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Using Fuzzy Risk Analysis Based on Similarity Measure of Generalized Fuzzy Numbers for Ranking Factories

N. Parandin[*1], M. Seidzadeh[2], S. Hamedi[3]

[1]Professor at the University, Department of Mathematics, "Kermanshah Branch, Islamic Azad University, Kermanshah, Iran"
[2]University student, Department of Mathematics, "Kermanshah Branch, Islamic Azad University, Kermanshah, Iran"
3Department of Soil Science, "Kermanshah Branch, Science and Research Islamic Azad University, Kermanshah, Iran"

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ABSTRACT

In this paper, we present a new approach based on similarity measure between extended fuzzy numbers for fuzzy risk analysis. First, a new method is proposed for calculating similarity measure of extended fuzzy numbers. By this method, fuzzy numbers are divided by two parts by 'α- cuts. Then, we can calculate perimeters of each fuzzy numbers produced by α- cuts separately.
In the follow, we introduce the index of similarity measure between fuzzy numbers by using the convex combination of distance between extended fuzzy number points. We apply fuzzy risk analysis algorithm to deal with fuzzy analysis problems.
KEYWORDS: Fuzzy risk analysis; Similarity measure; Convex combination; Fuzzy numbers.

1. INTRODUCTION

The task of handling risk analysis problems plays an important role. Chen and Chen (2006)[1] presented a method for ranking generalized fuzzy numbers for handling fuzzy risk analysis problems. Chen and Chen (2007)[2] presented a method for fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers. Schmucker(1984) [3] presented a method for fuzzy risk analysis based on fuzzy number arithmetic operations. In recent years, some similarity measure between fuzzy numbers have been presented Chen & Chen, (2001) [4]; Chen, (1996) [5]; lee, (1999[6]). Chen(1996) presented a similarity measure between fuzzy numbers for subjective mental workload assessment and fuzzy risk analysis. Hsieh and Chen (1999)[7] presented a similarity measure between fuzzy numbers using the " graded mean integration representation distance".

In this paper, we propose a new similarity measure between trapezoidal fuzzy numbers; It combines the concepts of the geometric distance, the convex combination of points and perimeter of generalized fuzzy numbers for calculating the degree of similarity between generalized fuzzy numbers. We also prove three properties of the proposed similarity measure. Based on the proposed similarity measure, we present a new fuzzy risk analysis algorithm for dealing with fuzzy risk analysis problems, where the value of the evaluating items are represented by generalized fuzzy numbers. The proposed method provides us a useful way for handling fuzzy risk analysis problems.

In the follow, this paper is organized by these sections. In section 2; we review some basic concept and arithmetic operations about extended fuzzy numbers. In Section 3, we briefly review some existing similarity measure of fuzzy numbers. In Section 4, we present a new similarity measure between generalized fuzzy numbers. In Section 5, we apply the proposed similarity measure to propose a fuzzy risk analysis algorithm to deal with fuzzy analysis problems.

2 Basic concepts of generalized fuzzy numbers

In this section, we briefly review basic concepts of generalized trapezoidal fuzzy numbers[5] A as A =(a, b, c, d; w), where a, b, c, d are real values and 0<w≤1. The membership function$\mu_A$of ageneralized fuzzy numbers A satisfies the following conditions:
$\mu_A$is a continuous mapping from the universe of discourse X to the closed interval in [0,1];
$\mu_A(x) = 0$, where $-\infty \le X \le a$;
$\mu_A(x)$ is monotonical increasing in [a,b];
$\mu_A(x) = w$, where $b \le X \le c$;
$\mu_A(x)$ is monotonical decreasing in [c,d];
$\mu_A(x) = 0$, where $d \le X \le -\infty$;
If w=1, then the generalized fuzzy numbers A is a normal fuzzy numbers, denoted as A = (a,b,c,d). If a=b and c=d, then the generalized fuzzy numbers A is a crisp interval. If a< $b = c < d$, then A is a triangular fuzzy number. If a< $b < c < d$, then A is a generalized trapezoidal fuzzy number. If a= b = c = d and w =1, then A is a crisp value.
Assume that there are two generalized trapezoidal fuzzy number A and B, where $A = (a_1, b_1, c_1, d_1; w_A)$ and $B = (b_1, b_2, b_3, b_4; w_B)$. The arithmetic operations between the generalized trapezoidal fuzzy numbers A and B are reviewed from Chen (1985)[8] and Chen (1999)[9] as follows:
Generalized fuzzy numbers addition ⊕ :

