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Pareto Optimal Decoupled Sliding Mode Controller Design for a Cart-Pendulum System

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ABSTRACT

In this work, a multi-objective optimization is used to design Pareto optimal decoupled sliding mode controllers for a cart-pendulum system with external disturbances. Three conflicting objective functions which have been used in Pareto design of the sliding mode controller are, namely, the integral of the absolute magnitude of cart position, and pendulum angle with the total energy consumption. The simulation results clearly show that an effective trade-off can be compromisingly achieved among the different sliding mode controllers obtained using the methodology of this work.
KEYWORDS: Pareto, Sliding mode controller, Decoupled, Cart-pendulum system

1. INTRODUCTION

A variable structure control which is called sliding mode controller has been used in many complex nonlinear problems in recent years. The sliding mode controllers are less sensitive with respect to external disturbances and parametric uncertainties; therefore, they are appropriate for many real applications [1]. The main part of the sliding mode controller is the sliding surface. This part causes the control structure to be varied. As a result, to choose appropriate sliding surface is the first step of the sliding mode controller design. It must satisfy many control characteristics, especially, stability. When the states of the systems lie on the sliding surface; the system acts as a linear and reduced order one. Providing that there are no uncertainties and disturbances, the system can reach the equilibrium point or desired trajectory smoothly; otherwise, in order to overcome the disturbances and uncertainties, the structure of the controller is switched from one structure to another. It should be noted that the switching should be accomplished very fast; however, it is often impossible, because of the switching delay computation and the limitation of physical actuators which cannot handle the switching of control signal at an infinite time. Therefore, the high-rate switching mechanism is unreachable, and it results in an undesirable phenomenon which is called chattering [2].

In order to reduce the chattering phenomenon, instead of keeping the sliding surface trajectory at the zero, a boundary layer thickness can be chosen. Although it leads to reducing the chattering, the performance of the controller reduces too, and the steady sate error occurs [3]. In order to increase the performance of the sliding mode controller with boundary layer, the parameters of sliding mode controller should be design properly.

In this paper, a multi-objective Pareto genetic algorithm is used to obtain Pareto frontiers of various non-commensurable objective functions in the design of decoupled sliding mode controllers for a cart-pendulum system. The obtained results demonstrate that compromise can be readily accomplished using graphical representations of the achieved trade-offs among the conflicting objectives.

Decoupled Sliding Mode Controller Design

The Decouple Sliding-Mode Control (DSMC) was first introduced by [4], which is used to control the under actuated systems. In this approach a fourth-order nonlinear system can be considered as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= f_1(\mathbf{x}) + b_1(\mathbf{x})u + d_1(t), \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= f_2(\mathbf{x}) + b_2(\mathbf{x})u + d_2(t), \end{aligned} \tag{1}$$

where, $\mathbf{x} = [x, \dot{x}, ..., x^{(n-1)}]^T = [x_1, x_2, ..., x_n]^T \in R^n$ is the state vector of the system, $f_1(.)$, $f_1(.)$ and $b_1(.)$, $b_2(.)$ are nonlinear functions $d_1(t)$ and $d_1(t)$ are the external disturbances which are the bounded ones ($|d_1(t)| < D_1, |d_2(t)| < D_2$), and u is the input the system. In the under-actuated system, such as a cart-pendulum system, the control signal u only acts on a cart; therefore, it is possible to control either a cart or a pendulum. In order to overcome this problem, DSMC method proposed intermediate variable z in sliding surface equation [5]. The sliding surface with the intermediate variable can be written as

$$s_1 = c_1(x_1 - z) + x_2, \tag{2}$$

and another sliding surface as

$$s_2 = c_2 x_3 + x_4. \tag{3}$$

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The aim of the control problem is to design the controller signal which causes the states of the system reach their original equilibrium. Also, the control signal drives both s_1 and s_2 to zeros gradually at the same time by an intermediate variable z. A sufficient condition which implies this condition is to choose the Lyapunov function such that

$$\frac{1}{2}\frac{d}{dt}(s_1)^2 \le -\eta|s_1|,\ \ \eta > 0\,. \tag{4}$$

In order to satisfy the sliding condition, the control input u must be defined as [6]

$$u = \frac{1}{b_1}\left(-c_1x_1 + c_1\dot{z} - f_1\right) - Ksat\left(s_1/\phi_1\right), \tag{5}$$

where $sat(.)$ is the saturation function which reduces the chattering phenoma, and ϕ_1 is the boundary layer thickness, and K is the control gain which should be chosen large enough to guarantee the condtion (4). The control gain can be written as follow

$$K = \frac{D_1}{|b_1|} + \eta\,. \tag{6}$$

In equation (2), the intermediate variable, z, has a value proportional to s_2. Therefore, the control objective in DSMC is to change the conditions x_1=0, x_2=0 to x_1=z, x_2=0.

