

Determining the Optimal Portfolio in Iran Stock Exchange by Value at Risk Approach

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ABSTRACT

Value at risk is a well-known concept in the measurement, prediction and management of risk which has received increasing attention in financial schemes. This method has been used in determining the optimum level of stocks of the investment portfolio. Value at risk minimization of the portfolio has been utilized to determine the optimal level of investment. Portfolio optimization aims to ascertain the optimal amount of assets so that the risk is minimized in a given level of return. The main objective of this study is introducing value at risk (VAR) as an appropriate pattern to manage the risk of the investments in Iran Stock Exchange and to select the optimal portfolio. Using parametric variance-covariance, the investment risk of a stock portfolio was calculated (selected through filtering technique). Finally, the optimal level of each stock in a portfolio was calculated by using a linear programming model. The findings reveal that there should be an investment in at least nine companies of the selected firms in which their stocks is determined.

KEYWORDS: Return, Risk, Value at Risk, Diversification, Optimal Portfolio.

1. INTRODUCTION

Vague future is the problem derived by economic and social activities of the human. Stock Exchanges are significant instruments which have the ability to concentrate the funds and lead different industries. The most essential task of this capital is the attraction of dispersion funds and optimal assigning of these resources to the units aimed to develop and support the expected benefits of the investors. Investors with different incentives play a key role in achieving more benefit and finance. However, the investors chiefly seek to gain more advantages and increase their utility through appropriate investment decisions. A wisdom investment works so that the highest probable and expected return is reached by accepting a given level of risk. The assumed or given return is one of the investment methods which might lower the risk and implement the diversification strategy of the portfolio.

The present study intends to analyze the market risk which is a spectrum of some risks such as inventory prices, stocks and currency rate. This evolution has been initiated by introducing a new indicator called value at risk that is a new method to measure the possible risk of the capital market. Value at risk is the response to the complexity of financial instruments and merely considers the numerical amount of the portfolio and ignores the risk exchange and expected return (Best, 1998). Value at risk summarizes the risk of the portfolio only in one figure. Hence top managers are not confronted with the risk calculations.

Value at risk is the concept applied specifically in securities stock exchanges to measure the risk level and optimal portfolio selection (Fusai and Luciano, 2000). It seems necessary to conduct studies about stock selection and optimum investment decisions according to the incremental extension of the Tehran Stock Exchange and the incremental number of investors in this market resulted by the implementation of privatization policies and disposal of governmental companies to the private ones. This technique is rarely used in Iran Stock Exchange, hence it is expected that applying this model causes a new horizon to be captured for the selection of optimum portfolio in Iran's capital market and consequently enhancing the investment culture. Utilization of this method contributes managing the investment risk and selecting an optimal portfolio. Therefore, we have first reviewed the literature and return, risk and value at risks of the sample firms were calculated and finally the optimal portfolio determined by using a linear programming model. Generally, the present paper seeks to find an answer to a question about whether by using a linear programming technique in the form of VaR, an optimum portfolio could be achieved.

2- THEORETICAL BACKGROUND AND LITERATURE REVIEW

This section is about the research variables and literature. First of all, the return and earnings per share are described and value at risk of each stock and of a portfolio is described in detail.

2-1- Theoretical Concepts Review

2-1-1- Return and Risk

Utility maximization is the main assumption of the financial theories. That is, the investors endeavor to maximize their expected utility. Consequently, the investors should select a proper investment option according to the different uncertain opportunities. Most of the investors agree upon the concept that the expected utility is a positive function of the expected return on investment and a negative function of the return variance. Therefore, the standard deviation of a return for a given stock is used as the quantitative measure applied to determine the risk.

It might be supposed that the portfolio risk is always measured through the weighted average of the standard deviation for different stocks. This will hold true when the stock prices change in the same direction. Portfolio risk has a direct relationship with the correlation coefficient. Portfolio risk depends on the characteristics of each stock and a variation in any of the characteristics will change the risk of the portfolio (Statman, 1987). Portfolio risk is then resulted from the impact of pair-wise covariance.