$$A_1 \oplus A_2 = (a_1, b_1, c_1, d_1; w_1) \oplus (a_2, b_2, c_2, d_2; w_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(w_1, w_2)). \quad (1)$$

Where $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$are real numbers.

Corresponding author: Noordin. Parandin, Department of Mathematics, Kermanshah Branch, University of Islamic Azad University, Kermanshah, Iran. Email address: n_parandin@yahoo.com

Generalized fuzzy numbers multiplication ⊗:

$$A_1 \otimes A_2 = (a_1, b_1, c_1, d_1; w_1) \otimes (a_2, b_2, c_2, d_2; w_2) = (a, b, c, d; \min(w_1, w_2)). \quad (2)$$

Where a = Min$(a_1 \times a_2, a_1 \times d_2, d_1 \times a_2, d_1 \times d_2)$, b = Min $(b_1 \times b_2, b_1 \times c_2, c_1 \times b_2, c_1 \times c_2)$, c = Max$(b_1 \times b_2, b_1 \times c_2, c_2, c_1 \times b_2, c_1 \times c_2)$,and d = Max $(a_1 \times a_2, a_1 \times d_2, d_1 \times d_2, d_1 \times d_2)$. It is obvious that if $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$are positive real numbers, then:

$$A_1 \otimes A_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2; \min(w_1, w_2)). (3)$$

Generalized fuzzy numbers division ∅ :

Let A_1 and A_2 be two generalized trapezoidal fuzzy numbers, where$A_1 = (a_1, b_1, c_1, d_1; w_1)$,$A_2 = (a_2, b_2, c_2, d_2; w_2)$, $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$arenonzero positive real numbers,$w_1 \in [0,1], w_2 \in [0,1]$. Then, the division between A_1 and $A_2$is defined as follows:

$$A_1 \varnothing A_2 = (a_1, b_1, c_1, d_1; w_1) \varnothing (a_2, b_2, c_2, d_2; w_2) = {}^{a_1}/_{d_2}, {}^{b_1}/_{c_2}, {}^{c_1}/_{b_2}, {}^{d_1}/_{a_2}; min(w_1, w_2)) \quad (4)$$

3. A Review of existing similarity measures between fuzzy numbers

In this section, we briefly review some existing similarity measures between fuzzy numbers from Chen (1996)[5], Chen and Chen (2001)[4], Hsieh and Chen (1999)[7] and Lee (1999)[9].

Let A and B be two trapezoidal fuzzy numbers, where $A = (a_1, b_1, c_1, d_1)$, $B = (b_1, b_2, b_3, b_4)$

Chen (1996)[5] presented a similarity measure between fuzzy numbers A and B based on the geometric distance, where the degree of similarity S(A,B) between the fuzzy numbers A and B is calculated as follows:

$$S(A,B) = 1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}, \quad (5)$$

Where S (A , B) ∈ [0 , 1]. The larger the value of S (A , B), the more the similarity between the fuzzy numbers A and B.

