A Multi-Period Model for Managing Used Products in Green Supply Chain Management under Uncertainty

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ABSTRACT

Green supply chain management in consequence of global warming, in particular, closed-loop logistics, has drawn the attention of researchers. Moreover, accounting of uncertainty in supply chain network design will lead to more realistic problems as a result of inherent uncertain parameters such as customer demand in establishment of closed-loop logistics network design problem. This paper proposes a \textit{multi-period-stage} closed-loop logistics network design under risk incorporating the strategic network design decisions along with tactical material flow to avoid sub-optimalities led from separated design in both parts. The proposed network structure includes suppliers, plants, distribution centers in forward direction and collection/inspection centers, and redistribution centers in reverse direction. The demands of first market customer zones are assumed stochastic. The problem is formulated in a mixed integer non-linear programming (SMINLP) decision making form as a multi-stage stochastic program with objective function maximizing the total expected profit.

KEYWORDS: Closed-loop supply chain network design, Logistics, Manufacturing/remanufacturing, Multi-stage stochastic programming.

INTRODUCTION

In order to agree to the environmental legislation, companies in many industries find themselves facing the challenge of developing their reverse logistics capabilities. In the other words, as a result of the increasing stringent pressures from social requirements, more and more manufacturers have adopted the practice of using returned products and incorporated product recovery activities into the manufacture. Performing of the reverse logistics operations needs set up additional appropriate logistics infrastructure for the arising flows of used and recovered products. Supply chain has been considered as a line, starting with the movement of goods from suppliers to manufacturers, and going ahead with wholesalers, retailers, and finally reaching consumer via distribution channels. Nowadays, by increasing the interaction between the ends of this line, supply chain designs tend to become circular to form the closed-loop rather than being linear. Therefore, industrial relationships prove the existence of material flows not only downstream but also upstream during the manufacture, distribution, and consumption stages.

Forward supply chain management covers all business functions connected to the flow of goods starting from supplier to ultimate customer, while reverse supply chain management covers the opposite flow which is oriented towards recovery processes starting with collection of end-of-life products from ultimate customers. Briefly, the forward supply chain is associated with the reverse supply chain by closing the material flow regarding environmental and economic worries. In the broadest sense, Reverse logistics consist of collection, inspection, recycling, refurbishing, and remanufacturing of used or returned products.

More specifically, Guide &Pokharel [1, 2] suggested some indicative strategies in establishing effective reverse logistics procedures: first of all, location of facilities close to the sources of used products, to disposal sites or even customers, Second, increasing availability of resources for reprocessing, as an illustration by exploiting products with small life cycles and large return rates, by accomplishing fast return of used products. Third, reducing the structure complexity of products, probably through re-engineering of the production process, and the last but not least, is balancing returns with demand and reducing inventories of used products to counterbalance their quick depreciation.

Moreover, considering uncertainty due to real world problems almost invariably cover some uncertain parameters, is a natural extension of a deterministic approach resulting in the more realistic problems in supply chain network design. Therefore, to cope with this uncertainty stochastic programming might be a proper approach. In stochastic programming approach, decisions need to be made prior to obtaining complete information. Stochastic programming is a structure for modeling optimization problems involved uncertainty while deterministic optimization problems are formulated with certain parameters.

The remainder of this paper is organized as follows. Most recent studies in closed-loop logistics network design problem is presented in section II. The concerned problem is defined in section III. The proposed stochastic mixed integer non-linear...
programming model is elaborated in section IV. Computational experiments are reported in section V and finally the concluding remarks as well as some future directions for future research are given in section VI.

I. LITERATURE REVIEW

Many models are developed for logistics network design founded upon facility location theory[3]. The majority of the literature about logistics network design contemplates on various facility location models base on MILP[4]. These models include a range of models from simple uncapacitated facility location models to more complex such as capacitated multi-objective models [5]. A general review of supply chain design model is presented by Dullaert et al [6] to support the development of supply chain models for future research directions. A smaller part of the literature is associated with reverse logistics network design and recently, a few papers have attended to integrated logistics network design. In integrated approach, the aim to integrating forward and reverse decisions is to avoid sub-optimality resulting from separated design of forward and reverse network. In the field of forward logistics network, many models have been proposed for variety of networks. A MILP model for a production-distribution network is proposed by Yeh[7] and to solve the intractable model an efficient hybrid heuristic is developed. As far as most of research in logistics network design were confined to considering a single capacity level for each facility, Amiri[8] proposed a MILP model for a multi-stage forward network with considering multiple capacity levels for each facility.

