J. Basic. Appl. Sci. Res., 3(3)1005-1008, 2013
© 2013, TextRoad Publication

ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

A Centralized Case of Cost Efficiency in Data Envelopment Analysis

Gholam Reza Jahanshahloo, Farhad Hosseinzadeh Lotfi, Mohsen Rostamy-Malkhalifeh, Ali Jamshidi

Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

ABSTRACT

In this paper, we proposed a centralized way for measuring cost efficiency (CE) in Data Envelopment Analysis (DEA). This approach based on Fare cost efficiency model to obtain CE. In Fare's method to obtain CE, a linear programming problem should solve for each DMU, separately. This procedure needs a long computational process. In this paper, we propose a new approach for measuring CE and calculate the cost efficiency of all DMUs by solving only one linear programming problem. This approach reduces the computational process and it will illustrated by an example.
KEYWORDS: Centralized approach; Decision Making Unit; Cost efficiency, Data Envelopment Analysis.

INTRODUCTION

Data envelopment analysis (DEA) is a non-parametric approach for evaluating peer decision making units (DMUs) that have the same inputs and the same outputs [1]. In DEA literature, the performance evaluation along with the efficiency of the included units of an organization is considered as a crucial fact, affecting the whole performance, either directly or indirectly. This happen is study by Jafarpour et al. [2]. Cost efficiency (CE) is a concept of DEA that evaluates the ability of a DMU to produce the current outputs at minimal cost when the prices of inputs are at hand. CE was first introduced by Farrell [3], and then developed by Fare, Grosskopf and Lovell [4] by using linear programming (LP) technique. This LP model requires input and output quantity data as well as input prices at each decision making unit.

Tone [5] pointed out the shortcomings of the cost efficiency measures in the presence of price differences between the DMUs. To overcome this limitation, he relaxed the fixed prices assumption and proposed the assessment of the DMUs in the cost space.

In any approach for measuring cost efficiency in DEA, we should solve a linear programming model for each DMU, separately and it is a difficult problem from a computational point of view.

In this paper, we produce a new approach for measuring cost efficiency using centralized data envelopment analysis [6] and our aim of this approach is decreasing the computational processes. Centralized DEA models aim at reallocating inputs and outputs among the units setting new input and output targets for each one by solving one linear programming problem. As was discussed about difficulty of obtain CE of all DMUs, in the new approach, we solve only one linear programming problem to obtain cost efficiencies of all DMUs. This approach decreases the computational process and it is a very important property of the proposed approach Compared to other methods.

This paper is organized as follows. Section 2 explains the Farrell cost efficiency and Fare model for measuring it and centralized data envelopment analysis [6]. Then, in section 3, we propose a new approach to obtain CE of DMUs and its validity. Section 4 contains an illustrative example. Finally, section 5 presents conclusions from this research.

1. Preliminaries

Farrell first introduced the concept of CE underlying a DEA assessment [3]. Fare et al. [4] operationalised cost efficiency measures based on the Farrell concept in the DEA literature.

Consider a set of n DMUs consisting of DMU_j; $j \in \{1,2,\dots,n\}$, each consuming inputs $X_j = (x_{1j}, x_{2j}, \dots, x_{mj}) \in \Re^m$ to produce outputs $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj}) \in \Re^s$ and Let C_i be the price of input i. The production possibility set under constant returns to scale assumption can be define as
$T_c = \{(X,Y) \in \Re^+ \mid X \geq \sum_{j=1}^n \lambda_j x_j,\ Y \leq \sum_{j=1}^n \lambda_j y_j, \lambda_j \geq 0\}$.

