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Modification of Schrödinger Equation in a media

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ABSTRACT

In this work Schrödinger quantum equation is modified by considering the electromagnetic waves propagating in a media instead of free space. Thus the equation can account for the prosperities of the medium and the absorption beside exchange of energy between the particles flux and the media. This modified equation is shown to explain inelastic scattering process better than the ordinary Schrödinger equation.

KEYWORDS: Schrödinger Equation, electromagnetic waves, absorption.

1. INTRODUCTION

Schrödinger quantum mechanical equation (SE) is shown to be successful in explaining many atomic phenomena [1]. It can explain the spectra of simple atoms, elastic scattering process beside explaining partially behavior of elementary particles in general [2,3]. Despite these successes SE suffers from noticeable setbacus. For instance it can not explain the interaction of elementary particles like quarks where it produces severe anomalies and divergences. It is also unable to explain the inelastic scattering process completely. The inelastic scattering process is explained by an optical potential which is inserted by hand in SE

In this paper can attempt was made to construct a model based on the electromagnetic wave equation in a medium instead of in free space as in SE.

The foundation of this new quantum mechanical equation requires finding the energy exchange relation between the medium and electromagnetic (E.M) waves and this is done in section (2.2). Section (2.3) is devoted to determine the expression of the wave number or the momentum in the medium.

The new quantum mechanical equation based on the E.M wave in a medium is exhibited in section (2.4)

2. The energy exchange relation

Where the electromagnetic field passed through a medium it exchanges energy and momentum with the medium .This cases the wave length , frequency and amplitude to change .In this chapter one tries to see how the electromagnetic field respond the properties of the medium. To see how the medium change the frequency and the energy of electromagnetic waves, we can write the wave [4,5,6]

$$E = E_0 e^{i(kx-\omega t)} \tag{1}$$

And the wave equation [7]

$$-\nabla^2 E + \mu\sigma \frac{\partial E}{\partial t} + \mu \varepsilon_T \frac{\partial^2 E}{\partial t^2} = 0 \tag{2}$$

Where $\varepsilon_T = (\varepsilon + \chi)$, the solution of this equation is:

$$\frac{\partial^2 E}{\partial x^2} = -k^2 E \quad , \quad \frac{\partial E}{\partial x} = ikE \quad , \quad \frac{\partial E}{\partial t} = -i\omega E \quad , \quad \frac{\partial^2 E}{\partial t^2} = -\omega^2 E \tag{3}$$

Substitute equations (3) in equation (2) to get

$$\mu \varepsilon_T \omega^2 + i\mu\sigma\omega - k^2 = 0 \tag{4}$$

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Using the relation, $x=\frac{-b\pm\sqrt{b^{2}-4ca}}{2a}$, with $x=\omega$ $a=\mu\varepsilon_{T}$ $b=i\mu\sigma$ $c=-k^{2}$ one gets

$$\omega=\frac{-i\mu\sigma\pm\sqrt{(i\mu\sigma)^{2}+4k^{2}\mu\varepsilon_{\mathrm{T}}}}{2\mu\varepsilon_{\mathrm{T}}} \tag{5}$$

For very small μ hence

$$\omega=\frac{-i\mu\sigma}{2\mu\varepsilon_{\mathrm{T}}}\pm\frac{2\sqrt{\mu\varepsilon_{\mathrm{T}}}}{2\mu\varepsilon_{\mathrm{T}}}k=\frac{-i\sigma}{2\varepsilon_{\mathrm{T}}}\pm\frac{k}{\sqrt{\mu\varepsilon_{\mathrm{T}}}} \tag{6}$$

This relation can be simplified further if one considers

$$\frac{1}{\sqrt{\mu\varepsilon_{\mathrm{T}}}}=\frac{1}{\sqrt{\mu(\varepsilon+\chi)}}=\frac{1}{\sqrt{\mu\varepsilon\left(1+\frac{\chi}{\varepsilon}\right)}}\qquad\rightarrow\qquad\frac{1}{\sqrt{\mu\varepsilon_{\mathrm{T}}}}=\frac{1}{\sqrt{\mu\varepsilon}\sqrt{1+\frac{\chi}{\varepsilon}}}$$

