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Robust Fuzzy Fractional-Order PID Controller Design using Multi-Objective Optimization

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ABSTRACT

In this paper, multi-objective optimization is used for Pareto optimum design of fuzzy fractional-order PID controllers for plants with parametric uncertainties. Two conflicting objective functions have been used in Pareto design of the fuzzy fractional-order PID controller. The results clearly show that an effective trade-off can be compromisingly achieved among the different fuzzy fractional-order PID controllers obtained using the methodology of this work and to achieve a robust design against the plant uncertainties.

KEYWORDS: Robust Control, Fuzzy, Fractional-order PID Controllers, Pareto

1. INTRODUCTION

The concept of fractional calculus has been used in many complex problems and research activities in recent years [1-3]. In order to obtain high performance-controllers, the control engineers are involved in using that concept to develop controllers which have outstanding performances in comparison with integer ones [1]. Inasmuch as the real dynamic systems are generally fractional [4], fractional calculus results in the exact modeling of dynamic systems. The complexity and tediousness of the fractional-order calculus caused engineers to use the integer-order calculus for modeling dynamic systems; however, nowadays, there are many methods to approximate fractional calculus [5]. The application of fractional-order calculus has been significantly increased and fractional-order controllers are attractively become a major topic of control engineering.

However, designing high-performance and cost effective controllers is a very complex subject which could be practically impossible to optimally accomplish by some conventional trial and error methods considering different design criterion in time and frequency domain. This can be more sophisticated when such fractional-order controllers with more complexity and design parameters are involved. There are many methods to design such controllers which can be regarded as optimization problems of certain performance measures of the controlled systems [5]. In this paper, a new control strategy is developed by combining fuzzy approach and multi-objective optimization to regulate the orders of fractional-order PID controllers, and other control parameters are obtained by multi-objective optimization. In order to show the robustness of proposed method, an uncertain process plant with probabilistic uncertainties is considered. The mean and variance of the stochastic step responses are considered as objective functions which should be minimized simultaneously. Whereas in robust design there is a trade-off among these objective functions; therefore, this problem can be formulated as a multi objective optimization problem (MOP) so that trade-offs between objectives can be found consequently.

Fractional-Order PID Controllers

One of the most widely used controllers in operation today is PID controllers. This is because of the fact that PID controllers are easy to design and to implement, and also performs reasonably robustly in the presence of uncertainties. In fractional-order PID controllers (FOPID), *I* and *D* operations are usually of fractional order; therefore, two extra degrees of freedom from the use of a fractional-order integrator and differentiator will be added to three gain design variables K_p, K_d, K_i. These two extra design variables are the order of fractional integration λ and that of fractional derivative μ. Such extra degree of freedom of design variables makes these FOPID to surpass the performance of integer-order PID (IOPID) controllers [6].

The transfer function of the $PI^{\lambda}D^{\mu}$ controller in the frequency domain can be defined as

$$C(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu} \qquad (1)$$

and the control effort of the these controllers in the time domain may be given as

$$u(t) = K_p e(t) + K_i \mathrm{D}^{-\lambda} e(t) + K_d \mathrm{D}^{\mu} e(t) \qquad (2)$$

Figure 1 depicts the FOPID controller and explains how the order of the integrator and the order of the differentiator can vary versus the horizontal and vertical axis. It can be concluded that $\lambda = 1$, $\mu = 1$ implies the ordinary

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PID controllers, for $\lambda = 1$, $\mu = 0$, the ordinary PI controller can be obtained, and $\lambda = 0$, $\mu = 1$ implies the ordinary PD controller. Evidently, ordinary PI, PD, and PID controller are special cases of the fractional-order PID controller when the values of λ and μ are integer values of 0 or 1.