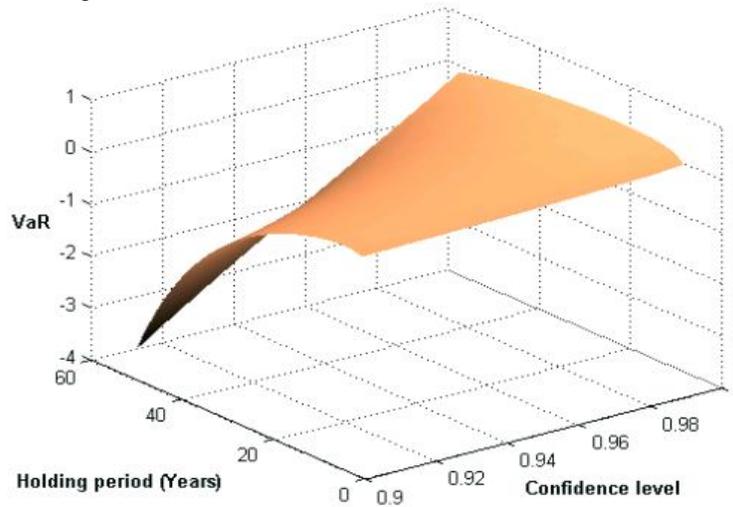
The periodic return of each stock shows the price growth (according to the beginning and ending prices) during the period. Periodic return has been also used to calculate the stock return. Annual portfolio return of the firms is simply measured as the weighted average of the annual returns. The weights are calculated as the ratio of the invested amounts in any stock to the total amount of investment and the total weight is proposed to be equal to one (Elton et al, 2010).

2-1-2- Value at Risk

There is a considerable advancement occurred in the risk management recently. This is initiated from the value at risk summarized as VaR. This is derived from the severe financial crisis occurred in the late 1990s. This crisis became an incentive for the financial institutions which is a reasonable method for quantifying the level of market risk (Jorion, 2000). However, it is not completely clear where the place of VaR is. During the 1990s, some similar titles were introduced such as Dollars at risk (DaR), Capital at risk (CaR), Income at risk (IaR), Earnings at risk (EaR) and Value at Risk (VaR). Users of these titles applied them the same as at Risk concept, but they didn't agree upon what is actually at risk. Anyway, value at risk is the title used by the researchers (Holton, 2002).

Value at risk as a statistical measure, reports the maximum expected loss of maintaining an asset or a portfolio in a given time period with definite probability (significance level) and is shown quantitatively (Aussenegg et al, 2006).

Figure (1): VaR and the Time Horizon (Dowd et al, 2003,7)



Simply, value at risk can be described as follows: there is X percent confidence which during the T future days, the firm will not be absolutely able to tolerate a loss more than V. This is a variable the same as the value at risk of an asset portfolio that includes two parameters of T (time horizon) and X (confidence level). For instance, if the value at risk of holding an asset at 99% of confidence level is 10million rials a day, it means that the average daily loss of declining the firm's market value, only in one day of 100 days, will be more than 10million rials.

2-1-2-1- Calculations of VAR

The calculation methods of value at risk fall into parametric and non-parametric categories. The former is summarized in variance-covariance and some other analyzing methods. Non-parametric method also includes historical simulation and Monte-Carlo simulation technique (Frang, 2004). The main approach of this study is based upon value at risk by parametric variance-covariance method.