With $v=\frac{1}{\sqrt{\mu\varepsilon}}$ then

$$\frac{1}{\sqrt{\mu\varepsilon_{\mathrm{T}}}}\approx v\left(1+\frac{\chi}{\varepsilon}\right)^{\frac{-1}{2}} \tag{7}$$

Since $\varepsilon=1+4\pi\chi$ and $\varepsilon>>\chi$ then $\frac{1}{\sqrt{\mu\varepsilon_{\mathrm{T}}}}\approx v\left(1-\frac{1}{2}\frac{\chi}{\varepsilon}\right)$ as a result are gets

$$\begin{aligned}&\frac{k}{\sqrt{\mu\varepsilon_{\mathrm{T}}}}\approx vk\left(1-\frac{1}{2}\frac{\chi}{\varepsilon}\right)\approx\omega_{0}\left(1-\frac{\chi}{2\varepsilon}\right)\\&vk=f_{0}\lambda\left(\frac{2\pi}{\lambda}\right)=\omega_{0}\end{aligned} \tag{8}$$

When no matter exists $\sigma=0$, $\chi=0$, $\varepsilon_{\mathrm{T}}=\varepsilon$, then the equation (8) become

$$\omega=\frac{k}{\sqrt{\mu\varepsilon}}=vk=\omega_{0} \tag{9}$$

Thus from (9) , (8) and (6)

$$\omega_{c}=\frac{-i\sigma}{2\varepsilon_{t}}\pm\frac{k}{\sqrt{\mu\varepsilon_{T}}}=\frac{-i\sigma}{2\varepsilon_{T}}\pm\omega_{0}\left(1-\frac{\chi_{c}}{2\varepsilon}\right)=\frac{-i\sigma}{2\varepsilon_{T}}-\omega_{0}c_{1} \tag{10}$$

Where $c_{1}=\pm\left(1-\frac{\chi_{0}}{2\varepsilon}\right)$ Since $\varepsilon>>\chi$ hence $\varepsilon_{T}\approx\varepsilon$ then $\omega_{c}=\omega_{0}-\frac{\omega_{0}\chi}{2\varepsilon}-\frac{i\sigma}{\varepsilon}$. Using the relations $\chi_{c}=\chi_{1}+i\chi_{2}$, $\omega_{0}\chi_{2}=\sigma_{1}=\sigma$, $\chi_{1}=\chi$ in equation (1) yield

$$E=E_{0}e^{-\frac{\sigma}{\varepsilon}t}e^{i(kx-\omega t)} \tag{11}$$

Where $\beta=\frac{\sigma}{\varepsilon}$, $\omega=\omega_{0}+\frac{\omega_{0}\chi}{2\varepsilon}$ (12)

In view of equation (11) it is apparent that the conductivity and electric permittivity affect the amplitude and frequency of the electromagnetic wave.

3. The wave number relation

The medium does not affect the frequency of frequency electromagnetic wave only, but may also affect the wave number k. This can be done by relating k to the refractive index of the medium n and then relating n to electric permittivity ε in susceptibility χ . Utilizing the definition of k one gets [6]

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{\omega}{v} = \frac{\omega}{c} n \tag{13}$$

Where the refractive index n is given as the ratio of the speed of light in free space c and the speed of light in the medium v,

$$n = \frac{c}{v} \tag{14}$$

While the term v is given by:

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \tag{15}$$

On the other hand the electric susceptibility is defined to be

$$\mathrm{P} = \chi E = (\chi_1 + i\,\chi_2)E \tag{16}$$

Where P represents the polarization while E stands for the electric field intensity. The electric flux density is also given by [8]

$$\begin{aligned} D &= \varepsilon E = E + 4\pi \mathrm{P} = E + 4\pi\chi E \\ D &= (1 + 4\pi\chi)E \end{aligned} \tag{17}$$