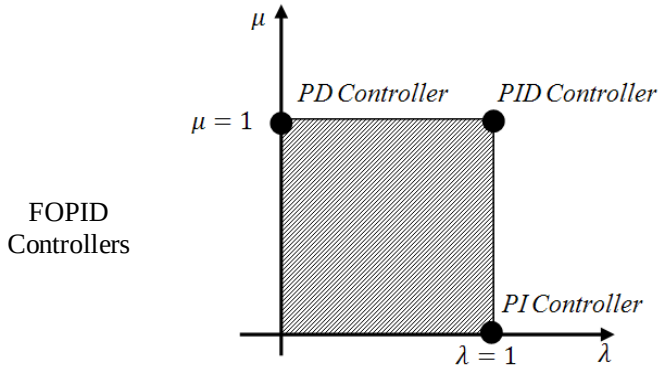

Fig. 1. FOPID and IOPID controllers

Process Model and Controller Design

In this paper, the parameters of FOPID including λ and μ can be regulated by a fuzzy supervisory system. In order to regulate them, two fuzzy logic systems must be defined. Each fuzzy logic system consists of four parts, the fuzzifier, the fuzzy rule base, the inference engine, and the defuzzifier. The fuzzy rule base of this can be written as follows

$$R^i : IF\ x_1\ is\ A_{1i}\ and\ x_2\ is\ A_{2i}\ THEN\ y\ is\ B \qquad (3)$$

where x_1 and x_2 are input linguistic variables, y is the output linguistic variable. A_{1i}, A_{2i}, and B are the values for each input linguistic variables and output linguistic variable, respectively. In this paper, x_1 is *error* and x_2 is *change in error*. For each input variable, five triangular membership functions are considered. In order to design these membership functions we consider that all universes of discourses are normalized to lie between -1 and 1, and the first and last membership functions have their apexes at -1 and 1 respectively. Also, the base vertices of membership functions are coincident with the apex of the adjacent membership functions. Therefore, membership functions can be designed by one parameter which is called spacing parameter [7]. Due to this parameter (*Ps*) we can obtain the apexes of each triangular membership functions as bellow

$$C_i = \left(\frac{i}{n}\right)^{ps}, n = \frac{N-1}{2}, i = 1,\ldots,n \qquad (4)$$

where N is the number of membership functions which is an odd number. It can be concluded that if *Ps* is less than one the centers are spaced out and if *Ps* is more than one, the centers are closed together in the center. In the multi-objective optimization design *Ps* is considered as a design variable to be found optimally. Figure 2 depicts five membership functions with three different *Ps*.

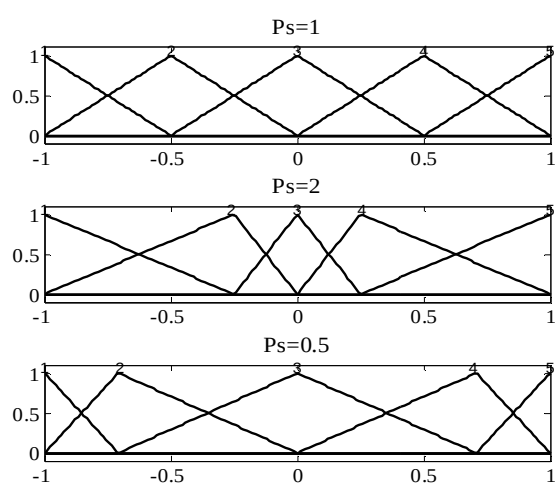

Fig. 2. Designed membership function with different Ps

In order to design the fuzzy rule base the ideas presented by Park and et al. [7] is used. In this way, two characteristic parameters are used to design a fuzzy rule base. One of them is spacing parameters for the inputs and the output, and another one is the angle for inputs. At first, it should be considered that extreme input values cause extreme output value, also, the mid-range of the input values result in the mid-range of the output value. In addition, similar combination of inputs leads to similar linguistic value of output.

