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Constructing the Minimal Steiner Tree Inside Simple Polygon

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ABSTRACT

The Steiner tree problem has numerous applications in urban transportation network, design of electronic integrated circuits, and computer network routing. This problem aims at finding a minimum Steiner tree in the Euclidean space, the distance between each two edges of which has the least cost. This problem is considered as an NP-hard one. Assuming the simple polygon P with m vertices and n terminals, the purpose of the minimum Steiner tree in the Euclidean space is to connect the n terminals existing in p. In the proposed algorithm, obtaining optimal responses will be sought by turning this problem into the Steiner tree problem on a graph.

KEYWORDS: Euclidean minimum steiner tree, Delaunay triangulation, Steiner tree in graph.

1. INTRODUCTION

The Steiner tree problem has applications in scientific and commercial fields such as computer networks, design of electronic integrated circuits, path finding for electrical power distribution and also for means of transportation in order for the distribution of goods [1, 2]. Moreover, several breakthroughs have been achieved in recent years in the field of cruise systems embedded within means of transportation. In these instances, there is a need for finding the shortest precise path from the start point to the determined destination or destinations. One of the best approaches for this is to use the Steiner tree. On the whole, different modes of this problem may be categorized into the three following general groups:

1. The Steiner tree problem in the Euclidean space: Having a set P of N points on the plane, the tree connecting the points of P and a set of points Q is called a Steiner tree. The points of P are called terminals and the points of Q are called Steiner points. A Steiner tree with minimum length is called a Steiner minimum tree.

2. The Steiner tree problem on graph: Consider the graph G(V,E), where V is the set of nodes and $e \subset v * v$ denotes the set of edges of the graph. A cost c(j,i) is attributed to each edge (j,i). A subset of nodes called T is also defined as the set of terminals. The purpose of the Steiner tree is to find a tree on the graph which embraces the set of terminals and has the minimum cost. This problem is used for implementing the transfer of multiple sections in computer networks.

3. The rectilinear Steiner tree problem: A set of n points in the space are considered. The goal is to find a Steiner tree with rectilinear edges which connect these points. This problem has applications in the physical design of electronic integrated circuits.

Cohon et al. investigated a case in which there are N points on the exterior border of a rectilinear polygon [3]. Furthermore, Julstrom presented a method for obtaining the Steiner tree for N points within a rectilinear polygon using the genetic algorithm technique. This algorithm required the time O(n). In this paper, an algorithm for constructing the Euclidean Steiner tree within a simple polygon is proposed. In the end, we compare our results on the Euclidean plane with those presented in [5]. The paper is organized as follows: In section 2, construction of the Steiner tree for three points is presented. In section 3, the proposed algorithm is explained. In section 4, the calculation results are presented. The final section is the conclusion.

2. Construction of the Steiner tree for three points

Torricelli came up with a solution for three points in 1640 [6]. In this solution, the three points are labeled A, B, and C. If we connect them, we have a triangle. If we build three equilateral triangles outside of the ABC triangle, each of which has AB, AC, and BC as one of its sides and we inscribe each of these triangles within a circle, we will have Figure 1.

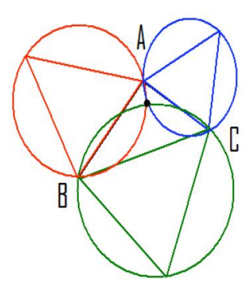

Figure 1. Torricelli's solution for three points

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The intersection point of these three circles is the Steiner point we seek which is called the Torricelli point [7]. In 1750, Simpson presented another solution in order to find the Torricelli point [7]. Similar to the previous method, the equilateral triangles outside of the ABC triangle are built. Afterward, the Simpson line is drawn between the vertices of the equilateral triangles. The intersection point of these three lines is the Torricelli point [6]. See Figure 2.