Thus the electric permittivity is given by

$$\varepsilon = 1 + 4\pi\chi \tag{18}$$

If it happens that the polarization P is not parallel to E , then it is better to decompose P to two components , $\chi_1 E$ which is parallel to E , and imaginary part $\chi_2 E$ which is perpendicular to E i.e. [9]

$$\mathrm{P} = \chi E = (\chi_1 + i\,\chi_2)E$$

As the result the permittivity ε can be written also in a complex form;[10]

$$\varepsilon = \varepsilon_1 + i\,\varepsilon_2 \tag{19}$$

Thus $\varepsilon_1 + i\,\varepsilon_2 = 1 + 4\pi\chi_1 + 4\pi\chi_2 i$ Then

$$\varepsilon_1 = 1 + 4\pi\chi_1 \quad \text{and} \quad \varepsilon_2 = 4\pi\chi_2 \tag{20}$$

In view of relations (4-13), (4-15) and (4-20), it is clear that n also is a complex parameter, i.e.

$$n = n_1 + in_2 = \frac{c}{v} = c\sqrt{\mu\varepsilon} \tag{21}$$

Squaring both sides yield $n^2 = (n_1 + in_2)^2 = c^2\mu\varepsilon = c^2\mu(\varepsilon_1 + i\,\varepsilon_2)$

$$n_1^2 - n_2^2 + 2n_1 n_2 = c^2\mu\varepsilon_1 + ic^2\mu\varepsilon_2 \tag{22}$$

From equation (22) we get

$$n_1^2 - n_2^2 = c^2\mu\varepsilon_1 \quad , \quad n_1 n_2 = c^2\mu\varepsilon_2 \tag{23}$$

Eliminating n_2 from the two relations we obtain:

$$n_1^2 - \left(\frac{c^2\mu\varepsilon_2}{2n_1}\right)^2 = c^2\mu\varepsilon_1 \rightarrow n_1^2 - \frac{\mu^2\varepsilon_2^2 c^4}{4n_1^2} = c^2\mu\varepsilon_1 \rightarrow$$

$$4n_1^4 - 4c^2\mu\varepsilon_1 n_1^2 - \mu^2\varepsilon_2^2 c^4 = 0 \tag{24}$$

Utilizing the identity , $x = \dfrac{-b \pm \sqrt{b^2 - 4ca}}{2a}$, with $x = n_1^2$, $a = 4$, $b = -4c^2\mu\varepsilon_1$, $c = -\mu^2\varepsilon_2^2 c^4$ yields

$$n_1^2 = \frac{4c^2\mu\varepsilon_1 \pm \sqrt{16c^4\mu^2\varepsilon_1^2 + 16c^4\mu^2\varepsilon_2^2}}{8} \rightarrow n_1^2 = \frac{\mu c}{2}\left[\varepsilon_1 \pm \sqrt{\varepsilon_1^2 + \varepsilon_2^2}\right] \tag{25}$$

Hence relations (25) and (23) yield

$$\begin{aligned} n_2^2 &= n_1^2 - c^2\mu\varepsilon_1 \\ &= \frac{\mu c^2\varepsilon_1}{2} \pm \frac{\mu c^2}{2}\sqrt{\varepsilon_1^2 + \varepsilon_2^2} - \mu c^2\varepsilon_1 \\ &= \frac{1}{2}\mu c^2\left[-\varepsilon_1 \pm \sqrt{\varepsilon_1^2 + \varepsilon_2^2}\right] \end{aligned} \tag{5.26}$$

Thus the wave number k can be given from (22) and (13) to be

$$\begin{aligned} k &= \frac{\omega}{c} n = \frac{\omega}{c}(n_1 + in_2) \\ k &= k_1 + ik_2 = \frac{\omega}{c} n_1 + i\frac{\omega}{c} n_2 \end{aligned} \tag{27}$$