In order to obtain a measure of cost efficiency for DMUs, Fare et al. solved the following linear problem:

$$\begin{aligned} Min\ & \sum_{i=1}^m C_i x_i^p \\ \text{s.t.}\ & \sum_{j=1}^n \lambda_{pj} x_{ij} = x_i^p, \forall i, \\ & \sum_{j=1}^n \lambda_{pj} y_{rj} \geq y_{rp}, \forall r, \\ & \lambda_{pj} \geq 0, \forall j,\ x_i \geq 0, \forall i. \end{aligned} \tag{1}$$

***Corresponding Author:** Ali Jamshidi, Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran. E-mail address: alijamshidi63@yahoo.com.Phone number: +989177049067

In the model above, x_i^p is a variable that, at the optimal solution, gives the amount of input i employed by DMU_p in order to produce the current outputs at minimal cost and the technological restrictions of the production possibility set.

Then, the cost efficiency of DMU_p obtained as the ratio of minimum cost with current price to the current cost at DMU_p as follows:

$$\text{Cost efficiency of } DMU_p = CE_p = \frac{\sum_{i=1}^{m} C_i x_i^*}{\sum_{i=1}^{m} C_i x_{ip}} \tag{2}$$

Definition 1: DMU_p is named cost efficient if $CE_p = 1$.

For obtain the cost efficiency of all DMUs, we must solve model (1) for each of them and then calculate CE by using the optimal solution of model (1) and relation (2).

The centralized DEA model proposed by Lozano and Villa [6] for project all DMUs on efficient frontier simultaneously, is written as follows:

$$\begin{aligned} &Min\ \ \theta_p \\ &s.t. \\ &\quad \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_{pj}x_{ij} \le \theta \sum_{j=1}^{n} x_{ij} \quad ,\forall i \\ &\quad \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_{pj}y_{rj} \ge \sum_{j=1}^{n} y_{rj} \quad ,\forall r \\ &\quad \sum_{j=1}^{n}\lambda_{pj} = 1 \quad ,\forall p \\ &\quad \lambda_{pj} \ge 0 \ \ \forall p, \forall j. \end{aligned} \tag{3}$$

This model projected all DMUs on efficient frontier, simultaneously and only one linear programming problem is been solved. However, to do it by Charnes et al. method n linear programming problem should solve.

2. A centralized approach

In this section, we use a centralized approach to measure cost efficiency of all DMUs under evaluation.

For this purpose, by use the equality constraints in model (1) and substitution x_i^p in objective function, the following model that is equivalent to model (1) with respect to DMU_p is at hand:

$$\begin{aligned} &Min\ \ \sum_{j=1}^{n}\lambda_{pj}\left(\sum_{i=1}^{m} C_i x_{ij}\right) \\ &s.t.\ \ \sum_{j=1}^{n}\lambda_{pj}y_{rj} \ge y_{rp}\ ,\ \forall r, \\ &\qquad \lambda_{pj} \ge 0, \forall j. \end{aligned} \tag{4}$$

Then:

$$\text{Cost efficiency of } DMU_p = CE_p = \frac{\sum_{i=1}^{m} C_i\left(\sum_{j=1}^{n}\lambda_{pj}^* x_{ij}\right)}{\sum_{i=1}^{m} C_i x_{ip}}. \tag{5}$$

While $\lambda_p^* = \left(\lambda_{p1}^*,\ \lambda_{p2}^*, \dots, \lambda_{pn}^*\right)$ is an optimal solution of model (4).

Now consider the following multiple objective linear problem:

$$\begin{aligned} &Min\ \left\{\sum_{j=1}^{n}\lambda_{1j}\left(\sum_{i=1}^{m} C_i x_{ij}\right), \sum_{j=1}^{n}\lambda_{2j}\left(\sum_{i=1}^{m} C_i x_{ij}\right), \dots,\ \sum_{j=1}^{n}\lambda_{nj}\left(\sum_{i=1}^{m} C_i x_{ij}\right)\right\} \\ &s.t.\ \ \sum_{j=1}^{n}\lambda_{pj}y_{rj} \ge y_{rp}\ ,\quad \forall r,\ \forall p, \\ &\qquad \lambda_{pj} \ge 0,\ j = 1, \dots, n,\ p = 1, \dots, n. \end{aligned} \tag{6}$$

Definition 2: Norm 1 of a vector $(x_1, x_2, \dots, x_n)$ defines as sum of all its components, i.e. $\sum_{j=1}^{n} x_j$.