Hsieh and Chen (1999) [7] presented a similarity measure between fuzzy numbers using the "graded mean integration representation distance", where the degree of similarity S (A,B) between the fuzzy numbers A and B is calculated as follows:

$$S(A , B) = \frac{1}{1+d(A ,B)}, \quad (6)$$

Where d (A , B) = |P(A) , P(B)| and P(A) and P(B) are the graded mean integration representations of A and B, respectively. If A and B are triangular fuzzy numbers, where $A=(a_1, a_2, a_3)$ and $B=(b_1, b_2, b_3)$, then the graded mean integration representations P(A) and P(B) of A and, respectively, are defined as follows:

$$P(A) = \frac{a_1+4a_2+a_3}{6}, \quad (7)$$

$$P(B) = \frac{b_1+4b_2+b_3}{6}, (8)$$

If A and B are trapezoidal fuzzy numbers, then the graded mean integration representations P(A) and P(B) of A and, respectively, are defined as follows:

$$P(A) = \frac{a_1+2a_2+2a_3+a_4}{6}, \quad (9)$$

$$P(B) = \frac{b_1+2b_2+2b_3+b_4}{6}. (10)$$

The larger the value of S (A , B), the more the similarity between the fuzzy numbers A and B.

Chen and Chen(2001) [4], presented a similarity measure between generalized trapezoidal fuzzy numbers. First, they calculate the COG points (x_A^*, y_A^*) and $(x_B^*, y_B^*)$of the generalized trapezoidal fuzzy numbers A and B, respectively.

If A is a generalized trapezoidal fuzzy number,$A = (a_1, b_1, c_1, d_1; w_A)$, then the COG point (x_A^*, y_A^*) of the generalized fuzzy numbers A is calculated as follows:

$$y_A^* = \begin{cases} \dfrac{w_A \times (\frac{a_3 - a_2}{a_4 - a_1} + 2)}{6} & \text{if } a_1 \neq a_4 \quad \text{and} \quad 0 < W_A \leq 1, \\ \dfrac{w_A}{2} & \text{if } a_1 = a_4 \quad \text{and} \quad 0 < W_A \leq 1, \end{cases} \quad (11)$$

$$x_A^* = \frac{y_A^*(a_3 + a_2) + (a_4 + a_1)(W_A - y_A^*)}{2}. (12)$$

Then the degree of similarity S (A , B) between fuzzy numbers A and B is calculated as follows:

$$, \quad (13) \frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)} \times (|x_A^* - y_A^*| - 1)^{[\frac{S_A + S_B}{2}]} (1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}) \times = S(A , B)$$

Where $S_A = a_4 - a_1$and$S_B = b_4 - b_1$. The larger the value of S (A , B), the more the similarity between the fuzzy numbers A and B.

4-A new method for calculating the degree of similarity between generalized fuzzy numbers

In this section, we propose a new method based on the convex combination of distance between fuzzy numbers points for calculating the similarity measure between extended fuzzy numbers.[10]

Suppose that there are extended trapezoidal fuzzy numbers $A_1, A_2, A_3 \ldots, A_n$, where= $A_i(a_{i1}, a_{i2}, a_{i3}, a_{i4}; w_{Ai})$,and $1 < i \leq n$, and $w_{Ai} \in [0,1]$,$-\infty < a_{i1} \leq a_{i2} \leq a_{i3} \leq a_{i4} < w_{Ai} < \propto$ for calculating the similarity measure between extended fuzzy numbers, we present a new method as follows:

Step1: Transform each extended fuzzy numbers into the upper part A_2, and lower part A_1 by using α-cut.

Step2: Assume that there are two extended trapezoidal fuzzy numbers A and B, where $A = (a_1, a_2, a_3, a_4; w_A)$ and $B = (b_1, b_2, b_3, b_4; w_A)$, $0 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq 1$, $0 \leq b_1 \leq b_2 \leq b_3 \leq b_4 \leq 1$.