In last decade, various models have been developed in the context of reverse logistics. Jayaraman et al [9] proposed a MILP model for reverse logistics network design under a pull system on the basis of costumers’ demand for recovered products with objective function minimizing the total costs. In accordance to the reverse logistics literature, incorporating uncertainty in the network design is unavoidable. Both markets for forward products and supply of used products by customers involve many uncertainties such as the uncertainty of the quantity and quality of the returned products. Moreover, Fleischmann et al [10] also pointed out that uncertainty is an inevitable characteristic of product recovery. Hence, this issue seems to deserve additional research effort. To this end, Listes and Dekker [11] develop a stochastic mixed integer programming model in a sand recycling network maximizing the total profit. They developed their model for different situations considering several scenarios. Salema et al [12] developed a stochastic model for multi-product networks under uncertain demand using stochastic mixed integer programming. The proposed capacitated reverse logistics network includes capacity constraints imposing on the total production/storage capacity of facilities. However, it is also mentioned that as the problem size increases, the computational burden might be grow accordingly. Therefore, Santos et al [13] applied a sampling strategy on the basis of crude Monte Carlo samples to deal with such stochastic large-scale network design problem. Qi & Shen[14] developed a stochastic closed-loop supply chain network design model that takes into consideration location, inventory and routing costs.

Uster et al [15] presented a multi-product forward/reverse logistics network design model and they solved the model using bender decomposition approach which manufacturing and remanufacturing was considered separately and a single sourcing was assumed for customers. A closed-loop MILP was developed by Kannan at al [16] to determine raw material, production, distribution and inventory, disposal and recycling at different facilities and in order to minimizing the total supply chain costs, they proposed a heuristics based genetic algorithm for their model. Pishvaee[17] suggested a bi-objective integrated forward/reverse supply chain design model in which the cost and responsiveness were considered as objective of the model. A dynamic reverse logistics network design under demand uncertainty was presented by Lee and Dong [18] with a simulated annealing (SA) to obtain solution.

A. Conclusion

Reviewing the above mentioned literature on closed-loop logistics, the area of integrated forward/reverse logistics network design still requires intensive research. Moreover, the literature of uncertainty in closed-loop logistics network design is scarce.

In the present study, we address the area of integrated combined manufacturing/remanufacturing system within an integrated forward/reverse supply chain subject to uncertainties. To meet this end, a multi-period multi-stage integrated forward/reverse logistics network is formulated in a stochastic mixed integer non-linear programming with objective function maximizing total expected profit.

II. PROBLEM DEFINITION

The integrated forward/reverse logistics network discussed in this paper is a multi-period multi-stage logistics network including suppliers, plants, distribution centers and first market customer zones in forward direction and collection/inspection, redistribution centers, disposal centers, and second market customer zones in reverse direction. As illustrated in Fig.1 in forward direction, the suppliers are responsible for providing raw material for plants. The new products are shipped from plants to first market customer zones via distribution centers to meet the customer demands. In reverse direction, the returned products are collected in collection/inspection centers, after inspection; the returned products are divided into four groups: recyclable products, manufactureable products, repairable products and scrapped products. Recyclable products are shipped to suppliers for recycling. In continues, manufactureable products are shipped to plants for manufacturing. It should be noted that those returned products needing trivial repairing (repairable products) are repaired in inspection center and then they are shipped to redistribution centers and finally scrapped products are shipped to disposal centers for safe disposing.
Respectively, by means of this strategy returned products are directly transferred to relevant facility. Remanufactured and repaired products are conveyed to second market customer zones via redistribution centers to meet their demand. In concordance to mentioned flow, the following assumptions and limitations are considered in the present model:

1) The location of first and second market customer zones is fixed and predetermined.
2) First market customers’ demand is stochastic.
3) The quantity of returned products is stochastic depending on the first market customer demand.
4) Remanufacturing and repaired products have different quality in comparison with new ones.
5) The potential location of suppliers, plants, distribution, collection/inspection, disposal centers and redistribution centers are known.
6) Cost parameters are known for each location and period.
7) Predefined percentages are determined as an average recycling, remanufacture and disposal rates.

Under above situation, the main issues to be addressed by this study are to choose the location and determine the number of suppliers, plants, distribution centers, collection/inspection centers, redistribution centers and disposal centers and to determine the quality of flow between network facilities.

III. MODEL FORMULATION

To describe the aforementioned logistics network, the following notations are used in the model formulation of the SMINLP model.

Indices
- $SU$: set of potential supplier center locations $su \in SU$
- $PL$: set of potential plant center locations $pl \in PL$
- $DI$: set of potential distribution center locations $di \in DI$
- $FM$: set of fixed location of first market customer zones $fm \in FM$
- $SM$: set of fixed location of second market customer zones $sm \in SM$
- $CI$: set of potential collection/inspection center locations $ci \in CI$
- $RD$: set of potential redistribution center locations $rd \in RD$
- $DP$: set of potential disposal center locations $dp \in DP$
- $T$: set of number of periods $t \in T$

