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Free Vibration Analysis of Cable Using Adomian Decomposition Method

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ABSTRACT

Adomian decomposition method is employed to investigate the free vibration of cable consisting of two uniform sections. Each section is considered a substructure which can be modeled using ADM. applying boundary conditions and continuity condition, the dimensionless natural frequencies and corresponding mode shapes may be obtained simultaneously. Computed results for different boundary conditions are presented which agree well with those analytical ones given in the literature. Also the effect of amount of mass and spring stiffness at the boundary on variation of natural frequencies is obtained and an effective procedure is suggested for encountering with these kinds of problems.
KEYWORDS: Cable; Adomian Decomposition Method; Vibration; Natural frequency; Mode shape.

INTRODUCTION

Nonlinear phenomena play an important role in applied science. Many of the analytical solution methods to solve nonlinear differential equations use linearization method or neglect the nonlinearity effects and assume they are relatively insignificant; however, this assumption can change the real solution of the mathematical model which describes the physical phenomena very seriously. Also numerical methods are based on discretization technique which is not effective in all states of time and space variable ranges.
A large class of analytical solution methods and numerical solution methods have been used to solve these problems such as Hirota' s bilinear method[1], The symmetry method[2], the Darboun transformation[3], the Homotopy perturbation method[4-6], variational iteration method[7], Adomian decomposition method[8, 9] and other numerical methods.

In this study, a new computed approach called Adomian decomposition method (ADM) is introduced to solve the vibration problem of uniform cables. ADM have been proved to be effective and reliable for solving linear or nonlinear equations. [8, 10-16]. Recently, the ADM has been applied to the problem of vibration of mechanical and structural systems [17-21].

Unlike the perturbation method, ADM does not depend on any parameter and gets the solution as an infinite series which converges to the exact solution [22]. The chief advantage of ADM is that it does not need any linearization, descretization and any other restrictive assumptions; consequently the reality of the problem does not change due to linearization, also the solution is not affected by errors related to descretization. Another advantage of ADM is its rapid convergence. That it only needs a small number of terms of series to get an approximation of the solution with high accuracy [23, 24, 25]

In this article we have used this method to solve the transverse vibration of uniform cables which governs numerous engineering experimentations.

Using ADM, the two coupled governing differential equations become two recursive algebraic equations and the boundary conditions at the right and the left ends; become two simple algebraic frequency equations which are suitable for symbolic computation. Moreover, after some simple algebraic operations on these frequency equations, any i*th* natural frequency and the corresponding mode shape may be obtained one at the same time. Finally some problems of uniform cables are solved both analytically and by using ADM. Results obtained from ADM, show excellent agreement with analytical ones which verify the accuracy and efficiency of the method.

ADM has been the focus of many researches in the last decades. Ismail et al[26] used ADM to solve Burger's-Huxley and Burger's-Fisher equations. The Shakeri[27] and Dehghan have employed ADM to solve a system of two nonlinear integral-differential equations which has special importants in biology. These equations describe a model of biological species living.

Adomian and Ratch [28] introduced the phenomena of the so called noise terms. They concluded that noise terms always appear in inhomogeneous equations and if a term or terms in the part y_0 are cancelled by a term or terms in the part y_1, then the remaining terms in y_0 can get the exact solution y(x). This conclusion may be useful in reaching a fast convergence of the series of solutions.

Hosseini[29] and Nasabzadeh, have introduced an efficient modification of Adomian decomposition method for solving second order ordinary differential equations. They showed that the proposed method can be applied to singular and nonsingular problems.

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Wazwaz [13] has introduced the modified Adomian decomposition method to solve some kinds of nonlinear problems.

Nuguleswaran [30], obtained an approximate solution for the transverse vibration of a uniform Euler-Bernoulli beam under linearly varying axial force. The converges of method is discussed be Tatari and Dehghan [31].

2. METHODS

2-1. Adomian Decomposition Method

In this section, the ADM is briefly explained. Consider a general differential equation:

$$Ly + Ry = g(x) \tag{1}$$

Where L is an invertible operator which contains highest order of derivatives and R contains reminder orders of derivatives and $g(x)$ is the specific function. Solving for Ly, one can obtain:

$$y = \Phi + L^{-1}g - L^{-1}Ry \tag{2}$$

In Eq. (2), Φ is integration constant such that $L\Phi = 0$. For solving Eq. (2) by ADM, y can be written as series $y = \sum_{k=0}^{\infty} y_k$. Substituting this series into Eq. (2) yields:

$$\sum_{k=0}^{\infty} y_k = \Phi + L^{-1}g - L^{-1}R\sum_{k=0}^{\infty} y_k \tag{3}$$

In the above equation, by assuming $y_0 = \Phi + L^{-1}g$, the recursive formula is obtained as follows:

$$y_k = -L^{-1}Ry_{k-1} \qquad k \geq 1 \tag{4}$$

2.2. Cable vibration theorem

Consider a pre-tensioned uniform cable with length l. For this cable which has the constant tension P and is subjected to the lateral force $f(x,t)$, the governing equation of motion is as follows:

$$P\frac{\partial^2 w(x,t)}{\partial x^2} + f(x,t) = \rho\frac{\partial^2 w(x,t)}{\partial t^2} \tag{5}$$

Where $w(x,t)$ and ρ, are transverse displacement and density of the cable respectively, and t shows the time. For $f(x,t) = 0$, free vibration equation may be written as:

$$P\frac{\partial^2 w(x,t)}{\partial x^2} = \rho\frac{\partial^2 w(x,t)}{\partial t^2} \tag{6}$$

Or

$$c^2\frac{\partial^2 w(x,t)}{\partial x^2} = \frac{\partial^2 w(x,t)}{\partial t^2} \tag{7}$$

In which:

$$c = (\frac{P}{\rho})^{\frac{1}{2}} \tag{8}$$

Using separation of variables technique, Eq. (7) may be written as Eqs. (9) and (10).

$$\frac{d^2 y(x)}{dx^2} + \frac{\omega^2}{c^2}y(x) = 0 \tag{9}$$

$$\frac{d^2 T(t)}{dt^2} + \omega^2 T(t) = 0 \tag{10}$$

In which ω is the frequency of vibration.Eqs. (11) and (12) are deduced by solving Eqs. (9) and (10):

$$y(x) = A\cos\frac{\omega x}{c} + B\sin\frac{\omega x}{c} \tag{11}$$

$$T(t) = C\cos\omega t + D\sin\omega t \tag{12}$$

Constants A, B, C and D will be obtained from boundary/initial conditions.

