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Inverted Pendulum control with pole assignment, LQR and multiple layers sliding mode control

Hamed Ebrahimi Mollabashi[1], Marziyeh Rajabpoor[2], Soroush Rastegarpour[3]

[1,3]MSc student in Department of Electrical Engineering, Islamic Azad University (IAU), South Tehran Branch, Tehran, Iran.
[2]MSc student in Department of Electrical Engineering, Islamic Azad University (IAU), science and Research, Tehran, Iran.

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ABSTRACT

Inverted pendulum system is one of popular and important laboratory models for teaching control system engineering. This paper presents a multiple layers sliding mode controller and pole assignment controller and LQR for inverted pendulum system. In multiple layers sliding mode control, firstly, the given system is divided into several subsystems. Then, one subsystem is selected to construct the first layer sliding mode surface and it is used to construct the second layer sliding mode surface with the sliding mode surface of another subsystem. This process continues till all states of all subsystems are included. The controller is designed according to this multiple layers structure. For optimization of sliding surfaces constants are used genetic algorithm.

KEYWORD: Pendulum, LQR, sliding mode

I. INTRODUCTION

The inverted pendulum system is a standard problem in the area of control systems. They are often useful to demonstrate concepts in linear control such as the stabilization of unstable systems. The sliding mode controller is a powerful nonlinear controller, which has been developed and applied to feedback control systems for the last three decades [1]. For under actuated systems, designing a conventional sliding mode surface is not appropriate, because the parameters of the sliding mode surface can't be obtained directly according to the Hurwitz condition. Many papers on the control of inverted pendulum(s) systems have been published. Niemann [1] designed a u controller for a double inverted pendulums system. In [2], Omatu presented two PID controllers for single and double inverted pendulums systems, whose parameters were regulated by genetic algorithm (GA) and neural network (NN), respectively. Yi [3] proposed a fuzzy controller for this class of under-actuated systems. The structure characteristic of a class of under actuated systems, such as inverted pendulum(s) systems, is that they can be divided into several subsystems. According to this structure characteristic of inverted pendulum(s) systems, Mon and Lin [5] presented a hierarchical sliding mode controller (HSMC) for a double inverted pendulums system and a single inverted pendulum system, respectively, whose parameters were regulated by fuzzy logic. But both of them only guaranteed the second level sliding mode surface was stable. Ebrahimi [4] give the stability analysis for under-actuated systems with more subsystems. For optimization of response is used genetic algorithm. (GA) improved the coefficient of sliding mode surface. In this paper, three methods are used for control of linear model of Inverted Pendulum, pole assignment, LQR and multiple layers sliding mode control

II. MODELING

A schematic of the inverted pendulum is shown in Figure 1. Self-balancing robots heavy cranes lifting containers in shipyards rockets and missiles future transport systems like: Segway's and jetpacks real examples of inverted pendulum system.

For the analysis of system dynamic equations, Newton's second law of motion was applied [7].

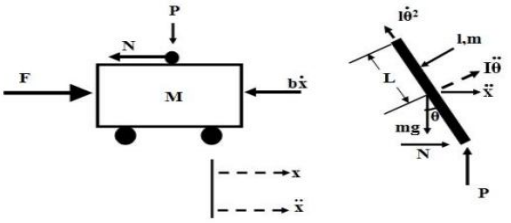

Fig. 1: inverted pendulum

*Corresponding Author: Hamed Ebrahimi Mollabashi, MSc student in Department of Electrical Engineering, Islamic Azad University (IAU), South Tehran Branch, Tehran, Iran. hamed_ebrahimi@aol.com

These equations are obtained by equating Horizontal Forces – Cart,

$$\begin{cases} \sum F_x = M\ddot{x}_M \rightarrow F - N - b\dot{x} = M\ddot{x} \\ \sum F_x = m\ddot{x}_m \rightarrow N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\cos\theta \end{cases} \Rightarrow \quad (M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \tag{1}$$

And these equations are obtained by equating Forces acting Perpendicular to the Pendulum

$$\begin{cases} \sum F_N = m\ddot{x}_N \rightarrow p\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta \\ \sum M = I.\ddot{\theta} \rightarrow -pl\sin\theta - Nl\cos\theta = I.\ddot{\theta} \rightarrow p\sin\theta + N\cos\theta = -\dfrac{I.\ddot{\theta}}{l} \end{cases} \Rightarrow \quad (I+ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \tag{2}$$