Parameters
- $D_{fmt}$: demand of first market customer zone $fm$ in period $t$
- $\sigma_{fmt}$: demand standard deviation of first market customer zone $fm$ in period $t$
- $\mu_{fmt}$: demand mean of first market customer zone $fm$ in period $t$
- $D_{smt}$: demand of second market customer zone $sm$
- $PR_{fmt}$: Unit price at first market customer zone $fm$ in period $t$
- $PRR_{smt}$: unit price at second market customer zone $sm$ in period $t$
Fixed costs

- $FSU_{su}$: fixed cost of opening supplier center $su$
- $FPL_{pl}$: fixed cost of opening plant center $pl$
- $FDI_{di}$: cost of opening distribution center $di$
- $FCI_{ci}$: fixed cost of opening collection/inspection center $ci$
- $FRD_{rd}$: fixed cost of opening redistribution center $rd$
- $FDP_{dp}$: fixed cost of opening disposal center $dp$

Distance between facilities

$DS_{ij}$: distance between two locations $i$ and $j$

$DS_{ij} = \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2}$, where $X_i$ and $X_j$ represent Cartesian coordinates of location $i$.

Capacity of facilities

- $SUC_{su}$: capacity of supplier $su$
- $PLC_{pl}$: capacity of plant $pl$
- $PLS_{pl}$: storage capacity of plant $pl$
- $DIC_{di}$: capacity of distribution center $di$
- $CIC_{ci}$: capacity of collection/inspection center $ci$
- $RDC_{rd}$: capacity of redistribution center $rd$
- $DPC_{dp}$: capacity of disposal center $dp$

Variable costs

- $MSU_{su}$: unit material cost supplied by supplier $su$ in period $t$
- $RSU_{su}$: unit recycling cost recycled by supplier $su$ in period $t$
- $MPL_{pl}$: unit manufacturing cost manufactured by plant $pl$ in period $t$
- $RPL_{pl}$: unit remanufacturing cost manufactured by plant $pl$ in period $t$
- $ICI_{ci}$: unit inspection cost by collection/inspection $ci$ in period $t$
- $RCI_{ci}$: unit repairing cost by collection/inspection $ci$ in period $t$
- $DP_{dp}$: unit disposal cost by disposal location $dp$ in period $t$

Unused capacity costs

- $UPL_{pl}$: unused manufacturing capacity cost per hour of plant $pl$
- $URPL_{pl}$: unused remanufacturing capacity cost per hour of plant $pl$

Holding costs

- $HPL_{pl}$: unit holding cost per period at the store of plant $pl$
- $HDI_{di}$: unit holding cost per period at the store of distribution $di$

Ratio

- $RR$: return ratio at the first market customer zone
- $RC$: recycling ratio
- $RM$: remanufacturing ratio
- $RP$: repairing ratio
- $RDP$: disposal ratio
- $RSU_{su}$: capacity utilization rate for supplying material of supplier $su$
- $PLR_{pl}$: capacity utilization rate for manufacturing of plant $pl$

Other

- $TC$: transportation cost per unit per km
- $SC$: unit shortage cost per period
- $MPL_{pl}$: manufacturing time per unit in hours at plant $pl$
- $RMPL_{pl}$: remanufacturing time per unit in hours at plant $pl$
- $P_{fm}$: unit purchasing cost at first market customer zone $fm$
Decision variables
Binary variables (related to the foundation of facilities)

\[
X_{su} = \begin{cases} 
1 & \text{if a supplier center is opened at location } su \\
0 & \text{otherwise}
\end{cases}
\]

\[
Y_{pl} = \begin{cases} 
1 & \text{if a plant center is opened at location } pl \\
0 & \text{otherwise}
\end{cases}
\]

\[
Z_{di} = \begin{cases} 
1 & \text{if a distribution center is opened at location } di \\
0 & \text{otherwise}
\end{cases}
\]

\[
O_{ci} = \begin{cases} 
1 & \text{if a collection/inspection center is opened at location } ci \\
0 & \text{otherwise}
\end{cases}
\]

\[
W_{rd} = \begin{cases} 
1 & \text{if a redistribution center is opened at location } rd \\
0 & \text{otherwise}
\end{cases}
\]

\[
L_{dp} = \begin{cases} 
1 & \text{if a disposal center is opened at location } dp \\
0 & \text{otherwise}
\end{cases}
\]

\[
L_{IJ} = \begin{cases} 
1 & \text{if shipment link is created between any two location } i \text{ and } j \\
0 & \text{otherwise}
\end{cases}
\]

Continues variables (related to the flow of the forward network)

\[
QSP_{stu}^{pl} \quad \text{quantity of raw materials shipped from supplier } su \text{ to plant } pl \text{ in period } t
\]

\[
QPD_{plt}^{di} \quad \text{quantity of products shipped from plant } pl \text{ to distribution centers } di \text{ in period } t
\]

\[
QDC_{dit}^{fm} \quad \text{quantity of products shipped from distribution centers } di \text{ to first market customer zone } fm \text{ in period } t
\]