3. Formulation of cable vibration by ADM method:

The cable shown in Fig.1 is considered. The cable is divided in two sections with the same length and the midpoint of the cable is considered as origin of the coordinate system.

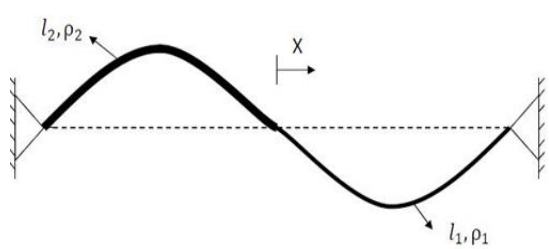

Figure 1. A two sectioned cable

Dimensionless equation of motion for each part of the cable can be written as:

$$\frac{d^2Y_i(X_i)}{dX_i^{\,2}} - \lambda_i Y_i(X_i) = 0 \qquad , \quad i = 1,2 \tag{13}$$

In which:

$$\lambda_i = -\frac{\omega^2 \rho_i l_i^2}{P_i}, \qquad X_i = \frac{x_i}{l_i}, \qquad Y_i = \frac{y_i}{l_i} \qquad , \quad i = 1,2 \tag{14}$$

Note that irefers to the section considered.

Operators L and R are defined as:

$$L_i Y_i = \frac{d^2Y_i(X_i)}{dX_i^{\,2}}, \qquad R_i Y_i = -\lambda_i Y_i(X_i) \qquad , \quad i = 1,2 \tag{15}$$

Applying ADM to the Eq. (13) we get:

$$\sum_{k=0}^{n-1} Y_{i,k}(X_i) = \phi_i + \lambda_i L_i^{-1} \sum_{k=0}^{n-1} Y_{i,k}(X_i) \qquad , \quad i = 1,2 \tag{16}$$

In which:

$$L_i^{-1} = \iint dX_i \, dX_i \qquad , \quad i = 1,2 \tag{17}$$

As mentioned before, the first term of the left side series is considered equal to Φ_i:

$$Y_{i,0}(X_i) = \phi_i = Y_i(0) + Y_i^{(1)}(0) X_i \qquad , \quad i = 1,2 \tag{18}$$

Therefore, the recursive relation is obtained as:

$$Y_{i,k}(X_i) = \lambda_i \int_0^X \int_0^X Y_{i,k-1}(X_i) dX_i dX_i \qquad , \quad i = 1,2 \tag{19}$$

Substituting Eq. (18) as the first term into Eq. (19) and expanding other terms, $Y_{i,k}(X_i)$ is obtained as follows:

$$Y_{i,k}(X_i) = \lambda_i L_i^{-1} Y_{i,k-1}(X_i) = \lambda_i^k \sum_{m=0}^{1} \frac{X_i^{(2k+m)}}{(2k+m)!} Y_i^{(m)}(0) \qquad , \quad i = 1,2 \tag{20}$$

After achieving the general relations of each equations, $Y_i(X_i)$ can be approximated as follows:

$$Y_i(X_i) \approx \sum_{k=0}^{n-1} \lambda_i^k \sum_{m=0}^{1} \frac{X_i^{(2k+m)}}{(2k+m)!} Y_i^{(m)}(0) \qquad , \quad i = 1,2 \tag{21}$$

Continuity conditions at midpoint are expressed as:

$$y_1(0) = y_2(0) \qquad , \qquad y_1^{(1)}(0) = y_2^{(1)}(0) \tag{22}$$

According to Eq. (14), dimensionless Eq. (22) would be:

$$Y_2(0) = \frac{1}{\alpha} Y_1(0) \qquad , \qquad Y_2^{(1)}(0) = Y_1^{(1)}(0) \tag{23}$$

In which:

$$\alpha = \frac{l_2}{l_1} \tag{24}$$

Substituting Eq. (23) into Eq. (21) yields:

$$Y_1(X_1) \approx \sum_{k=0}^{n-1} \lambda_1^k \sum_{m=0}^{1} \frac{X_1^{(2k+m)}}{(2k+m)!} Y_1^{(m)}(0) \tag{25}$$

$$Y_2(X_2) \approx \sum_{k=0}^{n-1} \lambda_2^k \sum_{m=0}^{1} \alpha^{m-1} \frac{X_2^{(2k+m)}}{(2k+m)!} Y_1^{(m)}(0) \tag{26}$$

Applying boundary conditions at both ends, a homogenous system of equations with two unknowns is got. In order to have a nontrivial solution for the system of equations, the determinant of the matrix of coefficients must be zero. The natural frequencies of system can be obtained by solving the resultant equation. By substituting any *i*th natural frequency into the aforementioned homogenous system of equations, the corresponding eigen vectors is calculated. The *i*th mode shape is also gained by substituting these vectors into Eqs. (25) and (26).

4. Examples

4.1. Free vibration of a cable with both ends pinned

4.1.1. Analytical solution

According to Fig.2, boundary conditions are:

$$w(0,t) = w(l,t) = 0 \tag{27}$$

Or

$$y(0) = y(l) = 0 \tag{28}$$

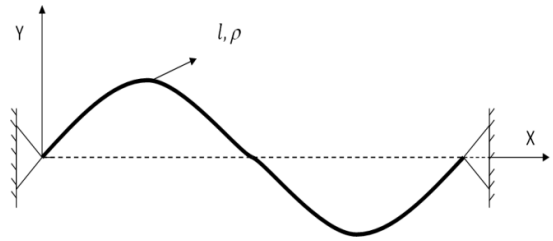

Figure 2. The uniform cable with both ends pinned used in analytical solution

Frequency equation is obtained by applying Eq. (28) to Eq. (11):

$$\omega_n = \frac{nc\pi}{l}, \qquad , \qquad n = 1,2,..... \tag{29}$$

4.1.2. Solution using ADM

As it is shown in Fig. 3, dimensionless boundary conditions are:

$$Y_1(1) = 0, \qquad , \qquad Y_2(-1) = 0 \tag{30}$$

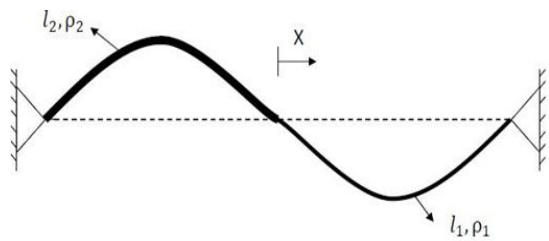

Figure 3. The cable with both ends pinned used in ADM solution

Applying Eq. (30) to Eqs. (25) and (26), matrix form of governing equations will be got as follows:

$$\begin{bmatrix} \sum_{k=0}^{n-1} \frac{(-\eta_1)^k \omega^{2k}}{2k!} & \sum_{k=0}^{n-1} \frac{(-\eta_1)^k \omega^{2k}}{(2k+1)!} \\ \sum_{k=0}^{n-1} \frac{(-\eta_2)^k \omega^{2k}}{\alpha(2k!)} & -\sum_{k=0}^{n-1} \frac{(-\eta_2)^k \omega^{2k}}{(2k+1)!} \end{bmatrix} \cdot \begin{Bmatrix} Y_1(0) \\ Y_1^{(1)}(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{31}$$