The intermediate variable is selected as a decaying oscillation signal; therefore, the whole system is driven to the equilibrium points. In order to prepare this condition, z is written as follow

$$z = Z_u sat\left(\frac{s_2}{\phi_z}\right),\ \ 0 < Z_u < 1, \tag{7}$$

where Z_u is the upper bound of $|z|$. This parameter causes the maximum absolute value of x_1 to be limited [6].

The performance of the sliding mode controller is very sensitive to these parameters. Therefore, it is absolutely imperative to find optimal value of each parameter. In this work, multi-objective optimization is used to design optimal DSMC using genetic algorithms.

Optimal DSMC Design for a Cart-Pendulum System

Consider a cart-pendulum system which is shown in Figure 1. If the state vector of the cart-pendulum system is chosen by $\mathbf{x} == [x_1, x_2, x_3, x_4] = [\theta, \dot{\theta}, x, \dot{x}]$, its dynamic equations can be described as

$$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= \frac{m_t g \sin x_1 - m_p L x_2^2 \sin x_1 \cos x_1}{L\left(\frac{4}{3}m_t - m_p \cos^2 x_1\right)}\\
&\quad + \frac{\cos x_1}{L\left(\frac{4}{3}m_t - m_p \cos^2 x_1\right)}u + d,\\
\dot{x}_3 &= x_4,\\
\dot{x}_4 &= \frac{\frac{4}{3}m_p L x_2^2 \sin x_1 + m_p g \sin x_1 \cos x_1}{\frac{4}{3}m_t - m_p \cos^2 x_1}\\
&\quad + \frac{4}{3\left(\frac{4}{3}m_t - m_p \cos^2 x_1\right)}u + d,
\end{aligned} \tag{8}$$

where m_p, m_c, and L are the mass of the pendulum, the mass of the cart, and the half-length of the pendulum, respectively, and m_t=m_p+m_c. In the simulation, the physical parameters are used as

$$\begin{aligned}
&m_p = 0.05kg, m_c = 1kg, |d| \le 0.088,\\
&L = 1m, g = 9.81m/s^2,
\end{aligned} \tag{9}$$

and initial values are

$$\theta_0 = -60^\circ, \dot{\theta}_0 = 0, x_0 = 0, \dot{x}_0 = 0. \quad (10)$$

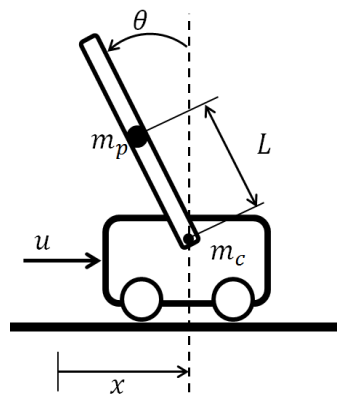


Fig. 1. A cart-pendulum system

In order to design optimal sliding mode control three objective functions are considered. They are defined as follows:

- $J_1 = \int_0^{10} |\theta(t)| dt$
- $J_2 = \int_0^{10} |x(t)| dt$
- $J_3 = \int_0^{10} |u(t)| dt$

These objective functions are related to time responses of the pendulum, the cart, and the control effort, respectively. Now these objective functions are considered simultaneously in a Pareto optimization process to obtain some important trade-offs among the conflicting objectives. The design variable vector is defined as follow

$$\vec{\mathbf{d}} = [c_1, c_2, \phi_1, \phi_z, Z_u, \eta]. \quad (11)$$

The evolutionary process of the multi-objective optimization is accomplished with a population size of 120 which has been chosen with crossover probability P_c and mutation probability P_m as 0.85 and 0.09, respectively. A total number of 78 non-dominated optimum design points have been obtained.