Continues variable (related to the flow of the reverse network)

\[
QCD_{fmt}^{ci} \quad \text{quantity of returned products shipped from first market customer zone } fm \text{ to collection/inspection center } ci \text{ in period } t
\]

\[
QDS_{cit}^{su} \quad \text{quantity of recyclable products shipped from collection/inspection centers } ci \text{ to suppliers } su \text{ in period } t
\]

\[
QDP_{cit}^{pl} \quad \text{quantity of remanufacturing products shipped from collection/inspection center } ci \text{ to plant } pl \text{ in period } t
\]

\[
QDR_{cit}^{rd} \quad \text{quantity of repaired products shipped from collection/inspection center } ci \text{ to redistribution center } rd \text{ in period } t
\]

\[
QPR_{plt}^{rd} \quad \text{quantity of remanufactured products shipped from plant } pl \text{ to redistribution } rd \text{ in period } t
\]

\[
QDD_{cit}^{dp} \quad \text{quantity of scrapped products shipped from collection/inspection centers } ci \text{ to disposal centers } dp \text{ in period } t
\]

\[
QRS_{rdt}^{sm} \quad \text{quantity of recovered products shipped from redistribution centers } rd \text{ to second market customer zone } sm \text{ in period } t
\]

\[
QIP_{plt}^{di} \quad \text{quantity of products shipped from plant store } pl \text{ to distribution } di \text{ in period } t
\]

\[
QIPP_{plt}^{pl} \quad \text{quantity of products shipped from plant, } pl \text{ to its store } pl \text{ in period } t
\]

\[
RQP_{plt} \quad \text{the residual inventory of the period } t \text{ at plant store } pl
\]

\[
RQD_{dit} \quad \text{the residual inventory of the period } t \text{ at distribution centers } di
\]

In the terms of above-mentioned notations, the multi-period multi-stage stochastic close-loop logistic network design problem can be formulated as follows:
Objective function

The objective function of the proposed model is to maximize the total expected profit of the closed-loop network.

Total expected profit = Total expected income – Total expected costs

Total expected income including total expected income through selling new products to first market customer zones and total expected income through selling recovered products to second market customer zones. Total expected cost including fixed costs, material costs, manufacturing costs, unused capacity costs, purchasing costs, shortage costs, inspection costs, recycling costs, disposal costs, remanufacturing costs, repairing costs, inventory holding costs and transportation costs.

\[ \text{MAX } Z = \]

\[ \sum_{di \in DI} \sum_{fm \in FM} \sum_{t \in T} QDC_{dit} \times PR_{fmt} + \sum_{rd \in RD} \sum_{sm \in SM} \sum_{t \in T} QRS_{rdt} \times PRR_{smt} - \sum_{su \in SU} \sum_{su \in SU} \sum_{pl \in PL} FPL_{pl} \times Y_{pl} \]

\[ + \sum_{di \in DI} FD_{di} \times Z_{di} + \sum_{ci \in CI} FCI_{ci} \times O_{ci} + \sum_{rd \in RD} FRD_{rd} \times W_{rd} + \sum_{dp \in DP} FDP_{dp} \times L_{dp} + \sum_{su \in SU} \sum_{pl \in PL} \sum_{t \in T} QSP_{sut} \times MSU_{sut} \]

\[ - \sum_{ci \in CI} \sum_{su \in SU} \sum_{t \in T} QDS_{sut} \times (MSU_{sut} - RSU_{sut}) + \sum_{pl \in PL} \sum_{di \in DI} \sum_{t \in T} QPD_{dit} \times MPL_{plt} + \sum_{pl \in PL} \sum_{di \in DI} \sum_{t \in T} QIPD_{dit} \times MPL_{plt} \]

\[ + \sum_{pl \in PL} \left( \left( (PLC_{pl} \times PLR_{pl} \times MPL_{pl}) \times Y_{pl} - \sum_{di \in DI} (QPD_{dit}) \times UPL_{plt} \right) + \right) \]

\[ + \sum_{pl \in PL} \sum_{ci \in CI} \sum_{t \in T} QCD_{cmt} \times P_{fm} + \sum_{fm \in FM} \sum_{ci \in CI} \sum_{t \in T} QCD_{cmt} \times ICI_{cit} + \sum_{ci \in CI} \sum_{su \in SU} \sum_{t \in T} QDS_{cit} \times RSU_{sut} \]

\[ + \sum_{pl \in PL} \sum_{rd \in RD} \sum_{t \in T} QPR_{rdt} \times RPL_{plt} + \sum_{pl \in PL} \sum_{di \in DI} \sum_{t \in T} QDD_{dit} \times D_{dp} + \sum_{su \in SU} \sum_{pl \in PL} QSP_{sut} \times DS_{su} \]

\[ + \sum_{ci \in CI} \sum_{dp \in DP} QDD_{cit} \times DS_{ci} + \sum_{ci \in CI} \sum_{rd \in RD} \sum_{t \in T} QPR_{rdt} \times RSU_{sut} \]

\[ + \sum_{rd \in RD} \sum_{sm \in SM} QRS_{sm} \times DS_{rd} + \sum_{t \in T} \left( \sum_{pl \in PL} QRP_{plt} \times HPL_{plt} + \sum_{di \in DI} RQD_{dit} \times H{D}_{d} \right) \]

Constraints

This sub-section is a representation of the constraints of the proposed model.