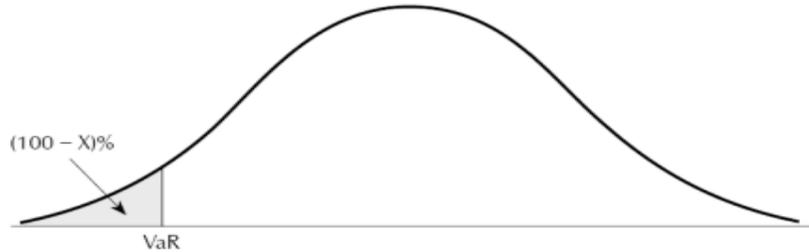
2-1-2-1-1- Variance-Covariance Method

Suppose a return distribution belongs to a special group of distributions like normal distribution, then calculating value at risk will be considerably simple. In this way, assuming that risk factor returns are always (jointly) normally distributed and that the change in portfolio value is linearly dependent on all risk factor returns. According to the normal distribution, the probability of placing a return on the left corner of the normal distribution equals to the standard normal probability:

$$Z_{\alpha} = P[Z < z]$$

Suppose that our time agreement is N days and the confidence level is X percent, value at risk is the level of loss equal to (100-X) percent of the probability distribution of the changes in the portfolio during the N future days. For example, when N=95 and X=97, value at risk is a loss equal to 3% of the probability distribution of the variations in the portfolio during the five future days. The following graph depicts the VaR for a situation in which the variations of portfolio is normally distributed.

Figure (2): VaR Distributary Position



2-2- Literature Review

In a general approach, the theories of creating a portfolio are classified into modern and postmodern theories. In the modern theory, the optimal portfolio is considered by trading off the return and risk and usually the risk, variance and standard deviation are regarded. In the postmodern theory, optimal portfolio is determined according to the relationship of return and undesired risk (Estrada, 2006). Markowitz model relates to the modern theory. He is the first one who formally introduced the concept of portfolio diversification. He assumed that risk is the same significant as the return. Optimal portfolio is a portfolio which has the least risk for a given return and a given risk for the most return. He proposed that the expected return is a random variable with the normal distribution which is completely described by two parameters of mean and variance. However, normal distribution of the return assumption does not always hold true and the probability of abnormal gains and losses is more than what the normal distribution anticipated (Giorgi and Post, 2008). On the other hand, variance as a risk measure is not tangible and requires statistical information (Giorhi, 2002). These problems caused new studies about portfolio. Integrated risk theory is one of the new approaches.

Huisman, Koedijk and Pownall (1999) used this method for optimizing a portfolio including the properties of stock and security markets in the United States of America. Fan et al (2004) found the value at risk of the China Stock Exchange by using variance-covariance method at 95 percent of significance level. The findings revealed the comparison of anticipated value at risk and real return which was significant at 95 percent. Pulz (1999) conducted the optimization of a portfolio by four methods of mean-variance, Minimax, random programming and aggregated convergence and confirmed the mean-variance method as the best one. Jorion (1985) and Xu (2003) suggested a reduction method of the risk related to the investment portfolios which is called portfolio diversification. De Vassal (2001) applied Monte-Carlo technique to show the stock risk reduction by maintaining a portfolio of different stocks. Raganathan and Mitchell (1997) analyzed the correlation between the stock returns as a base for reducing the portfolio risk and showed that portfolios with the negative correlation or very low positive correlation will contribute risk declining. This pattern was tested empirically by Gilmorea and McManu in 2002.

METHODOLOGY

The present paper is an applied study with the non-experimental correlation orientation. The statistical sample covers all companies listed on Tehran Stock Exchange for a one year period during 2010 by using a filtering technique and considering the following criteria:

1. The companies should have been listed on the stock exchange until the end of 2005.
2. The companies should not be classified as insurance, investment and financial intermediaries,
3. There should have been no change of the fiscal year during the investigation period.
4. The required data about calculating the research variables should have been available.
5. There should have been no transaction cease for more than three months.

Therefore, 171 companies were selected from those firms with the above characteristics. Finally, 15 companies with the highest positive returns were selected among the limited society after sorting according to the real return. Different software was applied to analyze the data including the daily prices and returns of the software packages of Minitab, EXCELL and Lingo.