From equation (5.27) we get

$$k_1 = \frac{\omega}{c} n_1 \quad \text{and} \quad k_2 = \frac{\omega}{c} n_2 \tag{28}$$

But $k_0 = \frac{\omega}{c}$ (29)

$$k_1 = k_0 n_1 \tag{30}$$

$$n_1 = \frac{c}{v} \tag{31}$$

Then $k_1 = \frac{\omega}{c} \times \frac{c}{v} = \frac{\omega}{v} = \frac{2\pi f}{\lambda f} = \frac{2\pi}{\lambda}$ (32)

where, k_0 is the wave number in vacuum while k_1 stands for the wave number in the medium. Using relation (27), (31) in equation (2) we obtain

$$E = E_0 e^{i((k_1 + ik_2)x - \omega t)} \rightarrow E = E_0 e^{-k_2 x} e^{i(k_1 x - \omega t)} \tag{33}$$

In view of equation (28) let

$$\alpha = k_2 = \frac{\omega}{c} n_2 \tag{34}$$

where n_2 is given from equation (23) to be $n_2 = \frac{c^2\mu\varepsilon_2}{n_1}$, where $\varepsilon_2 = \frac{4\pi\sigma_1}{\omega}$

Utilizing equation (20) one find

$$n_2 = \frac{c^2\mu}{2n_1}(4\pi\chi_2) \tag{35}$$

But since $J = \frac{\partial P}{\partial t} = \chi\frac{\partial}{\partial t}E_0 e^{-i\omega t}$

$$J = -i\omega(\chi_1 - i\chi_2)E = (\omega\chi_2 - i\omega\chi_1)E = \sigma E = (\sigma_1 + i\sigma_2)E \tag{36}$$

Hence $\chi_2 = \frac{\sigma_1}{\omega}$ (37)

This equation (34) can be written with mid of (35) in the form

$$\alpha = k_2 = \frac{\omega}{c} n_2 = \frac{2\pi c\mu\chi_2\omega}{n_1} = \frac{2\pi\mu\chi_2\sigma_1}{n_1} \tag{38}$$

In view of equation (28), (37) and (38) one can decide that the wave number and electromagnetic amplitude are affected by the refractive index n_1 , magnetic susceptibility μ as well as conductivity σ_1 and relaxation time τ .

4. Modified Schrödinger equation

The ordinary Schrödinger equation describes as; [11,12]

$$\psi = Ae^{i(kx-\omega t)} \tag{39}$$

In a free space when it enter vacuum in which the field distribute it self through the expression [13,14]

$$E = \frac{P^2}{2m} + V \tag{40}$$

The above expressions form which the ordinary Schrödinger equation are derived from any term which feel the existence of matter .This defect can be cured if are takes in to account the expression for the wave which is propagated a medium, where the effect of the medium manifests it self through the change in and amplitude energy, momentum through, the refractive index, susceptibility and conductivity. Thus expression

$$\psi = e^{-\beta t} e^{-\alpha x} e^{\frac{i}{\hbar}(Px - Et)} \tag{41}$$

Is more convenient to describe the effect derived of the medium on the atomic entities (particles) than equation (39) . [15]

$$-\frac{\hbar}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} \tag{42}$$

Free electron $V = 0$, then equation (42) becomes [16,17]

$$-\frac{\hbar}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t} \tag{43}$$

Where in view of equations (12) , (30) and (37) one gets

$$E = \hbar\omega = \hbar\omega_0 - \frac{\hbar\omega_0\chi}{2\varepsilon} = E_0 - \frac{\chi}{2\varepsilon}E_0 = c_1 E_0 \tag{44}$$

Where $c_1 = \left(1 - \frac{\chi}{2\varepsilon}\right) = 1 - c_0$ and $c_0 = \frac{\chi}{2\varepsilon}$, $P = i\hbar k_1 = \hbar k_0 n_1 = P_0 n_1$