Now we calculate the similarity measure index S(A , B)between extended fuzzy numbers by using the convex combination of distances between fuzzy numbers points as follows:

$$S_\lambda(A , B) = \left(1-\frac{\lambda(\sum_{i=1}^{2}|a_i-b_i|+\sum_{i=5}^{6}|a_i-b_i|)}{4} + \frac{(1-\lambda)(\sum_{i=2}^{5}|a_i-b_i|)}{3}\right)$$

$$\times \frac{\min\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right) + \min(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2})}{\max\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right) + \max(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2})}$$

Where,$S_\lambda(A . B)\in [0,1]$,$P(A_i)$ and $P(B_i)$, $(1 \le i \le 2)$ are defined as follows:

$+(a_6 - a_1).$ (15)$(a_5 - a_2) + +\sqrt{(a_5-a_6)^2 + w_{A_1}{}^2}\sqrt{(a_1-a_2)^2 + w_{A_1}{}^2}=P_\lambda(A_1)$

$+(a_5 - a_2).$ (16)$(a_4 - a_3) + +\sqrt{(a_4-a_5)^2 + (w_{A_2}-w_{A_1})^2}\sqrt{(a_2-a_3)^2 + (w_{A_2}-w_{A_1})^2}=P_\lambda(A_2)$

$+(b_6 - b_1).$ (17)$(b_5 - b_2) + +\sqrt{(b_5-b_6)^2 + w_{B_1}{}^2}\sqrt{(b_1-b_2)^2 + w_{B_1}{}^2}=P_\lambda(B_1)$

$+(b_5 - b_2)$. (18)$(b_4 - b_3) + +\sqrt{(b_4-b_5)^2 + (w_{B_2}-w_{B_1})^2}\sqrt{(b_2-b_3)^2 + (w_{B_2}-w_{B_1})^2}=P_\lambda(B_2)$

$P(A_i)$ and $P(B_i)$ $(1\le i \le 2)$denote perimeters of extended fuzzy numbers A and B.

Assume that A and B denote two extended trapezoidal fuzzy numbers The proposed similarity measure have following properties:

Property 4.1. The extended trapezoidal fuzzy numbers A and B are identical if and if S(A,B)=1.

proof

(i) If A and B are identical, then if fuzzy numbers Cut to for the same α - cut In the case of fuzzy numbers A and B Than In two parts bottom and topA_1 and A_2 , B_1 and B_2 respectively. In this case, we have:

$A_1 = (a_1, a_2, a_5, a_6; w_{A_1}), B_1 = (b_1, b_2, b_5, b_6; w_{B_1})$, $\longrightarrow$ $A_1 = B_1$.

$B_2 = (b_2, b_3, b_4, b_5; w_{B_2}), A_2 = (a_2, a_3, a_4, a_5; w_{A_2})$, $\longrightarrow$ $A_2 = B_2$

w_{B_1}, $= b_6$,and $w_{A_1} = b_5, a_6 = b_4, a_5 = b_3, a_4 = b_2, a_3 = b_1, a_2 =$ Then, a_1

Thus, $P_\lambda(A_1)= P_\lambda(B_1)$ and $P_\lambda(A_2) = P_\lambda(B_2)$,Thus, for different λ:

$$min\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right)$$
$$= max\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right).$$

$\min(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2}) = \max(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2})$.

The degree of similarity between A and B is calculated as follows:

$$S_\lambda(A , B) = \left(1-\frac{\lambda(\sum_{i=1}^{2}|a_i-b_i|+\sum_{i=5}^{6}|a_i-b_i|)}{4} + \frac{(1-\lambda)(\sum_{i=2}^{5}|a_i-b_i|)}{3}\right)$$

$$\times \frac{\min\left((\lambda P(A_1)+(1-\lambda)P(A_2)),(\lambda P(B_1)+(1-\lambda)P(B_2))\right)+\min(\lambda W_{A_1}+(1-\lambda)W_{A_2}, \lambda W_{B1}+(1-\lambda)W_{B2})}{\max\left((\lambda P(A_1)+(1-\lambda)P(A_2)),(\lambda P(B_1)+(1-\lambda)P(B_2))\right)+\max(\lambda W_{A_1}+(1-\lambda)W_{A_2}, \lambda W_{B1}+(1-\lambda)W_{B2})} = 1-0=1$$

