSS. So for this reason was calculated the spectral density function. The applications of time-space noise is the estimated channel in MIMO communication. In this communication, noise can be variable with ambient and time, So Can have different effects on sent the signal. Therefore, the noise should be considered as a variable of time and place.

As was shown, time-space noise and sample noise (time or place) are the same and have the same characteristics and the only difference is the dependence of the time and place.

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