Capacity constraints

\[ \sum_{pl \in PL} QSP_{sut} < SUC_{su} \times RSU_{sut} \times X_{su} \quad \forall t, su \quad (2) \]

\[ \sum_{di \in DI} (QPD_{plt} + QIPD_{plt}) < (PLC_{pl} \times PLR_{pl} \times Y_{pl}) \quad \forall t, pl \quad (3) \]

\[ RQP_{plt} \leq PLS \times Y_{pl} \quad \forall t, pl \quad (4) \]

\[ \sum_{pl \in PL} (QPD_{plt} + QIPD_{plt}) + RQD_{di} < D_{di} \times Z_{di} \quad \forall t, di \quad (5) \]

\[ \sum_{su \in SU} QDS_{cit} + \sum_{pl \in PL} QDF_{cit} + \sum_{rd \in RD} QDR_{rdt} + \sum_{dp \in DP} QDD_{dp} \leq DPC_{dp} \quad \forall t, ci \quad (6) \]

\[ \sum_{sm \in SM} QRS_{rdt} \leq RDC_{rd} \quad \forall t, rd \quad (7) \]

\[ \sum_{ci \in CI} QDS_{cit} \leq SUC_{su} \times (1 - RSU_{sut}) \quad \forall t, su \quad (8) \]

\[ \sum_{ci \in CI} QDD_{dp} \leq DPC_{dp} \quad \forall t, dp \quad (9) \]

Linking shipping constraints:

\[ LSP_{supl} + \sum_{t \in T} QSP_{sul} \leq M \times LSP_{supl} \times \sum_{t \in T} QSP_{sul} \quad \forall su, pl \quad (10) \]

\[ LPD_{pldi} + \sum_{t \in T} (QPD_{dlit} + QIPD_{dlit}) \leq M \times LPD_{pldi} \times \sum_{t \in T} (QPD_{dlit} + QIPD_{dlit}) \quad \forall pl, di \quad (11) \]
\begin{align}
LDF_{difm} + \sum_{t \in T} QDC_{dit}^{fm} & \leq M \times LDF_{difm} + \sum_{t \in T} QDC_{dit}^{fm} \forall di, fm \\
LFI_{fmc} + \sum_{t \in T} QCP_{fmt}^{ci} & \leq M \times LFI_{fmc} + \sum_{t \in T} QCD_{fmt}^{ci} \forall ci, fm \\
LIS_{cisu} + \sum_{t \in T} QDS_{sucit}^{su} & \leq M \times LIS_{cisu} + \sum_{t \in T} QDS_{sucit}^{su} \forall su, ci \\
LIP_{cipl} + \sum_{t \in T} QDF_{citi}^{pl} & \leq M \times LIP_{cipl} + \sum_{t \in T} QDF_{citi}^{pl} \forall pl, ci \\
LIR_{cird} + \sum_{t \in T} QDR_{rdcit}^{nd} & \leq M \times LIR_{cird} + \sum_{t \in T} QDR_{rdcit}^{nd} \forall nd, ci \\
LD_{cidp} + \sum_{t \in T} QDD_{dpicit}^{dp} & \leq M \times LD_{cidp} + \sum_{t \in T} QDD_{dpicit}^{dp} \forall dp, ci \\
LPR_{plrd} + \sum_{t \in T} QPR_{plt}^{rd} & \leq M \times LPR_{plrd} + \sum_{t \in T} QPR_{plt}^{rd} \forall pl, rd \\
LRS_{dsm} + \sum_{t \in T} QRS_{rdt}^{sm} & \leq M \times LRS_{dsm} + \sum_{t \in T} QRS_{rdt}^{sm} \forall nd, sm \\
\end{align}