Nonlinear form of Inverted Pendulum system can be depicted by:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1 + b_1 u \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2 + b_2 u \end{aligned} \tag{3}$$

$x_1 = x$ is the cart position, $x_4 = \dot{x}$ is the velocity of the cart, $x_3 = \theta$ is the pendulum angle with respect to the vertical line; $x_2 = \dot{\theta}$ is the angle velocity of the pendulum with respect to the vertical line, and the expressions of f_1, f_2, b_1 and b_2 are given in [1]

We linearize the system about the unstable equilibrium $\theta = \pi$:

$$\begin{aligned} (I + ml^2)\ddot{\theta} - mgl\theta &= -ml\ddot{x} \\ (M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta} &= F \end{aligned} \tag{4}$$

The linearization of the cart-pendulum system around the upright position is:

$$\dot{x} = Ax + Bu$$

$$y = Cx + D$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-b(I+ml^2)}{I(M+m)+mMl^2} & \dfrac{m^2gl^2}{I(M+m)+mMl^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-mbl}{I(M+m)+mMl^2} & \dfrac{mgl(M+m)}{I(M+m)+mMl^2} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \dfrac{(I+ml^2)}{I(M+m)+mMl^2} \\ 0 \\ \dfrac{ml}{I(M+m)+mMl^2} \end{bmatrix} \tag{5}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

III. MULTIPLE LAYERS SLIDING MODE CONTROL DESIGN

From [4-5] we can design multiple layers sliding mode control for inverted pendulum. For SIMO under actuated mechanical systems, the mathematical model can be translated into the following form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1 + b_1 u \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2 + b_2 u \end{aligned} \tag{6}$$

Where $X = [x_1, x_2, ...x_{2n}]^T$ the state variables and u is the input of the system, f_n and b_n are bounded nominal. As the Multiple layers sliding structure has been shown in Figure 2.

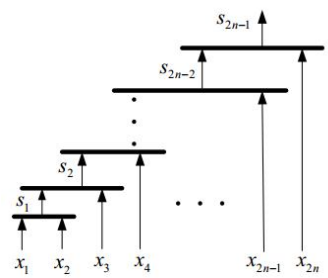

Fig. 2: sliding mode surfaces

First layer can be defined by [4-5] $s_1 = c_1 x_1 + c_2 x_2$ the equivalent law control obtain from $\dot{s}_1 = 0$

$$0 = c_1 \dot{x}_1 + c_2 \dot{x}_2 \rightarrow c_1 x_2 + c_2 (f_1 + b_1 u) \rightarrow \qquad u_{eq(1)} = -\frac{c_2 f_1 + c_1 x_2}{c_2 b_1} \tag{7}$$

The second layer defined as $s_2 = c_3 x_3 + s_1$ and $s_3 = c_4 x_4 + s_2$ so the total U_{eq} is:

$$u_{eq(i)} = -\frac{\sum_{j=1}^{m} c_{2j-1}.x_{2j} + \sum_{j=1}^{m} c_{2j} f_j}{\sum_{j=1}^{m} c_{2j} b_j} \tag{8}$$

$u_{sw(i)}$ is the switch control law for every layer sliding surface, $u_{sw(i)}$ can improved the response time.[4]
The switch control law is defined as:

$$u_{sw(i)} = \begin{cases} 0 & i = 1 \\ \sum_{j=1}^{i} \eta_j \operatorname{sgn}(s_j) / den(i) & i > 1 \end{cases} \tag{9}$$

Where η is a positive constant, $\eta_j = 2\eta_{j-1}$

$$den(i) = c_2 b_1 + \sum_{j=2}^{m} (c_{2j}.b_j.\operatorname{sgn}(s_{2j-1})) \tag{10}$$

c_i and η are constants which have the same sign but this parameters and other sliding surface can't be obtain according Hurwitz condition so for this reason and optimizations, using genetic algorithm.

$$U_{sm(i)} = u_{eq(i)} + u_{sw(i)} \tag{11}$$

IV. The GA Optimization

Genetic algorithms, introduced by Holland [6], are based on the idea of engendering new solutions from parent solutions, employing mechanisms inspired by genetics. In the following text the controller will be optimized by genetic algorithm. The optimization parameters are c_i. The number of variables is 8. The fitness function is shown as formula. Evaluate the fitness value of each output.

$$j = \sum_{i=0}^{n} (x_i^T Rx + u_i^T qu) \tag{12}$$

V. POLE ASSIGNMENT CONTROL DESIGN

Pole placement method is one of the classic control theories and has an advantage in system control for desired performance. Theoretically pole placement is to set the desired pole location and to move the pole location of the system to that desired pole location to get the desired system response.