In which:

$$\lambda_i = -\frac{l_i^2 \rho_i \omega^2}{P_i} = (-\eta_i)\omega^2 \qquad , \qquad i = 1,2 \tag{32}$$

Assuming $P = 4000N, \rho_1 = \rho_2 = 5Kg/m, l_1 = l_2 = 1m,$ natural frequencies can be calculated**.** They are presented in table 1 along with the exact solutions. The value of n in the table is chosen arbitrary. According to table 1, it is found that the number of obtained frequencies and their accuracy is increased and converged to the exact value by increasing the value of n. The three first mode shapes are shown in Fig. 4.

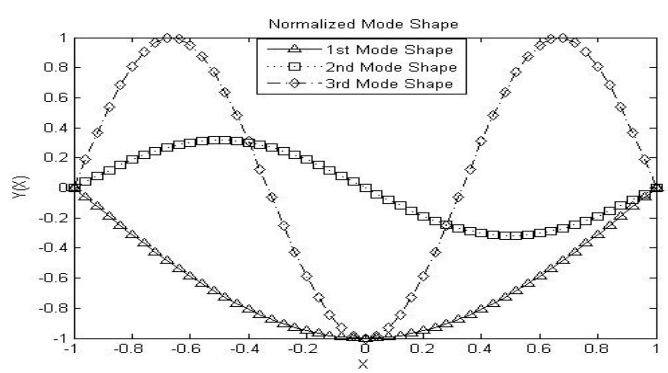

Figure 4. Three first mode shapes of the cable with both ends pinned

Table 1. Natural frequencies of the cable with both ends pinned

n	ω_1 (rad/s)	ω_2 (rad/s)	ω_3 (rad/s)	ω_4 (rad/s)	ω_5 (rad/s)
3	40				
4	45.0413	87.0131			
8	44.4288	88.8582	132.5547	169.0933	
15	44.4288	88.8576	133.2864	177.7153	222.1431
18	44.4288	88.8576	133.2864	177.7153	222.1441
exact	44.4288	88.8576	133.2864	177.7153	222.1441

4.2. Free vibration of a cable with one end pinned- one end free

4.2.1. Analytical solution

According to Fig.5, one end at *x=0* is fixed and its displacement is set to zero. Another end is pinned to a joint which can freely moves along vertical direction. This end cannot sustain lateral force. Thus the boundary conditions would be:

$$w(0,t)=0 \quad , \quad \frac{\partial w(l,t)}{\partial x}=0 \tag{33}$$

Or

$$y(0)=\frac{dy(l)}{dx}=0 \tag{34}$$

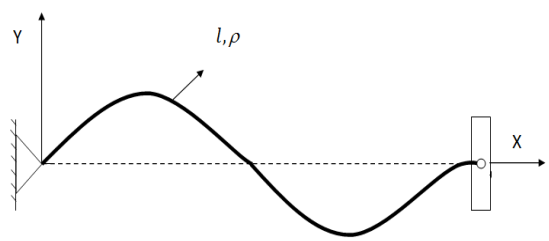

Figure 5. The uniform cable with one end pinned- one end free used in analytical solution

Applying Eq. (34) to Eq. (11), we get frequency equation as:

$$\omega_n=\frac{(2n+1)c\pi}{2l} \quad , \quad n=0,1,2,..... \tag{35}$$

4.2.2. Solution using ADM

Dimensionless boundary conditions as shown in Fig.6 are:

$$Y_1^{(1)}(1)=0 \quad , \quad Y_2(-1)=0 \tag{36}$$

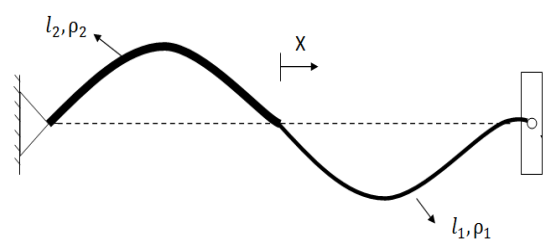

Figure 6. The cable with one end pinned- one end free used in ADM solution

$Y_1^{(1)}$ can be calculated by differentiating from Eq. (25). Matrix form of governing equations can be presented as Eq. (37) by applying Eq. (36) to Eqs. (25) and (26).

$$\begin{bmatrix} \sum_{k=1}^{n-1}\frac{(-\eta_1)^k\omega^{2k}}{(2k-1)!} & \sum_{k=0}^{n-1}\frac{(-\eta_1)^k\omega^{2k}}{(2k)!} \\ \sum_{k=0}^{n-1}\frac{(-\eta_2)^k\omega^{2k}}{\alpha(2k!)} & -\sum_{k=0}^{n-1}\frac{(-\eta_2)^k\omega^{2k}}{(2k+1)!} \end{bmatrix} \cdot \begin{Bmatrix} Y_1(0) \\ Y_1^{(1)}(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{37}$$

Imagining scalar parameters of Ex. 1, natural frequencies and related mode shapes (three first mode shapes) are gained and shown in table 2 and Fig. 7, respectively.