(ii) If S(A,B) =1, then

$$S_\lambda(A , B) = \left(1-\frac{\lambda(\sum_{i=1}^{2}|a_i-b_i|+\sum_{i=5}^{6}|a_i-b_i|)}{4} + \frac{(1-\lambda)(\sum_{i=2}^{5}|a_i-b_i|)}{3}\right)$$

$$\times \frac{\min\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right) + \min(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2})}{\max\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right) + \max(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2})}$$
$$= 1$$

a_4 It implies that $= a_3, b_4 = a_6, b_3 = a_5, b_6 = a_2, b_5 = a_1, b_2 = b_1$ and

$w_{A_1} = w_{B_1}$, $w_{A_2} = w_{B_2}$, also:

$$\min\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right)$$
$$= \max\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right)$$

And$\min(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2}) = \max\ (\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2})$

Therefore, the generalized trapezoidal fuzzy numbers A and B are identical.□

Property 4.2. $S_\lambda(B ,A). = S_\lambda(A , B)$

Proof: Because

$$S_\lambda(A , B) = \left(1-\frac{\lambda(\sum_{i=1}^{2}|a_i-b_i|+\sum_{i=5}^{6}|a_i-b_i|)}{4} + \frac{(1-\lambda)(\sum_{i=2}^{5}|a_i-b_i|)}{3}\right)$$

$$\times \frac{\min\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right) + \min(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2})}{\max\left((\lambda P(A_1) + (1-\lambda)P(A_2)),(\lambda P(B_1) + (1-\lambda)P(B_2))\right) + \max(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2})}$$

$$S_\lambda(B , A) = \left(1-\frac{\lambda(\sum_{i=1}^{2}|b_i-a_i|+\sum_{i=5}^{6}|b_i-a_i|)}{4} + \frac{(1-\lambda)(\sum_{i=2}^{5}|b_i-a_i|)}{3}\right)$$

$$\times \frac{\min\left((\lambda P(B_1) + (1-\lambda)P(B_2)),(\lambda P(A_1) + (1-\lambda)P(A_2))\right) + \min(\lambda W_{B1} + (1-\lambda)W_{B2}, \lambda W_{A_1} + (1-\lambda)W_{A_2})}{\max\left((\lambda P(B_1) + (1-\lambda)P(B_2)),(\lambda P(A_1) + (1-\lambda)P(A_2))\right) + \max(\lambda W_{B1} + (1-\lambda)W_{B2}, \lambda W_{A_1} + (1-\lambda)W_{A_2})}$$

And because $\sum_{i=1}^{2}|b_i - a_i| = \sum_{i=1}^{2}|a_i - b_i|$ and $\sum_{i=5}^{6}|a_i - b_i| = \sum_{i=2}^{5}|b_i - a_i|$ and $\sum_{i=2}^{5}|a_i - b_i| = \sum_{i=2}^{5}|b_i - a_i|$.

In conclusion, we can say

$\min\left(\left(\lambda P(B_1) + (1-\lambda)P(B_2)\right), \left(\lambda P(A_1) + (1-\lambda)P(A_2)\right)\right) = \min\left(\left(\lambda P(A_1) + (1-\lambda)P(A_2)\right), \left(\lambda P(B_1) + (1-\lambda)P(B_2)\right)\right)$

Also:

$\max\left(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2}\right) = \max\left(\lambda W_{B_1} + (1-\lambda)W_{B_2}, \lambda W_{A1} + (1-\lambda)W_{A2}\right)$.

Therefore, $S_\lambda(B, A). = S_\lambda(A, B)$.□

Property 4.3.IfA and B are two generalized trapezoidal fuzzy numbers with the same shape, the same scale (i.e.,$w_A = w_B$) and the same offset d, where $d = b_1 - a_1 = b_2 - a_2 = b_3 - a_3 = b_4 - a_4 = b_5 - a_5 = b_6 - a_6$.then ,$S_\lambda(A, B) = 1 - |d|$.