Balance constraints:

\begin{align}
\sum_{su \in SU} QSP_{sut}^{pl} &= \sum_{di \in DI} QPD_{di}^{pl} + QIP_{di}^{pl} \forall t, pl \\
QIP_{di}^{pl} + RQP_{pl}(t-1) &= RQP_{pl} + \sum_{di \in DI} QPD_{di}^{pl} \forall t, pl \\
\sum_{pl \in PL} (QPD_{di}^{pl} + QIP_{di}^{pl}) + RQD_{di}(t-1) &= RQD_{di} + \sum_{fm \in FM} QDC_{dit}^{fm} \forall t, di \\
\sum_{ci \in CI} QCD_{fmt}^{ci} &\leq (\sum_{di \in DI} QDC_{dit}^{fm}) \times RR \forall t, fm \\
\sum_{fm \in FM} QCD_{fmt}^{ci} \times RC &= \sum_{su \in SU} QDS_{sucit}^{su} \forall t, ci \\
\sum_{fm \in FM} QCD_{fmt}^{ci} \times RM &= \sum_{pl \in PL} QDF_{citi}^{pl} \forall t, ci \\
\sum_{fm \in FM} QCD_{fmt}^{ci} \times RP &= \sum_{su \in SU} QDR_{rdcit}^{nd} \forall t, ci \\
\sum_{fm \in FM} QCD_{fmt}^{ci} \times RDP &= \sum_{dp \in DP} QDD_{dpicit}^{dp} \forall t, ci \\
\sum_{ci \in CI} QDF_{citi}^{pl} &= \sum_{nd \in RD} QPR_{plt}^{nd} \forall t, pl \\
\sum_{ci \in CI} QDR_{rdcit}^{nd} + \sum_{pl \in PL} QPR_{plt}^{nd} &= \sum_{sm \in SM} QRS_{sm}^{ndt} \forall t, nd \\
\sum_{nd \in RD} QRS_{sm}^{ndt} &\leq D_{smt} \forall t, sm \\
\end{align}

Maximum number of facilities constraints:

\begin{align}
\sum_{su \in SU} x_{su} &\leq SU \\
\sum_{pl \in PL} y_{pl} &\leq PL \\
\sum_{di \in DI} z_{di} &\leq DI \\
\sum_{ci \in CI} o_{ci} &\leq CI \\
\sum_{nd \in RD} w_{nt} &\leq RD \\
\end{align}
\[
\sum_{dp \in DP} L_{dp} \leq DP
\]  
(38)

\[
X_{su} \cdot Y_{pl} \cdot Z_{di} \cdot O_{ci} \cdot W_{rd} \in [0,1]
\]  
(39)

\[\forall su \in SU, pl \in PL, di \in DI, ci \in CI \text{ and } nd \in RD \text{ all } Q \geq 0\]  
(40)

Equations (2-9) are related to capacity constraints such that (2) ensure that the sum of the flow exiting from each supplier to all plants does not exceed the supplier capacity at each period. Equation (3) makes sure that the sum of new product shipped to plants stores and to all distribution centers does not exceed the plant capacity at each period. Equation (4) prohibits from transferring the units of new product to plant stores over their capacities at each period. Equation (5) ensures that residual inventory at each distribution center and flow entering to it at the existing period does not exceed this distribution center capacity. Equation (6) ensures that the returned products shipped to the collection/inspection centers does not exceed this collection/inspection capacity at each period. Equation (7) makes sure that the flow exiting from each redistribution center to second market customer zones does not exceed this redistribution capacity at each period. Equation (8) makes sure that the recyclable product shipped from collection/inspection center to each supplier does not exceed the supplier recycling capacity at each period. Equation (9) ensures that the scrapped products entering to each disposal center from all collection/inspection centers do not exceed its disposing capacity at each period. Equation (10-19) are related to linking shipping constraint in a way that there is no link between any location without actual shipment and ensure that there is no shipping between any non-linked locations simultaneously. Equation (20-32) are related to balance constraints such that (20) ensures that the flow entering to each plant from all suppliers is equal to sum of the new products shipped from this plants to its store and to all distribution centers at each period. Equation(21) ensures that the sum of the flow entering to each plant store and its residual inventory from the previous period is equal to the sum of the exiting flow to each distribution and residual inventory of the existing period. Equation(22) assures that the sum of the flow entering to each distribution center form each plant of plant store and its residual inventory from the previous period is equal to the sum of the exiting to each first market customer zone and the residual inventory of the existing period. Equation(23) prohibit that the sum of the flow entering to each first market customer zone from exceeding the sum of the demand at each first market customer zone and the previous agglomerated back orders at each periods. Equation(24) prohibits that the returned products shipped from each first market customer zone to all collection/inspection centers do not exceed the sum of new product entering to each first market customer zone at each period. Equation(25) make sure that the flow entering to each redistribution/inspection from all first market customer zones is equal to the sum of flow exiting to each supplier for recycling, to each plant for remanufacturing, to each redistribution center or to each disposal center for disposing at each period. Equations (26-29) assure the flow balance at the suppliers, plants redistribution centers, disposal centers and collection/inspection centers at each period considering recycling ratio, remanufacturing ratio, repairing ratio and disposing ratio. Equation(30) assures that all remanufactured products entering to each plants from each collection/inspection center is equal to the sum of exiting to each redistribution center at each period. Equation(31) make sure that all remanufactured products entering to each redistribution center from all plants and all repaired products entering to it from all collection/inspection centers is equal to the sum of recovered products exiting to each second market customer zone at each period. Equation(32) make sure that the recovered product shipped to each second market customer zone does not exceed the second market customer demand at each period.