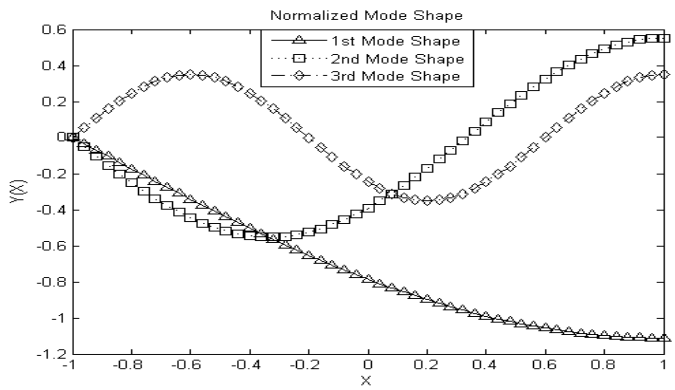

Figure 7. Three first mode shapes of the cable with one end pinned- one end free

Table 2. Natural frequencies of the cable with one end pinned- one end free

n	ω_1 (rad/s)	ω_2 (rad/s)	ω_3 (rad/s)	ω_4 (rad/s)	ω_5 (rad/s)
3	21.2973	58.1929			
4	22.2452	64.8998			
8	22.2144	66.6431	110.9562	154.7517	
10	22.2144	66.6432	111.0716	155.5296	
16	22.2144	66.6432	111.0720	155.5009	199.9297
exact	22.2144	66.6432	111.0720	155.5009	199.9297

4.3. Free vibration of a cable with one end pinned- one end sprung

4.3.1. Analytical solution

The cable is pinned at the left end and is connected to a spring with stiffness constant K at the other end as shown in Fig. 8. Boundary conditions would be:

$$w(0,t)=0 \quad , \quad P\frac{\partial w(l,t)}{\partial x}=-Kw(l,t) \tag{38}$$

Or

$$y(0)=0 \quad , \quad P\frac{dy(l)}{dx}=-Ky(l) \tag{39}$$

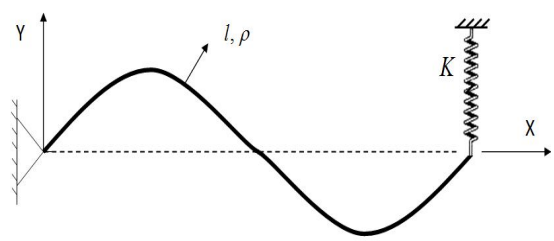

Figure 8. The uniform cable with one end pinned- one end sprung used in analytical solution

Natural frequencies can be obtained from solving the following frequency equation:

$$\tan\frac{\omega l}{c}+\frac{P\omega}{Kc}=0 \tag{40}$$

In which P is the constant tension of cable, K is stiffness constant and c is equal to $(P/\rho)^{0.5}$. It is evident that Eq. (40) is nonlinear which does not have an explicit solution and numerical techniques should be applied for solving it.

4.3.2. Solution using ADM

According to Fig. 9, boundary conditions are considered as follows:

$$PY_1^{(1)}(1)=-KY_1(1), \qquad , \qquad Y_2(-1)=0 \tag{41}$$

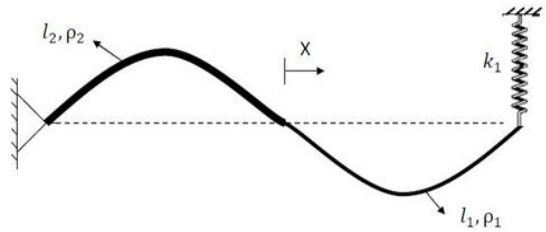

Figure 9. The cable with one end pinned- one end sprung used in ADM solution

Substituting Eq. (41) into Eqs. (25) and (26), we would have matrix form of governing equations:

$$\begin{bmatrix} \sum_{k=1}^{n-1}\frac{(-\eta_1)^k\omega^{2k}}{(2k-1)!}+\sum_{k=0}^{n-1}\frac{K_1}{P}\frac{(-\eta_1)^k\omega^{2k}}{(2k)!} & \sum_{k=0}^{n-1}\frac{(-\eta_1)^k\omega^{2k}}{(2k)!}+\sum_{k=0}^{n-1}\frac{K_1}{P}\frac{(-\eta_1)^k\omega^{2k}}{(2k+1)!} \\ \sum_{k=0}^{n-1}\frac{(-\eta_2)^k\omega^{2k}}{\alpha(2k)!} & -\sum_{k=0}^{n-1}\frac{(-\eta_2)^k\omega^{2k}}{(2k+1)!} \end{bmatrix}\begin{Bmatrix} Y_1(0) \\ Y_1^{(1)}(0) \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{42}$$

Considering scalar parameters of Ex. 1 and $K_1 = K = 16000 N/m$, natural frequencies and the three first mode shapes are obtained. The results are shown in table 3 and Fig. 10 respectively.

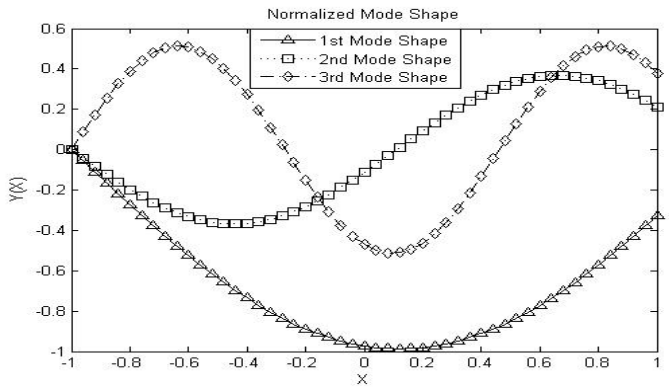

Figure 10. Three first mode shapes of the cable with one end pinned- one end sprung

Table 3. Natural frequencies of the cable with one end pinned-one end sprung

n	ω_1 (rad/s)	ω_2 (rad/s)	ω_3 (rad/s)	ω_4 (rad/s)	ω_5 (rad/s)
3	35.8928	64.2423			
5	39.6429	79.3406			
10	39.6605	80.1416	121.6634	164.0189	
15	39.6605	80.1416	121.6658	164.0395	207.0031
18	39.6605	80.1416	121.6658	164.0395	207.0034
approximate value	39.6605	80.1416	121.6658	164.0395	207.0034

Obtained results are validated by satisfying the natural frequencies derived from ADM in Eq. (40).