Proof.

Because $d = b_1 - a_1 = b_2 - a_2 = b_3 - a_3 = b_4 - a_4 = b_5 - a_5 = b_6 - a_6$.

Also:

$w_{A_2} = w_{A_2}$, $w_{B_1} = w_{A_1}$.

Based on Eqs. (16-23), we can get:

$$\min\left(\left(\lambda P(A_1) + (1-\lambda)P(A_2)\right), \left(\lambda P(B_1) + (1-\lambda)P(B_2)\right)\right)$$
$$= \max\left(\left(\lambda P(A_1) + (1-\lambda)P(A_2)\right), \left(\lambda P(B_1) + (1-\lambda)P(B_2)\right)\right)$$

$\min\left(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2}\right) = \max\left(\lambda W_{A_1} + (1-\lambda)W_{A_2}, \lambda W_{B1} + (1-\lambda)W_{B2}\right)$

The degree of similarity between A and B is calculated as follows:

$$S_\lambda(B, A) = \left(1 - \frac{\lambda\left(\sum_{i=1}^{2}|b_i - a_i| + \sum_{i=5}^{6}|b_i - a_i|\right)}{4} + \frac{(1-\lambda)\left(\sum_{i=2}^{5}|b_i - a_i|\right)}{3}\right)$$
$$\times \frac{\min\left(\left(\lambda P(B_1) + (1-\lambda)P(B_2)\right), \left(\lambda P(A_1) + (1-\lambda)P(A_2)\right)\right) + \min\left(\lambda W_{B1} + (1-\lambda)W_{B2}, \lambda W_{A_1} + (1-\lambda)W_{A_2}\right)}{\max\left(\left(\lambda P(B_1) + (1-\lambda)P(B_2)\right), \left(\lambda P(A_1) + (1-\lambda)P(A_2)\right)\right) + \max\left(\lambda W_{B1} + (1-\lambda)W_{B2}, \lambda W_{A_1} + (1-\lambda)W_{A_2}\right)}$$
$$= 1.$$

$$= 1 - \frac{\lambda\left(\sum_{i=1}^{2}|a_i - (a_i + d)| + \sum_{i=5}^{6}|a_i - (a_i + d)|\right)}{4} + 1 \times \frac{(1-\lambda)\left(\sum_{i=2}^{5}|a_i - (a_i + d)|\right)}{3} = \square.d \| 1 -$$

5. Fuzzy risk analysis based on the proposed similarity measure

In the section, we apply the proposed similarity measure of generalized fuzzy numbers to present a new fuzzy risk analysis algorithm to deal with fuzzy risk analysis problems. Let us consider the structure of fuzzy risk analysis as shown in figure1 Schmucker, (1984) [3] According to (Schmucker 1984), each sub-component A_i is evaluated by two evaluating items, i.e., "probability of failure" and "severity of loss", where the linguistic term R_i denotes the probability of failure of the sub-component $A_i$and the linguistic term w_i denotes the severity of loss of the sub-componentA_i , and $1 \leq i \leq 3$. Zhang (1986) uses trapezoidal fuzzy numbers to represent linguistic terms. In this paper, a 9-memberlinguistic term set Zhang, (1986) [11] is used to represent linguistic terms. Each linguistic term in the 9-memberlinguistic term set is corresponding to a generalized trapezoidal fuzzy number, as shown in table 1.

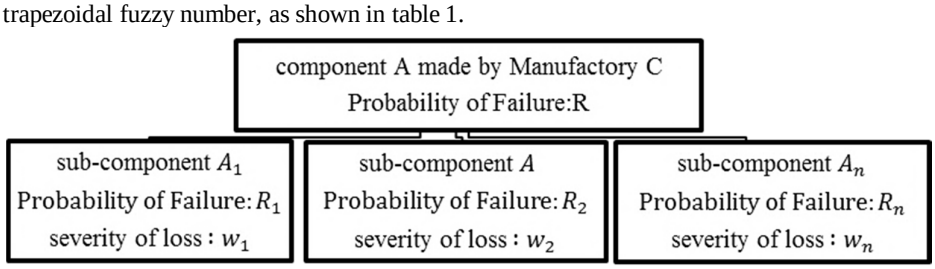

Fig. 1. Structure of fuzzy risk anakysis (Schmucker, 1984).