If we combine the objectives of MOLP (6) by norm 1, a single objective linear program produced as follows:

$$\begin{aligned} &Min\ \sum_{j=1}^{n}\lambda_{1j}\left(\sum_{i=1}^{m} C_i x_{ij}\right) + \sum_{j=1}^{n}\lambda_{2j}\left(\sum_{i=1}^{m} C_i x_{ij}\right) + \dots + \sum_{j=1}^{n}\lambda_{nj}\left(\sum_{i=1}^{m} C_i x_{ij}\right) \\ &s.t.\quad \sum_{j=1}^{n}\lambda_{pj}y_{rj} \ge y_{rp}\ ,\quad \forall r,\ \forall p, \\ &\qquad \lambda_{pj} \ge 0,\ \forall j,\ \forall p. \end{aligned} \tag{7}$$

Then we obtain the following single objective linear problem:

$$\begin{aligned} &Min\ \ \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_{pj}\left(\sum_{i=1}^{m} C_i x_{ij}\right) \\ &s.t.\ \ \sum_{j=1}^{n}\lambda_{pj}y_{rj} \ge y_{rp}\ ,\quad \forall r, \forall p \\ &\qquad \lambda_{pj} \ge 0,\ \forall j, \forall p. \end{aligned} \tag{8}$$

Theorem 1: If $\lambda^* = (\lambda_1^*, \lambda_2^*, \dots, \lambda_n^*)$ is an optimal solution for model (8), then $\lambda_p^* = (\lambda_{p1}^*, \lambda_{p2}^*, \dots, \lambda_{pn}^*)$ is optimal for model (1) with respect to DMU_p.

Proof: We first suppose that $\lambda^* = (\lambda_1^*, \lambda_2^*, \dots, \lambda_n^*)$ is an optimal solution for model (8). By contradiction, assume that there exists $k \in \{1,2,\dots,n\}$ such that $\lambda_k^* = (\lambda_{k1}^*, \lambda_{k2}^*, \dots, \lambda_{kn}^*)$ is not optimal for model (1). Let $\overline{\lambda_k} = (\overline{\lambda_{k1}}, \overline{\lambda_{k2}}, \dots, \overline{\lambda_{kn}})$ is optimal for model (1) with respect to DMU_k, then obviously

$$\sum_{j=1}^{n} \overline{\lambda_{kj}} \left(\sum_{i=1}^{m} C_i x_{ij}\right) < \sum_{j=1}^{n} \lambda_{kj}^* \left(\sum_{i=1}^{m} C_i x_{ij}\right).$$

By substitution λ_k^* by $\overline{\lambda_k}$ in λ^*, a new feasible solution $\bar{\lambda}$ for model (8) is at hand where

$$\sum_{p=1}^{n} \sum_{j=1}^{n} \overline{\lambda_{pj}} \left(\sum_{i=1}^{m} C_i x_{ij}\right) < \sum_{p=1}^{n} \sum_{j=1}^{n} \lambda_{pj}^* \left(\sum_{i=1}^{m} C_i x_{ij}\right),$$

and it is contradicted by optimality of λ^*.

To obtain the cost efficiency of all DMUs by Fare method, we have to solve a linear problem for each DMU with n variables and $m + s$ constraints. If we solve model (8) with n^2 variables and $n \times s$ constraints, then we can obtain the cost efficiency of all DMUs as follows:

$$\text{Cost efficiency of } DMU_p = CE_p = \frac{\sum_{i=1}^{m} C_i (\sum_{j=1}^{n} \lambda_{pj}^* x_{ij})}{\sum_{i=1}^{m} C_i x_{ip}}. \tag{9}$$

While $\left\{\lambda_{pj}^*,\ p, j \in \{1,2,\dots,n\}\right\}$ is optimal in model (8).