For using pole placement control design assumptions are: 1- The system is completely state controllable 2- The state variables are measurable and are available for feedback. 3- Control input is unconstrained.

The open loop poles are placed at 0, -0.0999, -7.2220, and 7.1430 from the imaginary axis. Now we want to move these poles to -8, -6, -1, and -1 from the axis. To move to the desired pole location we have to fine feedback gains. K is fine feedback gain.

VI. SIMULATION RESULTS

M	mass of the cart	1kg
m	mass of the pendulum	1kg
b	friction of the cart	0.1N/m/sec
l	length of the pendulum	0.1m
i	inertia of the pendulum	$0.006\, kg.m^2$
F	force applied to the cart	$kg.m/s^2$
g	gravity	$9.8\, m/s^2$
θ	Vertical pendulum angle	in degree

x_1 is cart position x_2 velocity of the cart, x_3 Vertical pendulum angle and x_4 angle velocity of the pendulum

Initial values are: $x_1 = 0, x_2 = 0, x_3 = pi/3, x_4 = 0$

With GA, constant is found: c= [-0.3643 -0.7448 3.9157 0.7355 -0.4195 0.0412 0.7938 5.1554]

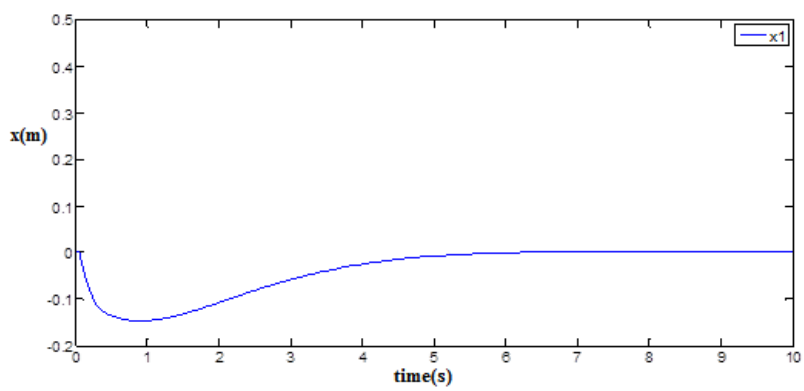

Fig. 3: Time response of carts positions (multiple layers sliding mode control).

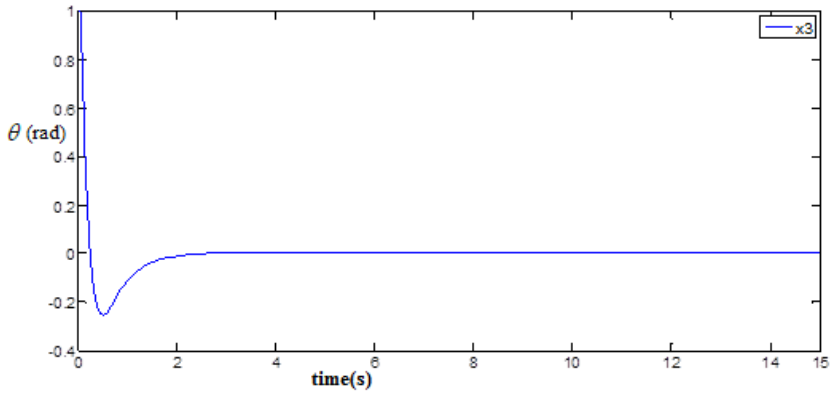

Fig. 4: Time response of pendulums angles (multiple layers sliding mode control).