The space parameters of the inputs are used to create possible combinations of linguistic variables which are called grid points, and the angle is used to divide this region into different regions. Each region contains one output linguist variable. Figure 3 shows how the fuzzy rule base can be designed. In Figure 3, each input and output have five membership functions whose space parameters are 1, and the angle is 45º.

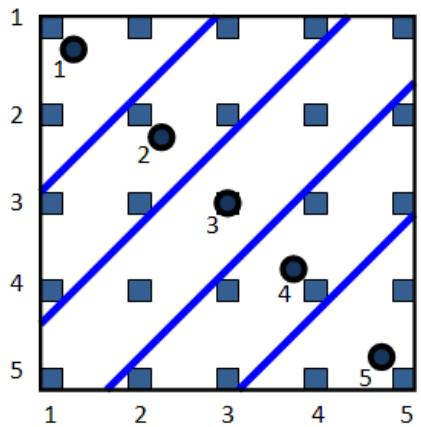

Fig. 3. Rule base construction

Table 1. The obtained fuzzy rule based from Figure 3

In2 \ In1	1	2	3	4	5
1	1	1	2	2	3
2	1	2	2	3	4
3	2	2	3	4	4
4	2	3	4	4	5
5	3	4	4	5	5

The fuzzy system for both λ and μ are defined as follows

$$\lambda = \lambda_0 + 0.2 \times F_\lambda(e, \dot{e}) \quad (5)$$

$$\mu = \mu_0 + 0.2 \times F_\mu(e, \dot{e}) \quad (6)$$

where, λ_0 and μ_0 are the base of fractional-orders and F_λ and F_μ are fuzzy functions. The space parameters, angle and the base of fractional-orders coupled with K_p, K_d, K_i are the design variables, which are all obtained using a multi-objective genetic algorithm.

Process Model and Controller Design

Many industrial systems can be adequately presented by a first-order plus in integrator as

$$G(s) = \frac{k}{s(1+Ts)} \quad (7)$$

In the case of stochastic robust design [8], parameters of the process plant vary according to *a priori* known probabilistic distribution functions around a nominal set of parameters. In this work, beta distributions with the coefficients of 2 and 2 with the limits of ±50% of the nominal values of plant parameters k=T=1 have been selected.

A good step response behavior of the system is one of the time domain performance metrics in controller design procedure that illustrates how system acts in transient and steady state periods. In order to obtain a good time-response, there are many criteria based on the integral of the step response error, such as the integral squared error (*ISE*) criterion, and integral time weighted squared error (*ITSE*) criterion. In this paper the ITSE which is given as follow is used to define the optimization criterion for a good step response.

$$ITSE = \int_0^{Tf} te^2(t)dt \quad (8)$$

where, $e(t)$ is the step response error of each random closed-loop system.

Whereas, the process plant in this paper has uncertain parameters therefore in the Monte-Carlo Simulation, the step responses are stochastic responses and *ITSE* for each response is stochastic variable. In the robust optimum design, both mean and variance of stochastic step responses should be minimized.

RESULTS

The two objective functions are now considered simultaneously in a Pareto optimization process to obtain some important trade-offs among the conflicting objectives. In a robust design approach, the vector of objective functions to be optimized in a Pareto sense is given as follow

$$\vec{J} = \left[mean(ITSE), \text{var}\, iance(ITSE)\right] \qquad (9)$$

which are computed in a quasi-Monte Carlo simulation process. The evolutionary process of the multi-objective optimization is accomplished with a population size of 150 which has been chosen with crossover probability P_c and mutation probability P_m as 0.8 and 0.01, respectively. The optimization process of the robust controller is accomplished for 100 Monte Carlo evaluations using Hammersley Sequence Sampling (HSS) [8] distribution for each candidate control law during the evolutionary process.

Pareto front of two objective functions is shown in Figure 4. Evidently, it can be seen that improving one objective will cause another objective deteriorates.