Broadly speaking, to visualize more than two objective functions is much more difficult. Hence, several multidimensional visualization methods are proposed, the *Level Diagrams* method premised on Blasco and et. al. is used to visualize a Pareto front [7]. In this method, each point of Pareto front must be normalized between 0 and 1 based on its minimum and maximum values [7]

$$J_i^M = \max J_i, J_i^m = \min J_i, i = 1, 2, 3, \quad (12)$$

$$\bar{J}_i = \frac{J_i - J_i^m}{J_i^M - J_i^m}. \quad (13)$$

Provided that the origin of the n-dimensional space is considered as ideal point, therefore, the distance of the each Pareto front point is used to choose optimum points. In this work, Euclidean norm (2-norm) is used for this purpose. Hence the point whose distance to the origin is the minimum, that is, the lowest value of 2-norm can be obtained as the most important trade-off point.

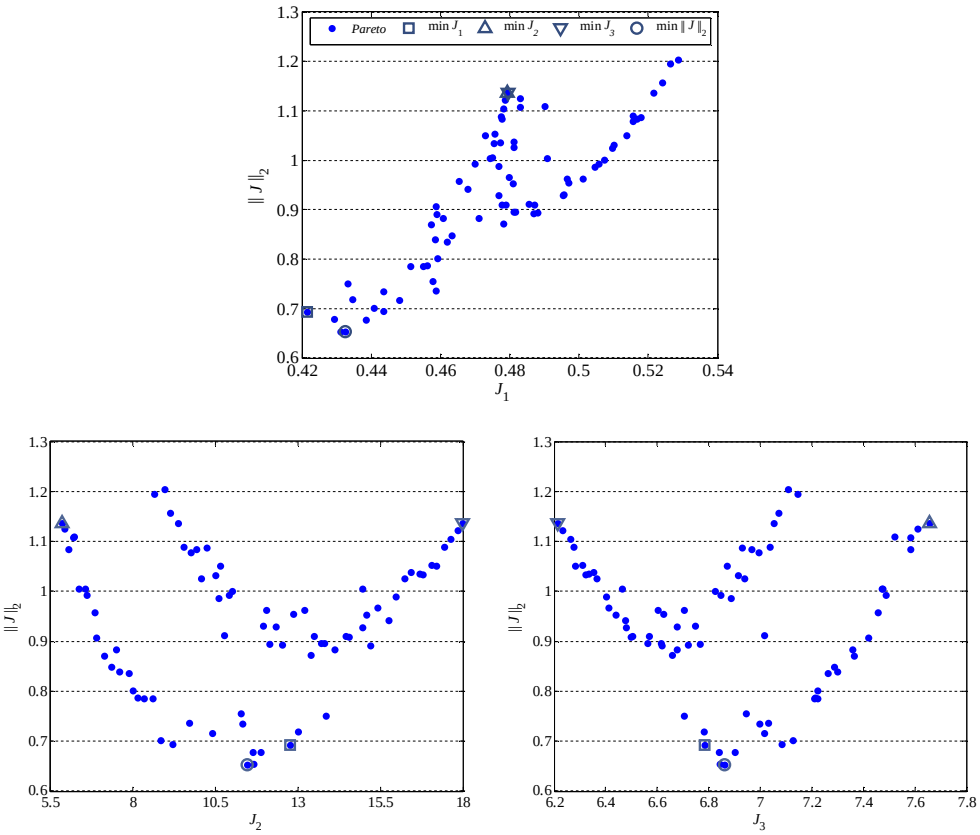


Fig. 2. 2-norm *Level Diagrams* of Pareto front.

The results of the 3-objective optimization process are shown in Figure 2. As it is shown, the point with the lowest vale of 2-norm has the low value of first objective function, and the average value of others. Also, it can be concluded that there is confliction between J_1 and J_2. Also, there is another confliction between J_2 and J_3, because the control signal directly acts on the cart and the system with low cost of energy should have the slow cart response.

To illustrate the results of the optimization process, four points are chosen which three of them have the minimum value of each objective function and the forth one has the minimum value of 2-norm. The values of the relevant design variables and objective functions are given in Table 1 and Table 2, respectively.