Assume that there is a component A consisting of n sub-components $A_1, A_2, A_3, \ldots A_n$, andassume that each sub-component is evaluated by two evaluating items "probability of failure" and "severity of loss", where R_i and w_i are linguistic values, shown in table 1, denoting "probability of failure" and "severity of loss", of sub-componentA_n, respectively, where $1 \leq i \leq n$. The proposed algorithm for dealing with fuzzy risk analysis is now presented as follows:

Step1: Based on the generalized fuzzy number arithmetic operations and the fuzzy weighted mean method, integrate the linguistic values R_i and w_i of each sub-component A_i to get the total risk R of component A, where

. $$(19)(r_1, r_2, r_3, r_4; w_R) = \sum_{i=1}^{n} w_i \varnothing R = \left(\sum_{i=1}^{n} w_i \otimes R_i\right)$$

Step2: Theprobability of failure (R) of above mentioned fuzzy numbers are divided two parts by using arbitraryα – cuts.

Step3: We can calculate the perimeters of each fuzzy numbers calculated in the previous step $P(R_i)$,$(1 \leq i \leq 2)$.

Step4: Eachfuzzy number shown in table 1 can be divided by using α – cuts, like previous step, We can calculate the perimeters of each parts.

Step5: Now, we can apply the proposed similarity measure to evaluate the similarity between fuzzy numbers R and each of fuzzy numbers presented in table 1.
In the following, we use an example to illustrate the fuzzy risk analysis process of the proposed method.
Example. Consider the structure of fuzzy risk analysis as shown in figure 1, where the component A consists of three sub-components A_1,A_2, $A_3$and we want to evaluate the probability of failure R of component A table 2 shows the linguistic values R_i and w_i of the evaluating items"probability of failure" and "severity of loss" of the sub-components A_1,A_2andA_3, respectivelySchmucker, (1984), where the linguistic values are represented by generalized trapezoidal fuzzy number as shown in table 3.
In the following, we use the proposed fuzzy risk analysis algorithm to deal with the fuzzy risk analysis problem.
Step1: Based on Eq.(19), tables 1 and 2, the probability of failure R of the component A can be calculated as follows:
R = [low ⊗low⊕ Medium ⊗ fairly – high ⊕high⊗ very – low] ∅ [low ⊕ fairly – high ⊕ very – low]
= [0.1614 , 0.2683 , 0.7052 , 1.095 ;1.0].

Table 2. linguistic values of the evaluating items R_i and w_i of the three sub-components A_1,A_2andA_3

sub-components A_i	linguistic values $w_i$of the severity of loss	linguistic values $R_i$of the probability of failure
A_1	Low	Low
A_2	Fairly – high	Medium
A_3	Very – low	High

Step2: We can divide fuzzy number calculated at step.1, by to parts, upper and lower, by using α – cuts. The obtained results can be seen in table 3.
Step3: Based on Eqs. (15-18) we can calculate perimeters of fuzzy numbers obtained in previous step as follows:
$\forall\ \alpha = 0.1,$

$$+(r_6 - r_1) = (r_5 - r_2) + +\sqrt{(r_5 - r_6)^2 + {w_{R_1}}^2}\sqrt{(r_1 - r_2)^2 + {w_{R_1}}^2} = P_{0/1}(R_1)$$

$$\sqrt{(1.056 - 1.095)^2 + (0.1)^2} + \sqrt{(0.1614 - 0.172)^2 + (0.1)^2} + (1.056 - 0.172) + (1.095 - 0.1614) = 2.0248.$$

$$+(r_5 - r_2) = \quad (r_4 - r_3) + +\sqrt{(r_4 - r_5)^2 + (w_{R_2} - w_{R_1})^2}\sqrt{(r_2 - r_3)^2 + (w_{R_2} - w_{R_1})^2} = P_\lambda(A_2)$$

$$+(1.056 - 0.172) = 3.192.\ (0.705 - 0.268) + +\sqrt{(0.705 - 1.056)^2 + (0.9)^2}\sqrt{(0.172 - 0.268)^2 + (0.9)^2}$$