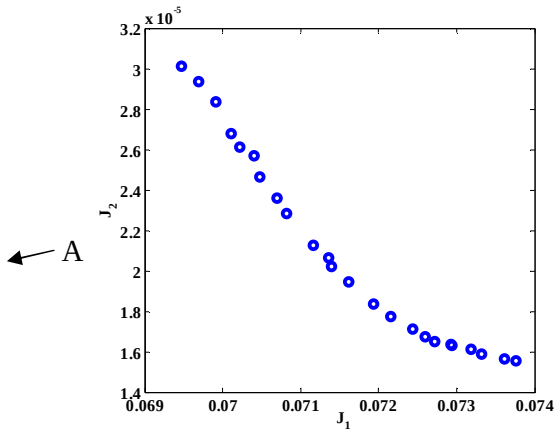

Fig. 4. Pareto front

The mean of step response for optimum trade-off point A for 100 HSS is shown in Figure 5. Also, the variance of the step response for this point is shown in Figure 6. It can be concluded from Figure 5 and Figure 6 that the proposed method causes the system to have a good step response with high robustness.

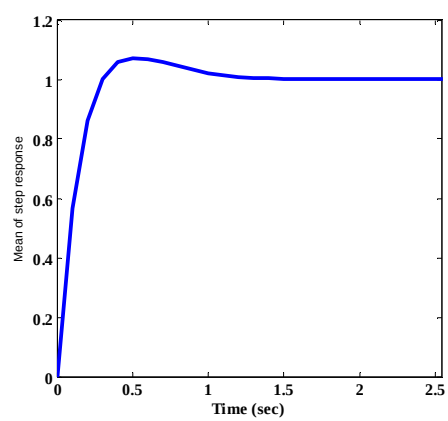

Fig. 5. Mean of step response of optimum point A

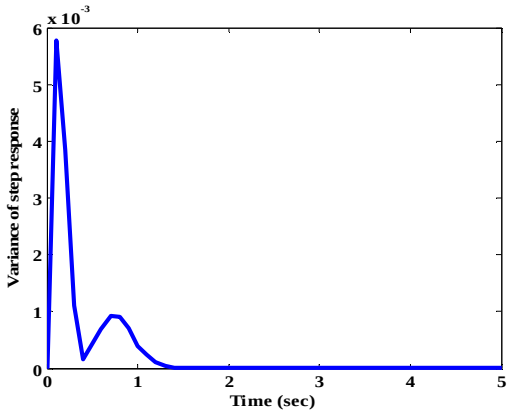

Fig. 6. Variance of step response of optimum point A

The stochastic step response for 1000 samples with the optimum controller A selected from the obtained Pareto front is shown in Figure 7. It can be concluded that the proposed method performs robustly in the presence of the plant probabilistic uncertainties.

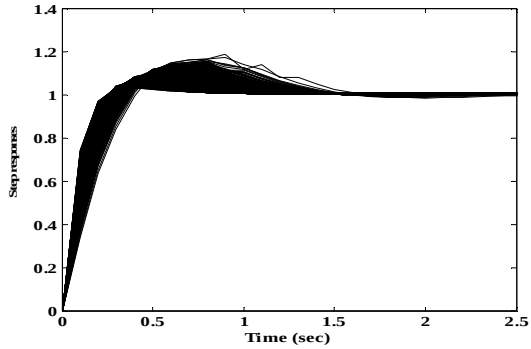

Fig. 7. Stochastic step responses of optimum point A for 1000 samples

Conclusions

In this work, multi-objective optimal robust fuzzy fractional-order PID controller has been proposed and successfully used for an uncertain process plant. The multi-objective optimization of fuzzy fractional-order PID controller led to the discovering the trade-off between those objective functions. The multi-objective GAs of this work for the Pareto optimization of fuzzy fractional-order PID controller using some non-commensurable objective functions is very promising and can be generally used in the optimum design of real-world complex control systems with uncertainties.

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