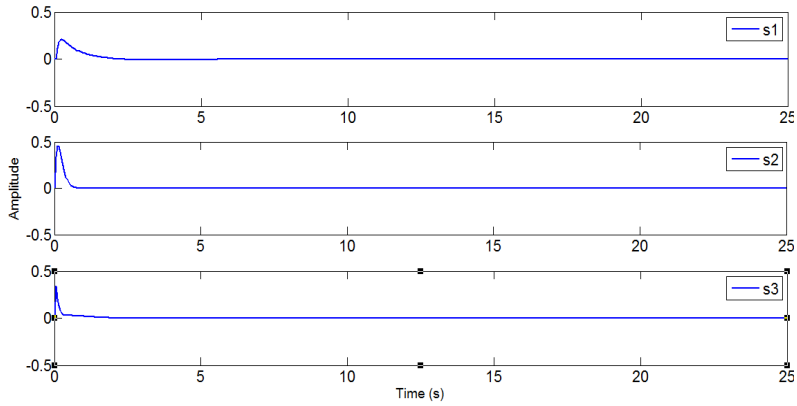

Fig. 5: sliding surfaces

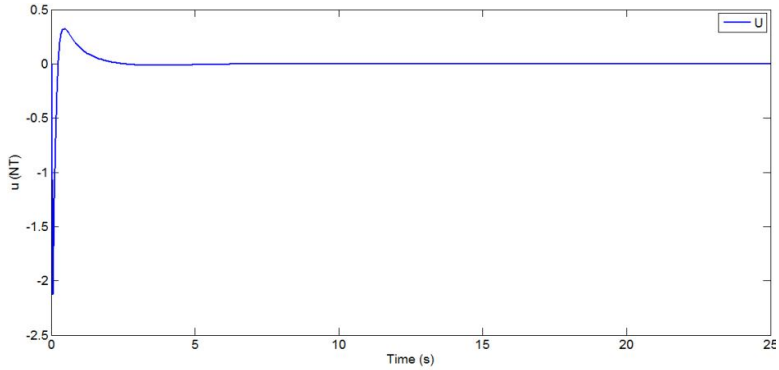

Fig. 6: Control output

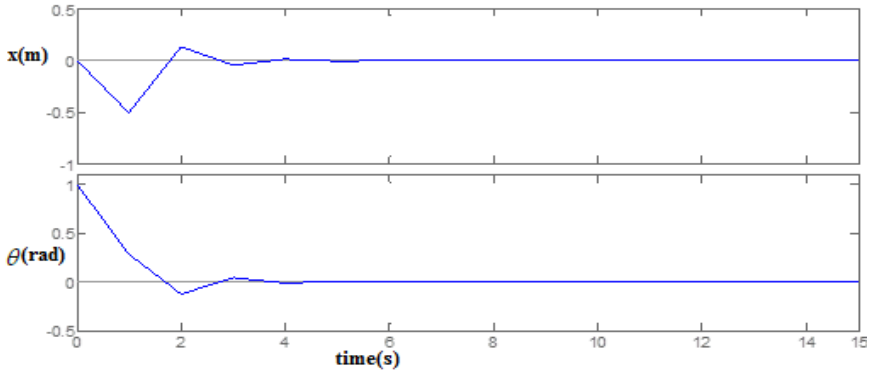

Fig. 7: Time response of carts positions and pendulums angles (Pole assignment control)

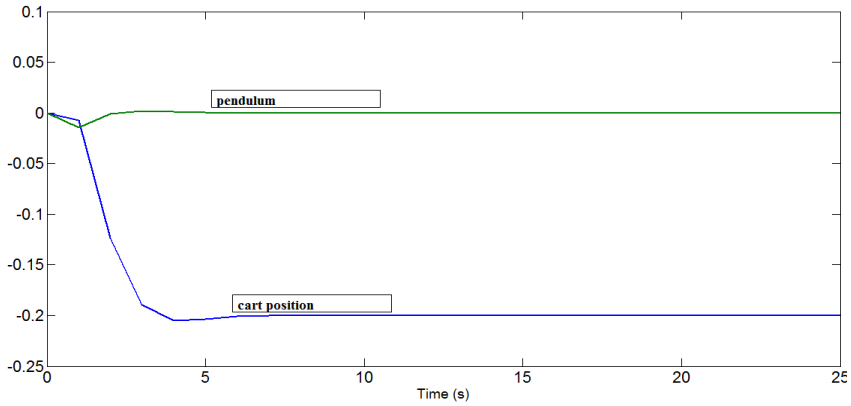

Fig. 8: Time response of carts positions and pendulums angles (LQR)

VII. CONCLUSION

Multiple layers sliding mode control (MLSMC), LQR and Pole assignment control method has been proposed in this paper. Simulation results were presented. In LQR method, SISO system has to be considered and the wagon (cart) position will be forgotten while in the multi-layer sliding mode method, cart position & pendulum are controlled simultaneously. Simulation results have shown that the all sliding surfaces are asymptotically stable. This approach has acceptable control effort.

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