4.4. Free vibration of cable with one end fixed- one end mass

4.4.1. Analytical solution

As it is shown in Fig.11, the cable is pinned at right and connected to mass at left. So boundary conditions would be:

$$P\frac{\partial w}{\partial x}(0,t) = m\frac{\partial^2 w}{\partial t^2}(0,t) \quad , \quad w(l,0)=0 \tag{43}$$

Or

$$P\frac{dy}{\partial x}(0) = -m\omega^2 y(0) \quad , \quad y(l)=0 \tag{44}$$

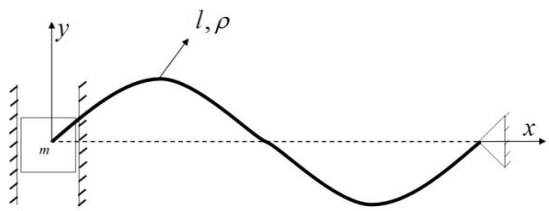

Figure 11. The cable with one end pinned-one end mass used in analytical solution

Applying boundary conditions to Eq. (11), gives the frequency equation as:

$$\tan\frac{\omega l}{c}-\left[\frac{P}{mc\omega}\right]=0 \tag{45}$$

4.4.2. ADM solution

According to Fig. 12, dimensionless boundary conditions are:

$$Y_1(1)=0 \qquad , \qquad PY_2^{(1)}(-1)=-m\omega^2 Y_2(-1) \tag{46}$$

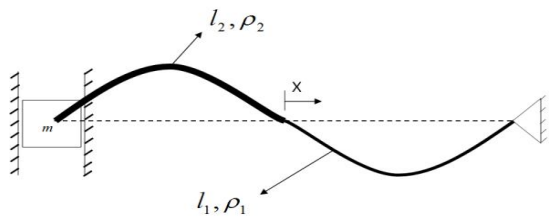

Figure 12. The cable with one pinned-one end mass used in ADM solution

Applying Eq. (66) to Eqs.(25) and (26), gives matrix form of governing equations as follow:

$$\begin{bmatrix} \sum_{k=0}^{n-1}\frac{(-\eta_1)^k\omega^{2k}}{(2k)!} & \sum_{k=0}^{n-1}\frac{(-\eta_1)^k\omega^{2k}}{(2k+1)!} \\ -\left[\sum_{k=1}^{n-1}\frac{(-\eta_2)^k\omega^{2k}}{\alpha(2k-1)!}-\sum_{k=0}^{n-1}\frac{M}{P}\frac{(-\eta_2)^k\omega^{2k+2}}{\alpha(2k)!}\right] & \sum_{k=0}^{n-1}\frac{(-\eta_2)^k\omega^{2k}}{(2k)!}-\sum_{k=0}^{n-1}\frac{M}{P}\frac{(-\eta_2)^k\omega^{2k+2}}{(2k+1)!} \end{bmatrix}\begin{Bmatrix} U^*_1(0) \\ U^{*(1)}_1(0) \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{47}$$

Assuming *m=10kg* and scalar parameters of the example 1, the natural frequencies and three first mode shapes are gained. The results are presented in table 4 and Fig.13, respectively.

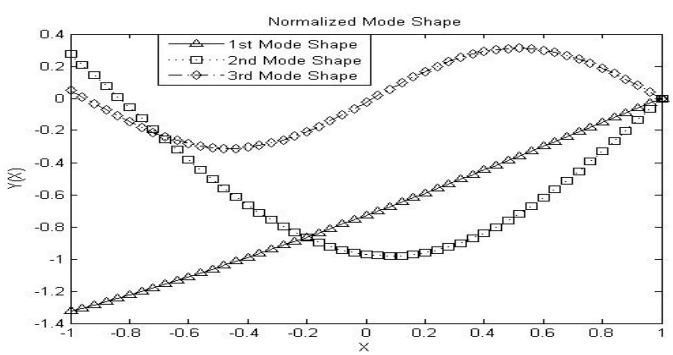

Figure 13. Three first mode shapes of the cable with one end pinned- one end mass

Table 4. Natural frequencies of the cable with one end fixed-one end mass

n	ω_1 (rad/s)	ω_2 (rad/s)	ω_3 (rad/s)	ω_4 (rad/s)	ω_5 (rad/s)
3	12.1673	48.9283	89.5102		
4	12.1669	48.4294	89.4344		
8	12.1669	48.4455	91.0371	134.8382	182.4680
12	12.1669	48.4455	91.0371	134.7651	178.8315
16	12.1669	48.4455	91.0371	134.7651	178.8313
approximate value	12.16695	48.4455	91.0371	134.7651	178.8313

4.5.Free vibration of a cable with one end free-one end mass

4.5.1. Analytical solution

As it is shown in Fig.14, the cable is free at left and connected to mass at right. So boundary conditions would be:

$$\frac{\partial w(0,t)}{\partial x}=0 \quad , \quad P\frac{\partial w}{\partial x}(l,t)=-m\ \frac{\partial^2 w}{\partial t^2}(l,t) \tag{48}$$

Or

$$\frac{dy(0)}{dx}=0 \quad , \quad P\frac{dy}{\partial x}(l)=m\ \omega^2 y(l) \tag{49}$$

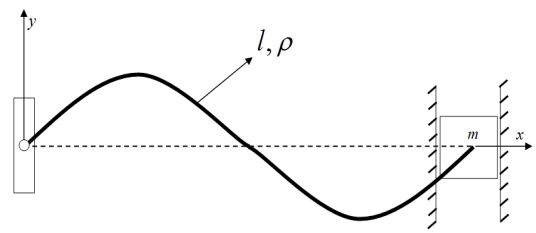

Figure 14. The cable with one end free-one end mass used in analytical solution

Applying boundary conditions to Eq. (11), gives the frequency equation as:

$$\tan\frac{\omega l}{c}+\left[\frac{mc\omega}{P}\right]=0 \tag{50}$$

4.5.2. ADM solution

According to Fig. 15, Dimensionless boundary conditions are:

$$PY_1^{(1)}(1)=m\ \omega^2 Y_1(1), \quad Y_2^{(1)}(-1)=0 \tag{51}$$

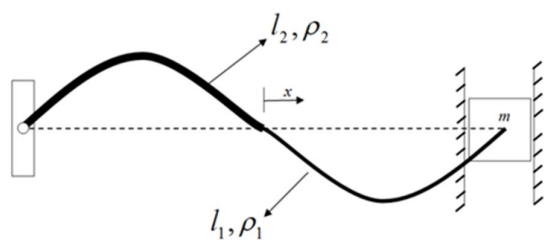

Figure 15. The cable with one end free- one end mass used in ADM solution

Applying Eq. (51) to Eqs. (25) and (26) yields:

$$\begin{bmatrix} \sum_{k=1}^{n-1}\frac{(-\eta_1)^k\omega^{2k}}{(2k-1)!}-\sum_{k=0}^{n-1}\frac{m}{P}\frac{(-\eta_1)^k\omega^{2k+2}}{(2k)!} & \sum_{k=0}^{n-1}\frac{(-\eta_1)^k\omega^{2k}}{(2k)!}-\sum_{k=0}^{n-1}\frac{m}{P}\frac{(-\eta_1)^k\omega^{2k+2}}{(2k+1)!} \\ -\sum_{k=1}^{n-1}\frac{(-\eta_2)^k\omega^{2k}}{\alpha(2k-1)!} & \sum_{k=0}^{n-1}\frac{(-\eta_2)^k\omega^{2k}}{(2k)!} \end{bmatrix}\begin{Bmatrix} Y_1(0) \\ Y_1^{(1)}(0)\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix} \tag{52}$$

Assuming *m=10kg* and scalar parameters of the example 1, the natural frequencies and three first mode shapes are obtained. The results are presented in table 5 and Fig.16, respectively.