Equations (33-38) all related to maximum number of facilities constraints and limit the number of activated located such that assure that the number of opened locations do not exceed the maximum number of facilities constraints.

Finally, (39-40) enforce the binary and non-negative limitation on the corresponding decision variables.

In term \( L_{ij} \times \sum_{t \in T} Q_{it} \) in linking shipped constraints where \( i \) and \( j \) are indices of any two location in the network is non-linear due to involving the multiplication of two binary integer variables. Hence, to avoid the complexity of such mixed integer non-linear programming (MINLP) model, the above model is linearized by defining to new constraint and formulating the mentioned function is as following.

\[ L_{ij} \leq \sum_{t \in T} Q_{it} \forall i, j \]

\[ \sum_{t \in T} Q_{it} \leq L_{ij} \times M \forall i, j \]

Therefore, the linearized linking shipped constraints can be formulated as follows:

\[ L_{S_{su}} \leq \sum_{t \in T} Q_{S_{su}} \]  
(39)

\[ \sum_{t \in T} Q_{S_{su}} \leq M \times L_{S_{su}} \forall su, pl \]  
(40)

\[ L_{PD} \leq \sum_{t \in T} (Q_{PD} + Q_{IPD}) \leq M \times L_{PD} \forall pl, di \]  
(40)

IV. COMPUTATIONAL RESULTS

In this section, in order to assess the performance of the proposed stochastic model, three test problems are designed and the values of test problems are given Table (I), as well as a brief overview of obtained results is provided in Fig. 2, 3 and 4. Pishvaee [3] mentioned that increasing the number of scenarios significantly increases the computational time with limited benefit in solution accuracy. The behavior of the model has been discussed with different model parameters. Demand mean is taken to represent the main affecting parameters. The nominal values of model parameters are given in Tables (I-VII).

<table>
<thead>
<tr>
<th>Problem no.</th>
<th>Size 1</th>
<th>Size 2</th>
<th>Size 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of potential suppliers</td>
<td>3</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>No. of potential plants</td>
<td>3</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>No. of potential distribution centers</td>
<td>3</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>No. of first market customer zones</td>
<td>4</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>No. of collection/inspection centers</td>
<td>3</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>No. of disposal centers</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>No. of redistribution centers</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>No. of second market customer zones</td>
<td>2</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>No. of periods</td>
<td>8</td>
<td>16</td>
<td>25</td>
</tr>
</tbody>
</table>

Nominal demand parameters which are demand mean and demand standard deviation of first market customer zones, unit price at first market customer zones, unit price at second market customer zones, demand of second market customer zones are given in Table (II).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{fmt}$</td>
<td>uniform(0,3000)</td>
</tr>
<tr>
<td>$\sigma_{fmt}$</td>
<td>50</td>
</tr>
<tr>
<td>$PR_{fmt}$</td>
<td>100</td>
</tr>
<tr>
<td>$PRR_{fmt}$</td>
<td>80</td>
</tr>
<tr>
<td>$D_{fmt}$</td>
<td>2000</td>
</tr>
</tbody>
</table>
Nominal fixed cost parameters which are fixed cost of opening supplier, fixed cost of opening plant, fixed cost of opening distribution, fixed cost of opening collection/inspection, fixed cost of opening redistribution and fixed cost of opening disposal centers are given in Table (III).

### TABLE III: Fixed Cost Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSU_{pr}</td>
<td>20000</td>
</tr>
<tr>
<td>FPL_{pl}</td>
<td>50000</td>
</tr>
<tr>
<td>FDI_{ri}</td>
<td>20000</td>
</tr>
<tr>
<td>FCI_{ct}</td>
<td>15000</td>
</tr>
<tr>
<td>FRD_{rd}</td>
<td>10000</td>
</tr>
<tr>
<td>FDP_{dp}</td>
<td>50000</td>
</tr>
</tbody>
</table>

Nominal capacity parameters of facilities which are suppliers, plants, plant stores, distribution centers, collection/inspection centers, redistribution centers and disposal center are given in Table (IV).

### TABLE IV: Location Capacity Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU_{su}</td>
<td>6000</td>
</tr>
<tr>
<td>PLC_{pl}</td>
<td>9000</td>
</tr>
<tr>
<td>PLS_{pt}</td>
<td>1000</td>
</tr>
<tr>
<td>DIC_{di}</td>
<td>4000</td>
</tr>
<tr>
<td>CIC_{ct}</td>
<td>4000</td>
</tr>
<tr>
<td>RDC_{rd}</td>
<td>3500</td>
</tr>
<tr>
<td>DPC_{dp}</td>
<td>3000</td>
</tr>
</tbody>
</table>

Ratio model parameters which are return ratio at the first market customer zones, recycling, remanufacturing, repairing and disposal ratio as well as capacity utilization rate for supplying material of suppliers and capacity utilization rate for manufacturing of plants are given in Table (V).