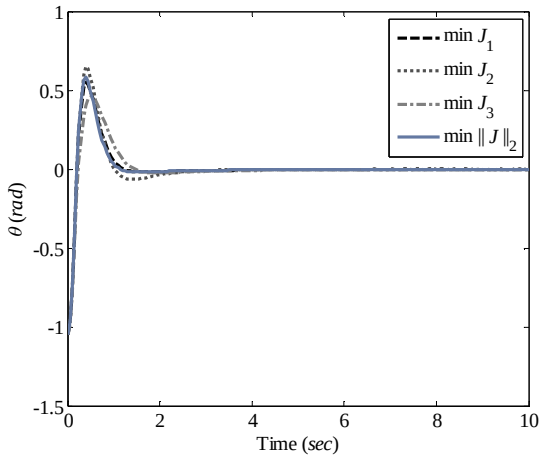
Table 1. The values of design variables of the selected optimal points

Points	c_1	c_2	ϕ_1	ϕ_z	Z_u	η
min J_1	7.24	0.49	0.72	16.90	0.95	11.92
min J_2	7.28	0.89	0.67	16.79	0.95	11.89
min J_3	5.73	0.43	0.70	20.23	0.87	11.95
min $\|J\|_2$	7.48	0.44	0.70	16.47	0.94	11.79

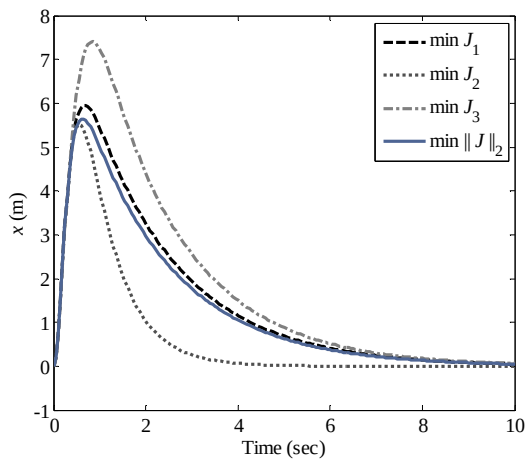
Table 2. The values of objective functions of the selected optimal points

Points	J_1	J_2	J_3	$\|J\|_2$
min J_1	0.42	13.76	7.78	0.69
min J_2	0.48	6.94	8.61	1.13
min J_3	0.48	18.97	7.21	1.14
min $\|J\|_2$	0.43	12.45	7.86	0.65

Figure 3 illustrates the simulation results of all optimum points. This figure clearly shows the trade-off among objective functions. For illustration, the system whose value of J_3 is low has the cart position response with high overshoot and settling time. Also, it can be concluded from Figure 3 that the system with low value of 2-norm has acceptable responses; therefore, it can be chosen by designers as an important trade-off point.



(a)



(b)

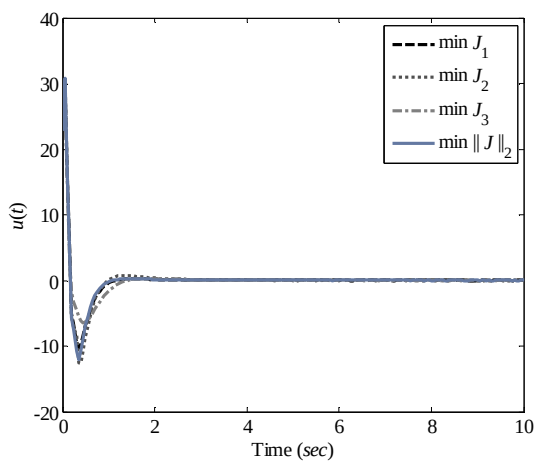


Fig. 3. The simulation results of the selected optimum points, (a) pendulum angle, (b) cart position, and (c) control signal

Conclusions

In this work multi-objective optimal sliding mode control has been proposed and successfully used to optimally design controller for a cart-pendulum system. The multi-objective optimization of sliding mode controller led to the discovering some important trade-offs among those objective functions. The multi-objective GAs of this work for the Pareto optimization of sliding mode controller using some non-commensurable objective functions is very promising and can be generally used in the optimum design of real-world complex control systems with external disturbances.

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