From equation (12) and equation(38) $\beta = \frac{\sigma}{\varepsilon}$, $\alpha = k_2 = \frac{2\pi\mu c}{n_1}\sigma$

Differentiating equation (41) yields

$$i\hbar\frac{\partial\psi}{\partial t} = (-i\hbar\beta + E)\psi = E_0\psi - \left(i\hbar\beta + \frac{\chi}{2\varepsilon}E_0\right)\psi = (-i\hbar\beta + c_1 E_0)\psi \quad) \tag{45}$$

$$\frac{\hbar}{i}\frac{\partial\psi}{\partial x} = (i\hbar\alpha + P)\psi = (i\alpha + n_1 P_0)\psi = (-i\hbar\alpha + n_1 P_0)\psi$$

$$-\hbar^2\frac{\partial^2\psi}{\partial x^2} = (i\hbar\alpha + P)^2\psi \tag{46}$$

Multiplying both sides of equation (40) after replace P by P_0 and E by E_0 one gets $E_0\psi = \frac{P_0^2}{2m}\psi + V\psi$ .Using equation (45) yields

$$\begin{aligned} \frac{i\hbar}{c_1}\frac{\partial\psi}{\partial t} + \frac{i\hbar\beta}{c_1}\psi &= \frac{1}{2m}\left(\frac{\hbar}{n_1 i}\frac{\partial}{\partial x} - \frac{i\alpha\hbar}{n_1}\right)^2\psi + V\psi \\ &= -\frac{\hbar^2}{2mn_1}\frac{\partial^2\psi}{\partial x^2} - \frac{\alpha\hbar}{2mn_1^2}\frac{\partial\psi}{\partial x} - \frac{\alpha^2\hbar^2}{n_1^2}\psi \end{aligned} \tag{47}$$

This equation represents the generalized Schrödinger equation (G.S.E) , which feel the existence of the medium via the refractive index n_1 and the terms β,α and c_0 which are dependent on the electrical and magnetic properties of the medium. It is un like ordinary Schrödinger equation (S.E)which does not feel the existence of the medium but feels the effect of the potential .The (S.E) this is non realistic for it gives the same result for particles in a certain field divided of any medium , and particles in the same field but inside a certain medium . This is in conflict with experiment where the behavior of the particles in a field only is different from behavior in a field inside matter at the same times.

5. Solution of generalized Schrödinger equation in free field space

G.S.E solution for a particle in a certain medium can be simplified by considering the field to be very weak inside the medium i.e. by sitting

$$V = 0 \qquad (48)$$

In this case the G.S.E (5.45) reduce to

$$i\hbar\frac{\partial\psi}{\partial t} + i\hbar\beta\psi = -\frac{\hbar^2 c_1}{2mn_1^2}\nabla^2\psi - \frac{\hbar^2\alpha c_1}{mn_1^2}\nabla\psi - \frac{\hbar^2\alpha^2}{2mn_1^2}c_1\psi \qquad (49)$$

Consider now solution in form

$$\psi = Ae^{\frac{i}{\hbar}(Px-Et)} \qquad (50)$$

Therefore $\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi \quad , \frac{\partial\psi}{\partial x} = \frac{i}{\hbar}P\psi \ , \frac{\partial^2\psi}{\partial x^2} = -\frac{P^2}{\hbar^2}\psi$ (51)

Utilizing equation (51) in equation (49) yields

$$i\hbar\left(-\frac{i}{\hbar}E\psi\right) + i\hbar\beta\psi = \frac{-\hbar^2 c_1}{2mn_1^2}\left(-\frac{P^2}{\hbar^2}\psi\right) - \frac{\hbar^2\alpha c_1}{mn_1^2}\left(\frac{i}{\hbar}P\right)\psi - \frac{\hbar^2\alpha^2 c_1}{2mn_1^2}\psi$$

$$E + i\hbar\beta = \frac{\hbar^2 c_1}{2mn_0^2}\frac{P^2}{\hbar^2} - \frac{\hbar\alpha c_1}{mn_1^2}Pi - \frac{\hbar^2\alpha^2 c_1}{2mn_1^2} = \frac{c_1}{2mn_0^2}P^2 - \frac{\hbar\alpha c_1}{mn_1^2}Pi - \frac{\hbar^2\alpha^2 c_1}{2mn_1^2} \qquad \mathbf{(52)}$$