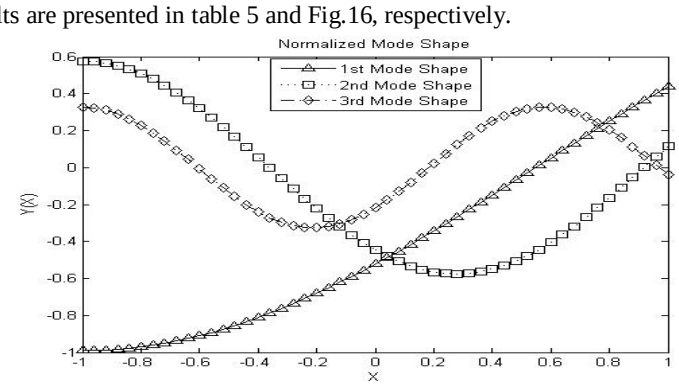

Figure 16. Three first mode shapes of the cable with one end free-one end mass

Table 5. Natural frequencies of the cable with one end free-one end mass

n	ω_1 (rad/s)	ω_2 (rad/s)	ω_3 (rad/s)	ω_4 (rad/s)	ω_5 (rad/s)
3	28.7502	65.7058			
4	28.6896	70.1187			
8	28.6909	69.4828	112.8448	156.3926	187.8959
12	28.6909	69.4828	112.8353	156.7731	200.9041
16	28.6909	69.4828	112.8353	156.7731	200.9234
approximate value	28.6909	69.4828	112.8353	156.7731	200.9234

5. Investigation of the effect of the spring stiffness and mass value at the boundary conditions on the value and variation of natural frequencies

5.1. Investigation of the effect of spring stiffness

5.1.1. Cable with both ends sprung

Consider a cable with both ends sprung as shown in Fig. 17 and suppose that $K_1 = K_2 = K$. For different values of *K*, the natural frequencies of the system is calculated and presented in table 6. Other scalar parameters are imagined identical to those provided in section 4.

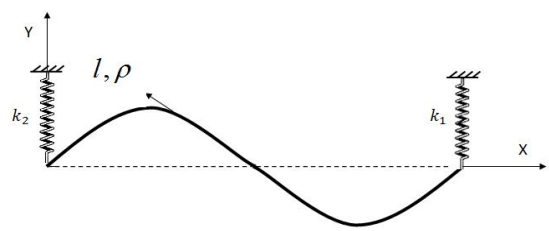

Figure 17. The cable with both ends sprung

Table 6. Natural frequencies of the both ends sprung cable for different values of spring stiffness

k	ω_1^*	ω_1	ω_2	ω_3	ω_4	ω_5
Free-Free		44.4288	88.8576	133.2864	177.7153	222.1441
1e-4	0.0045	0.00447	44.4288	88.857	133.2864	177.7153
1e-3	0.0141	0.0141	44.4288	88.8576	133.2864	177.7153
1e-2	0.0447	0.0447	44.4288	88.8576	133.2865	177.7153
1e-1	0.1414	0.1414	44.4292	88.8578	133.2866	177.7154
1	0.4472	0.4471	44.4333	88.8599	133.2879	177.7164
1e1	1.4142	1.4136	44.4737	88.8801	133.3014	177.7265
1e2	4.4721	4.4535	44.8744	89.0821	133.4363	177.8277
1e3	14.1421	13.5791	48.5218	91.0498	134.7691	178.8330
1e4	44.7214	32.3070	67.3347	105.5523	146.0403	187.8958
1e5	141.4214	42.7220	85.4559	128.2130	171.0040	213.8379
1e6	447.2136	44.2518	88.5036	132.7555	177.0074	221.2593
1e7	1.4142e+3	44.4110	88.8221	133.2331	177.6442	222.0553
1e8	4.4721e+3	44.4270	88.8541	133.2811	177.7082	222.1352
1e9	1.4142e+4	44.4286	88.8573	133.2859	177.7146	222.1432
1e10	4.4721e+4	44.4288	88.8576	133.2864	177.7153	222.1441
Fixed-Fixed		44.4288	88.8576	133.2864	177.7153	222.1441

In the second column, the first natural frequency is obtained from Eq. (53).

$$\omega_1^* = \sqrt{\frac{K_{eq}}{m}} \tag{53}$$

In which $K_{eq} = K_1 + K_2$ and m is the mass of the cable.

As it is evident from table 6, for $K_{eq} \prec 10^3$ or $\omega_1^* \prec 14$, Eq. (53) can be used for estimation of the first natural frequency. Also the i*th* natural frequency of the system is equal to the (i-1)*th* natural frequency of the cable with both ends free,for which the exact solution is available ($i \geq 2$).

Also for $K_{eq} \succ 10^5$ or $\omega_1^* \succ 140$, a both ends fixed cable can be assumed instead for achieving the natural frequencies. Fig. 18 shows the value of the first natural frequency obtained using ADM and the one obtained from Eq. (53) as a function of spring stiffness in logarithmic scale. Fig.19 shows the variation of the second, third, fourth and the fifth natural frequencies as a function of spring stiffness.