### TABLE V: Ratio Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR</td>
<td>60%</td>
</tr>
<tr>
<td>RC</td>
<td>30%</td>
</tr>
<tr>
<td>RM</td>
<td>40%</td>
</tr>
<tr>
<td>RP</td>
<td>10%</td>
</tr>
<tr>
<td>RDP</td>
<td>20%</td>
</tr>
<tr>
<td>RSU_{su}</td>
<td>70%</td>
</tr>
<tr>
<td>PLR_{pl}</td>
<td>80%</td>
</tr>
</tbody>
</table>

Variable cost model parameters including unit material cost and recycling cost by suppliers, unit manufacturing and remanufacturing cost by plants, unit inspection and repairing by collection/inspection centers and unit disposal cost by disposal locations are provided in Table (VI).

### TABLE VI: Variable Cost Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSU_{sat}</td>
<td>20</td>
</tr>
<tr>
<td>MPL_{plt}</td>
<td>25</td>
</tr>
<tr>
<td>ICI_{cst}</td>
<td>10</td>
</tr>
<tr>
<td>RCI_{ct}</td>
<td>15</td>
</tr>
<tr>
<td>RSU_{sat}</td>
<td>5</td>
</tr>
<tr>
<td>RPI_{plt}</td>
<td>20</td>
</tr>
<tr>
<td>DP_{dpt}</td>
<td>2</td>
</tr>
</tbody>
</table>

Other model parameters including unused manufacturing and remanufacturing capacity cost per hour at plants, unit holding cost per period at store of plants and distribution centers and manufacturing and remanufacturing time per unit in hours at plants and unit purchasing cost at first market customer zones are provided in Table (VII).
TABLE VII: Other Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$UPL_{pt}$</td>
<td>10</td>
</tr>
<tr>
<td>$URPL_{pt}$</td>
<td>10</td>
</tr>
<tr>
<td>$HPI_{pt}$</td>
<td>10</td>
</tr>
<tr>
<td>$HD_{di}$</td>
<td>5</td>
</tr>
<tr>
<td>$MPL_{pt}$</td>
<td>4</td>
</tr>
<tr>
<td>$RMP{\text{L}}_{pt}$</td>
<td>8</td>
</tr>
<tr>
<td>$P_{fm}$</td>
<td>40</td>
</tr>
<tr>
<td>$TC$</td>
<td>1%</td>
</tr>
<tr>
<td>$SC$</td>
<td>10</td>
</tr>
</tbody>
</table>

Initial experimentation shows that increasing the number of scenarios increases severely the processing time with limited benefit in solution accuracy such that the difference in results did not exceed 0.05% when increasing the number of scenarios from 20 to 100. Therefore, only 20 scenarios were chosen in the present work for 5 periods in test problem 1, 10 periods in test problem 2 and finally, 15 periods in test problem 3 for a normally distributed discretized demand.

The model is built by using AIMMS which is designed for modeling and solving large-scale optimization problems. Following this model language, the demand distribution can take any form since it accepts a large number function forms. All calculations were carried out using Windows 7 ultimate on Intel® core™ processor and 4.00 GB RAM laptop.

**Effects of demand**

Generally, the total expected profit is proportional to the total demand. The total expected profit increases with increase in demand as illustrated in Fig (2) to Fig(4). At certain instances, it decreases due to shortage costs as it is not profitable to open an extra facility.

![Fig 2](image2.png) Total expected profit vs. demand mean in Test problem 1

![Fig 3](image3.png) Total expected profit vs. demand mean in Test problem 2

![Fig 4](image4.png) Total expected profit vs. demand mean in Test problem 3
V. CONCLUSION

This paper proposed a stochastic optimization model to cope with issue of uncertainty in integrated forward/reverse logistics network design and is successful in designing closed-loop logistics networks while considering stochastic demand with three echelons (suppliers, plants, and distribution centers) in forward direction and two echelons (collection/inspection and redistribution centers) in reverse direction. The model is flexible to solve larger problems and handle data uncertainty. Therefore it can be concluded that SMINLP model can be used as powerful tool in practical cases.

Following possible future research directions can be defined in the area of logistics network design on the uncertainty: addressing uncertainty in multi-product integrated logistics networks or considering maximizing the responsiveness of the network as an objective besides maximizing total expected profit in uncertain conditions that result in multi-objective stochastic programming. Moreover, incorporating inventory and routing decision in integrated logistics network design problems are also an attractive direction for future research to take benefit of integrating strategic level with tactical an operational level decisions. Finally, as far as the computational time increases significantly when the size of problem and the number of scenarios increases, therefore developing efficient heuristic solution methods is also a critical need in this area.

VI. REFERENCES