3. Illustrative example

In this section, we explain our new approach for measuring cost efficiency with an example. Our discussion uses the data set in Table 1 as an illustrative example, and the CE results of DMUs by Fare approach are shown in the last column of this Table.

By solving model (8) for each DMU and calculating the cost efficiency of it by relation (9), we obtain the cost efficiency for each DMU as same as these Fare's cost efficiency measures.

In this paper, we use the GAMS software to solve linear programming problems.

We note that for assessing the cost efficiency of all DMU by Fare approach, we should solve 20 linear programming problems, and the required time for this is 3.885 seconds. However, if the CE of all DMUs is obtained by solving model (8) and relation (9), the required time is 0.172 seconds. In this approach, only one linear programming problem is solved. As theorem 1 shows, the CE of all DMUs calculated by Fare et al. approach and new approach are similar. Thus, the last column of table 1 shows the CE of all DMUs obtained by two approaches.

Table 1: Data for 20 DMUs and their cost efficiencies

DMU	x_1	x_2	y_1	y_2	CE
1	20	151	100	90	1
2	19	131	150	50	1
3	25	160	160	55	0.88
4	27	168	180	72	0.98
5	22	158	94	66	0.88
6	55	225	230	90	0.97
7	33	235	220	88	0.9
8	31	206	152	80	0.78
9	30	244	190	100	0.82
10	50	268	250	100	1
11	53	306	260	147	1
12	38	284	250	120	1
13	42	200	189	112	1
14	45	235	200	118	0.93
15	37	289	156	98	0.63
16	29	199	215	87	1
17	18	186	179	88	1
18	19	240	146	134	1
19	25	179	120	78	0.82
20	40	221	204	60	0.83

We also measure the cost efficiencies of 200 DMUs with two inputs and four outputs. The GAMS software needs 27.066 seconds to solve model (1) for all DMUs and to measure these cost efficiencies. However, the required time to solve model (8) to assess CE for all DMU is 0.715 seconds.

These computational results show that our new model requires less time than fare approach for measuring cost efficiency, which is an important advantage from a computational point of view.

4. Conclusion

In this paper, we provided a new approach for measuring cost efficiency in data envelopment analysis based on Fare et al. approach and centralized DEA. In this method to measure cost efficiency for all DMUs, only one linear programming problem should solved, instead of solving *n* linear programming problem. In problem with large number of decision making units, the number of linear programming problem that should solve is large and this is difficult to calculate CE. Our proposed approach reduces the computational process and that it is an important trait for it as compared other approaches.

REFERENCES

1. Charnes, A., W.W. Cooper and E. Rhodes, 1978. Measuring the efficiency of decision making units, European Journal of Operational Research 2: 429–444.
2. Jafarpour, E., N. Mollaverdi, F. MokhatabRafiei and S. M. Arabzad, 2012. Evaluation of the Suggestions System Performance Using DEA, The Case of Isfahan's Mobarakeh Steal Company, Journal of Basic and Applied Scientific Research 2 (11): 11717-11725.
3. Farrell, MJ. 1957. The measurement of productive efficiency, Journal of the Royal Statistical Society, Series A 120: 253–290.
4. Fare, R., S. Grosskopf and CAK. Lovell, 1985. The measurement of efficiency of production, Dordrecht MA, Kluwer, Boston.
5. Tone, K. 2002. A strange case of the cost and allocative efficiencies in DEA, Journal of the Operational Research Society 53: 1225–1231.
6. Lozano S. and G.Villa, 2004. Centralized resource allocation using data envelopment analysis. Journal of Productivity Analysis 22: 143–161.
7. Cooper, W.W., L.M. Seiford and K. Tone, 2002. Data envelopment analysis, A comprehensive text with models applications references, DEA solved software, Third printing, Kluwer Academic Publishers.