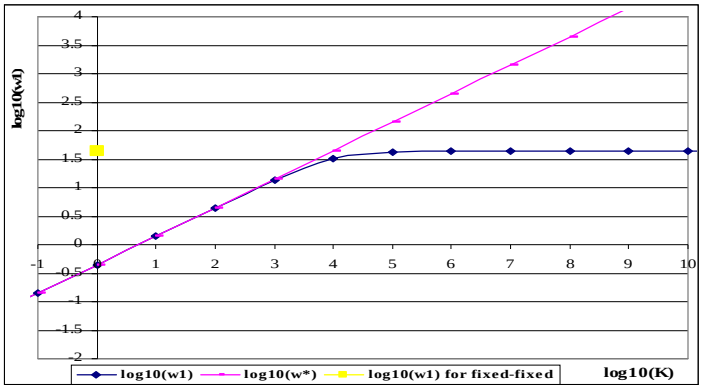

Figure 18. Comparison of the first natural frequency obtained from Eq. (53) and the ADM as a function of spring stiffness for the cable with both ends sprung

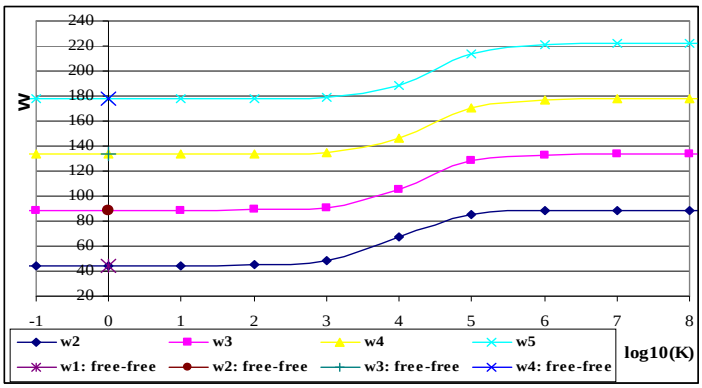

Figure 19. Variation of the second, third, fourth and the fifth natural frequencies as a function of spring stiffness for the cable with both ends sprung

5.1.2. Cable with one end fixed- one end sprung

Similar to the previous section, natural frequencies of the system is obtained as a function of spring stiffness and presented in table 7.

Table 7. Natural frequencies of the cable with one end fixed- one end sprung for different values of spring stiffness

k	ω_1	ω_2	ω_3	ω_4	ω_5
Fixed-Free	22.2144	66.6432	111.0720	155.5009	199.9297
1e-2	22.2144	66.6432	111.0720	155.5009	199.9297
1e-1	22.2148	66.6433	111.0721	155.5009	199.9297
1	22.2189	66.6447	111.0729	155.5015	199.9302
1e1	22.2593	66.6582	111.0810	155.5073	199.9347
1e2	22.6556	66.7929	111.1620	155.5651	199.9797
1e3	25.9734	68.1062	111.9640	156.1409	200.4284
1e4	37.5284	77.1362	118.6715	161.3423	204.6348
1e5	43.5587	87.1239	130.7016	174.2975	217.9164
1e6	44.3401	88.6803	133.0204	177.3606	221.7008
1e7	44.4199	88.8398	133.2598	177.6797	222.0997
1e8	44.4279	88.8558	133.2838	177.7117	222.1397
1e9	44.4287	88.8574	133.2862	177.7149	222.1437
1e10	44.4288	88.8576	133.2864	177.7152	222.1441
1e11	44.4288	88.8576	133.2864	177.7153	222.1441
Fixed-Fixed	44.4288	88.8576	133.2864	177.71531	222.1441

As it is evident from table 7, for $K \prec 10^3$ or $\omega_1^* \prec 10$, a cable with one end fixed-one end free can be assumed for calculating the natural frequencies of the system. Also for $K \succ 10^5$ or $\omega_1^* \succ 100$, the natural frequencies of the system are almost equal to the natural frequencies of the cable with both ends fixed. Figure 20, shows the variation of the first natural frequency andFigure 21shows the variation of the second, third, fourth and the fifth natural frequencies as a function ofspring stiffness.

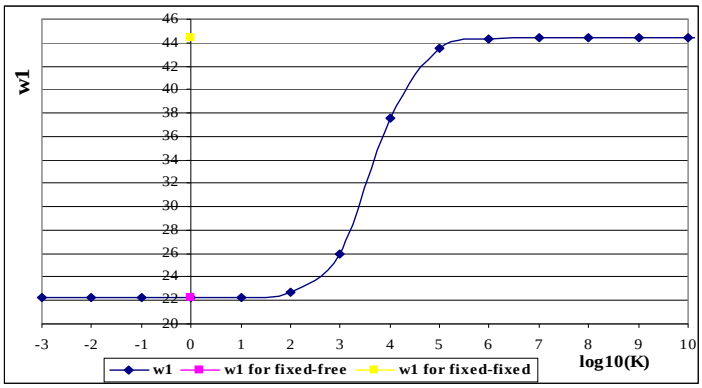

Figure 20. Variation of the first natural frequency as a function of spring stiffness for the cable with one end fixed- one end sprung

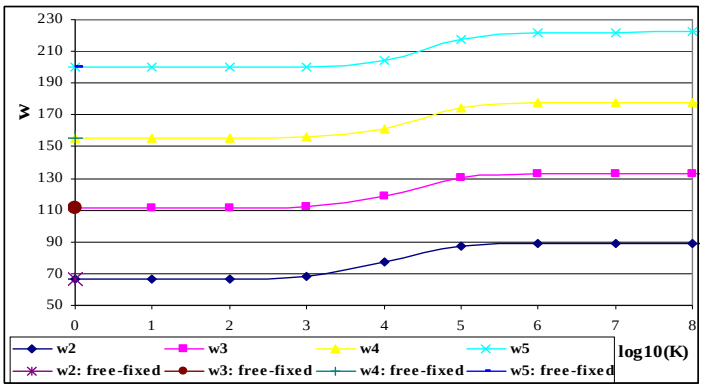

Figure 21. Variation of the second, third, fourth and the fifth natural frequencies as a function of spring stiffness for the cable with one end fixed- one end sprung

5.2. Investigation of the effects of mass value

5.2.1. Cable with one end fixed- one end mass

Similar to Fig. 12 in section 4, a cable with one end fixed- one end mass is considered. For different values of mass, the natural frequencies of the system is obtained and presented in table 8. In the second column of the table, the first natural frequency is calculated according to the Eq. (53), in which $K_{eq} = \frac{P}{l}$. As it is obvious from the table 8, for $m \succ 10^1$ or $\omega_1^* \prec 14$, Eq. (53) can be employed for calculating the first natural frequency. Also the i*th* natural frequency of the system is equal to (i-1)*th* natural frequency of the cable with both ends fixed which the exact solution is available ($i \geq 2$). Also for $m \prec 1$ or $\omega_1^* \succ 44$, a free-fixed condition can be considered instead.