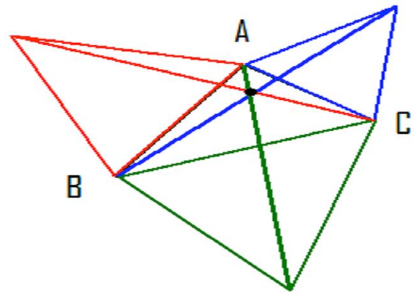


Figure 2. Simpson's solution for three points

The Steiner point is obtained from the intersection of three edges in Figure 2 which are at an angle of 120 degrees with each other. This condition is called the angular condition of the Steiner point (Figure 3).

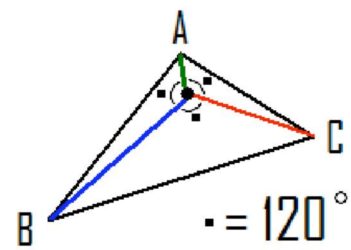
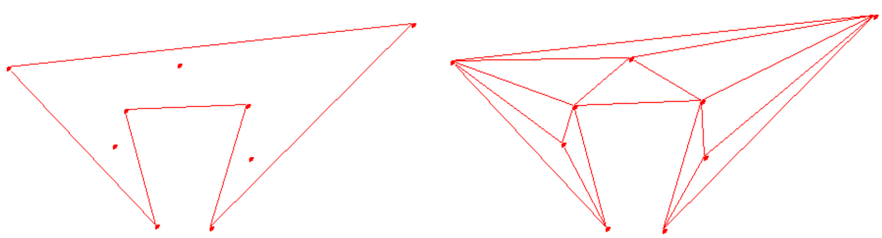


Figure 3. The edges connected to the Steiner point are at an angle of 120 degrees with one another.

3. The Proposed Algorithm

The proposed algorithm includes 3 steps:

Step 1: The set of terminals and vertices of the polygon of Figure 4-a are triangulated using the triangulation algorithm [8]. The result is Figure 4-b.



(a) (b)

Figure 4. Triangulation of the set of vertices and terminals of the polygon

Step 2: In each triangle constructed in the previous step whose angles are less than 120 degrees, we obtain the Steiner point using Torricelli or Simpson method (Figure 5).

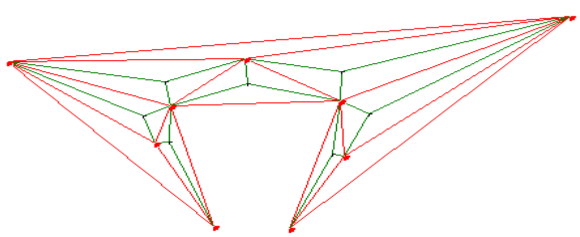


Figure 5. The Steiner point obtained in each triangle

Step 3: We obtain the Steiner tree on the graph using the algorithm of Milan et al. [9] (Figure 6).

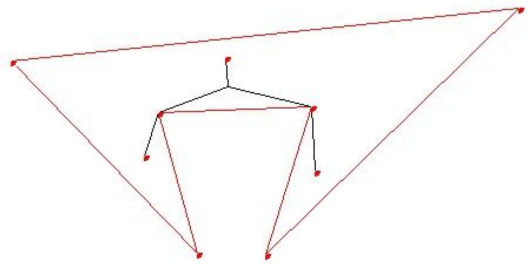


Figure 6. Steiner tree for three terminals

4. Calculation Results

The proposed algorithm was implemented using the Delphi programming language. The experiments were conducted using examples from Soukup [5]. A convex polygon was considered around all terminals. In Table 1, a number of the implemented results are compared with optimum results demonstrating the fact that the proposed algorithm has presented acceptable results.

Table 1. Our algorithm compared to Soukup's examples

Example Number	Optimum result	Our proposed algorithm
EX.2	150.05	150.50
EX.2A	207.77	208.03
EX.3	159.88	159.88
EX.11	382.80	382.80
EX.15	50.329	51.24
EX.23	76.603	77.43
EX.30	193.95	193.99

5. Conclusion

In this paper, the proposed algorithm is able to solve the Steiner tree problem within a simple polygon on the Euclidean plane. The calculation results of the mentioned algorithm are easy in terms of implementation and they lead to acceptable results.

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