Equaling real and imaginary parts yields

$$\hbar\beta = \frac{\hbar\alpha c_1}{mn_1^2}P \quad , \quad P = \frac{mn_1^2\beta}{\alpha c_1} \quad (imajenary) \qquad (53)$$

$$E = \frac{c_1 P^2}{2mn_1^2} - \frac{\hbar^2\alpha^2 c_1}{2mn_1^2} \quad (real) \qquad (54)$$

Rearranging to get the expression for P yields $p^2 = \frac{2mn_1^2}{c_1}\left[E + \frac{\hbar^2\alpha^2}{2mn_1^2}c_1\right]$

Taking now the square root of both sides the momentum becomes

$$P = \pm\sqrt{\frac{2mn_1^2}{c_1}E + \hbar^2\alpha^2} \qquad (55)$$

According to this expression the momentum may be negative or positive, real or imaginary .It is quite a obvious that the imaginary value of P is forbidden. This happens when $\frac{2mn_1^2}{c_1}E + \hbar^2\alpha^2 < 0 \ \rightarrow$ $\frac{E}{c_1} < -\frac{\hbar^2\alpha^2}{2mn_1^2}$ but according to equation (5.10.b) $c_1 = \pm|c_1| \quad where \quad |c_1| = 1 - \frac{\chi}{2\varepsilon} = 1 - c_0$ since $\varepsilon = 1 + 4\pi\chi$, when $\chi << 1$as usually observed in many materials thus $\varepsilon > \chi$ and $|c_0| > 0$ if then one takes $c_1 = -|c_1|$ in this case $\frac{E}{-|c_1|} < -\frac{\hbar^2\alpha^2}{2mn_1^2}$ Hence $\alpha = \frac{2\pi\mu c\sigma}{n_1}$

$$-E < -\frac{h^2}{4\pi^2}\left(4\pi^2\right)\frac{\mu^2c^2}{2mn_1^4}\sigma^2|c_1| \qquad (56)$$

Thus the energy ranges $E > \frac{h^2}{2mn_1^4}|c_1|\sigma^2$ are forbidden. While the energy ranges $E \le \frac{h^2}{2mn_1^4}|c_0|\sigma^2$ are allowed. Since one consider the potential vanishes, i.e. V=0 , hence the electron is free i.e. $E \ge 0$ .This means that the energy range of the conduction band to be

$$0 \le E \le \frac{h^2}{2mn_1^4}|c_1|\sigma^2 \qquad (57)$$

As a result a pure insulator where $\sigma = 0$ there is no conduction band since $0 \le E \le 0$.

6. RESULTS AND DISCUSSION

Unlike Schrödinger equation, there are additional terms which describe the exchange of energy and momentum between the particle and the medium other than the potential term.

Equation (44) shows that the second terms in this equation reflects the absorption or gain of the energy of the medium. Presence of χ and ε reflects the effects of the electric polarization field and the electric force of the bulk matter on the particle energy.

The effect of the relaxation time via σ as well as the number of free electrons, which at the same time represents the number of ionized atoms in the matter, as well as the electrical resistance which manifests it self through the term ε on energy absorption is apparent via equation (45) . The numbers of ions as well as the relaxation time τ beside magnetic resistance contribute to momentum losses as equation (46) reads

6.10 Conclusion

The above discussion shows that the GSE may certainly behave better than SE, since its expression for energy and momentum is sensitive and feels the effect of mechanical magnetic and electrical resistance. It's also sensitive to the number of atoms and electrons on the bulk matter.

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