Table 8. Natural frequencies of the cable with one end fixed- one end mass for different values of mass

m	ω_1^*	ω_1	ω_2	ω_3	ω_4	ω_5
Fixed-Free		22.2144	66.6432	111.0721	155.5009	199.9297
1e-4	4.4721e+3	22.2142	66.6425	111.0709	155.4993	199.9277
1e-3	1.4142e+3	22.2121	66.6365	111.0609	155.4853	199.9097
1e-2	447. 136	22.1922	66.5766	110.9611	155.3455	199.7300
1e-1	141.4214	21.9944	65.9838	109.9745	153.9672	197.9628
1	44.7214	20.2072	60.8932	102.2209	144.2535	186.8768
1e1	14.1421	12.1669	48.4455	91.0371	134.7651	178.8313
1e2	4.4721	4.3989	44.8743	89.0821	133.4363	177.8277
1e3	1.4142	1.4118	44.4738	88.8801	133.3014	177.7265
1e4	0.4472	0.4471	44.4333	88.8599	133.2879	177.7164
1e5	0.1414	0.1414	44.4292	88.8578	133.2866	177.7154
1e6	0.0447	0.0447	44.4288	88.8576	133.2865	177.7153
1e7	0.0141	0.0141	44.4288	88.8576	133.2864	177.7153
Fixed-Fixed		44.4288	88.8576	133.2864	177.7153	222.1441

Figure 22, compares the value of the first natural frequency obtained using ADM and obtained from Eq. (53) as a function ofmass in logarithmic scale. Figure 23, demonstrates the variation of the second, third, fourth and the fifth natural frequencies as a function of mass.

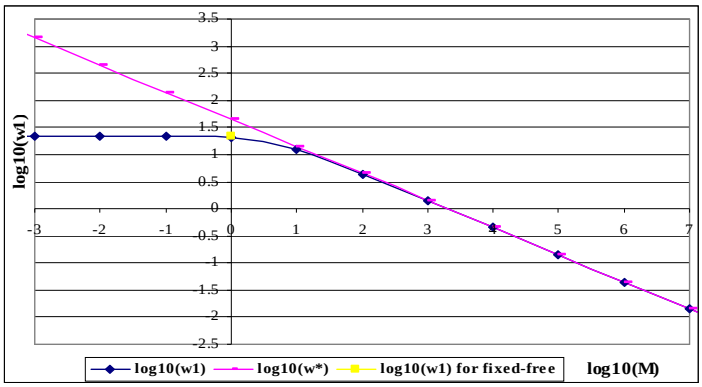

Figure 22. Comparison of the first natural frequency obtained from Eq. (53) and the ADM as a function of mass for the cable with one end fixed- one end mass

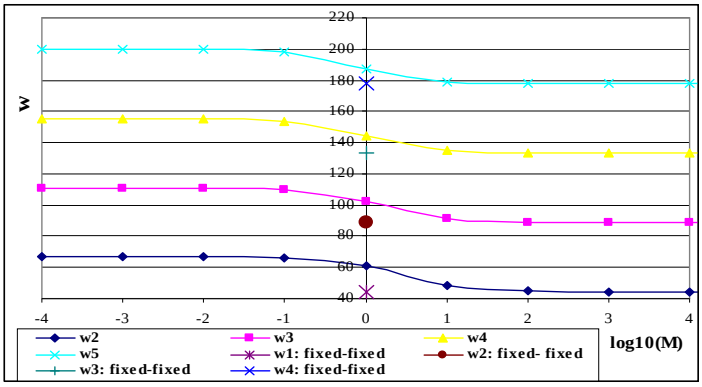

Figure 23. Variation of the second, third, fourth and the fifth natural frequencies as a function of mass for the cable with one end fixed- one end mass

6. RESULTS AND DISCUSSION

As it was shown in section 4, the series solution of the ADM converges to the exact value rapidly. Any natural frequencies and corresponding mode shapes would achieved by considering sufficient number of terms of the series solution.

Having analyzed the data in section 5, it was disclosed that for a large range of mass or spring stiffness, a fixed or free condition can be assumed instead. Also the approximate relation (53), generally gives an accurate result for the first natural frequency.

As examples, in the case of the cable with both ends sprung, for $\omega_1^* \prec 14$, the Eq. (53) can be used for calculating the first natural frequency. Also the second, third, fourth and the fifth natural frequencies, are the first, second, third and the fourth natural frequencies of the cable with both ends free respectively.

In the case of the cable with one end fixed- one end sprung, for $\omega_1^* \prec 10$, the natural frequencies can be approximated by the natural frequencies of the cable with one end fixed- one end free. Also for $\omega_1^* \succ 100$, the spring condition can be replaced by a fixed one.

For the cable with one end fixed- one end mass, Eq. (53) gives good results for $\omega_1^* \prec 14$. Also the second, third, fourth and the fifthnatural frequencies are approximately equal to the first, second, third and the fourth natural frequencies of the cable with both ends fixed respectively. Also for $\omega_1^* \succ 44$, a free- fixed condition can be assumed instead.

7. Conclusion

A cable with five different boundary conditions was analyzed by applying Adomian decomposition method. Using ADM, the governing differential equation is reduced to a recursive algebraic equation and the boundary conditions became simple algebraic frequency equations which are proper for symbolic computation. Any natural frequencies and corresponding mode shapes were obtained simultaneously by some simple mathematical operations.

This paper proposed an efficient procedure for analyzing vibration of cables with different boundary conditions. By increasing the number of terms, we got more natural frequencies and consequently more mode shapes with higher accuracy. The results with ADM are in excellent agreement with the analytical ones. However, for some cases, obtaining the natural frequencies from the frequency equation was not possible explicitly. In these cases the results were validated by satisfying the corresponding frequency equation.

It was revealed that if the amount of mass or spring stiffness considered in boundary conditions, gets an upper value from a limit, a fixed condition can be replaced instead. Also if it gets less, a free one can be considered instead.

Less computation effort and rapid convergence, in comparison with numerical methods,demonstrated that the ADM is a reliable and efficient procedure for solving physical differential equations.

It should be noted that the proposed method for the cable vibration analysis is generally applicable to arbitrary boundary conditions. On the basis of these results, the frequency equations derived in the present paper can be used in the design of cables with various boundary conditions.

- The authors have declared no conflict of interest.

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