We can calculate perimeters of other fuzzy numbers obtained in table 3 similarity. There are results shown in table 4.

Table 4. R perimeters of fuzzy numbers

α – cut	0.1	0.3	0.5	0.7	0.9
$P(R_1)$	2.024	2.429	2.659	2.978	3.292
$P(R_2)$	3.192	2.676	2.161	2.646	1.131

Step4: In this step, each fuzzy numbers shown in table 1 are divided two parts by arbitrary α – cuts. Then, by Eqs.(15-18), We can calculate the perimeters of each of the obtained numbers.
In the follow, we can calculate the perimeters of Absolutely – low fuzzy number.
In the same manner, the perimeters of other fuzzy numbers presented in table1 can be calculate. These result are shown in table 5.

Table 5.perimeters of fuzzy numbers Table 1

α – cut	0.1	0.3	0.5	0.7	0.9
$(P(A_1), P(A_2))$	(0.2,1.8)	(0.6,1.4)	(1,1)	(1.4,0.6)	(1.8,0.2)
$(P(B_1), P(B_2))$	(0.335,1.886)	(0.72,1.47)	(1.115,1.06)	(1.5,1.05)	(1.89,0.24)
$(P(C_1), P(C_2))$	(0.574,2.067)	(0.34,1.63)	(1.32,1.21)	(1.704,0.793)	(2.083,1.37)
$(P(D_1), P(D_2))$	(0.685,2.177)	(1.061,1.75)	(1.45,1.34)	(1.82,0.9)	(2.2,0.49)
$(P(E_1), P(E_2))$	(0.84,2.285)	(1.212,1.85)	(1.58,1.43)	(1.95,0.99)	(2.31,0.55)
$(P(F_1), P(F_2))$	(0.74,2.232)	(1.121,1.813)	(1.48,1.39)	(1.87,1.76)	(2.26,0.54)
$(P(G_1), P(G_2))$	(0.69,2.182)	(1.07,1.75)	(1.44,1.33)	(1.82,0.91)	(2.2,0.49)
$(P(H_1), P(H_2))$	(0.34,1.89)	(0.73,1.48)	(1.12,1.07)	(1.51,0.66)	(1.9,0.25)
$(P(I_1), P(I_2))$	(0.2,1.8)	(0.6,1.4)	(1,1)	(1.4,0.6)	(1.8,0.2)

Step5: We can calculate the similarity measureS_λ (A,B) between fuzzy number R and other fuzzy numbers presented in table 1 for different λ.
Therefore, we can calculate the similarity between measure trapezoidal fuzzy number R and fuzzy numbers shown in table 1. These results are shown in table 5.
From table 5, we can see that (s (R , Medium)) for different λ has the highest value, Therefore, the trapezoidal fuzzy number R can be transformed linguistic term medium. This means that the probability of failure of section A produced by manufactory C is to be medium. This result is the same as Schmucker [3] method.

Conclusion

In this paper, we have introduced a new similarity measure method for fuzzy numbers. Also, we can demonstrate some properties of the proposed similarity measure method. In the follow, we also have applied fuzzy risk analysis algorithm for the proposed method to deal with fuzzy risk analysis problems. The proposed method provides us with a useful way to deal with fuzzy risk analysis problems.
According to this algorithm based on the proposed index, we can obtain different result for different λ. Therefore, decision maker can make different decision selecting different λ.

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