4- Calculating the Variables

Stock return variance (standard deviation) is used as a quantitative measure to determine the risk per share. The following relationships are to calculate the individual stock return and the risk and return of the portfolio:

$$(Kiani, 2010,447) .(1)$$

$$\sigma_p^2 = \sum_{i=1}^n \sigma_i^2 W_i^2 + 2 \sum_{i=1}^n \sum_{j=1}^n W_i W_j \text{Cov}(i,j)$$

$$(Karan et. Al, 2010,1217) .(2)$$

$$R_{it} = \left(\frac{P_{it} - P_{it-1}}{P_{it-1}} \right) * 100$$

$$(Karan et. Al, 2010,1217) .(3)$$

$$R_p = \sum_{i=1}^n W_i R_i$$

Equation (1) indicates the portfolio risk as a weighted mean of the stock risk by applying the correlation impact. Where in it, the number of the stocks in a portfolio is n ; σ_i^2 is the variance of stock i , and W_i and W_j are the weights of stocks i and j , respectively. Equation (2) indicates R_{it} as the stock return of stock i in t period. P_{it} and P_{it-1} reveal the price of stock i in t and $t-1$ period. This relationship gains the periodic return of each stock. According to equation 3, the portfolio return is obtained by the weighted average of the stock returns. The elements are the same as the previous one.

The following relationships are used to calculate the value at risks of the individual stocks and portfolios. In this relationship, values at risk can be measured for different time horizons and confidence levels.

$$(Alexander, 2001,264) .(4)$$

$$VaR_i = M_i \sigma_i Z_\alpha \sqrt{T}$$

$$(Alexander, 2001,267) .(5)$$

$$VaR_p = X \sigma_p Z_\alpha \sqrt{T}$$

Equations 4 and 5 are used to calculate the value at risk per share and the stock portfolio. In any of these relationships, T is the future time horizon and Z_α indicates a point on the normal probability graph in which the error is α percent. In equation 5, σ_i is the standard deviation of the stock i , M_i is the market value of the stock i or the rate of the stock i in the last day with the available information. Moreover, σ_p is the standard deviation of the portfolio and X is the value of the stock portfolio.

5- Data Analysis and Determining the Optimal Portfolio

5-1- Calculating Return, Risk and VaR

In this section, we first calculate return, risk (standard deviation of the return) and VaR for 15 selected companies. In doing so, the return and standard deviation of the return (risk) is calculated for a daily time horizon. Additionally, value at risk for each share is also calculated for the above mentioned time horizon and significance level of 95 percent. These are shown in table 1.

Table (1): Mean and Standard Deviation and Value at Risk of the Return

| Company | Daily Return Mean | Daily Return S.d | Daily VaR (95%) |
|-----------|-------------------|------------------|-----------------|
| kama | 0.000751 | 0.01746 | 472.665 |
| Faravar | 0.000535 | 0.05177 | 571.88 |
| Defra | 0.000433 | 0.01673 | 129.24 |
| Haptro | 0.00372 | 0.02618 | 919.799 |
| Track | -0.002582 | 0.05506 | 239.292 |
| Kgol | -0.001330 | 0.05094 | 832.581 |
| Dzheravi | 0.001338 | 0.01502 | 392.118 |
| Foka | 0.000004 | 0.00041 | 3.199 |
| Codema | -0.000409 | 0.0219 | 98.9392 |
| Diran | 0.001129 | 0.02781 | 298.83 |
| Sabad | -0.002620 | 0.00723 | 13.4903 |
| Daro | 0.000173 | 0.03808 | 476.12 |
| Fasmin | -0.001377 | 0.06265 | 1193.2 |
| Floleh | -0.003298 | 0.03773 | 196.195 |
| Ghanghash | 0.004429 | 0.0238 | 642.159 |

5-2- Covariance Matrix of the Returns

The covariance matrix of the returns is formed in a pair format for a daily time horizon to determine the stock portfolio and finally measure the risk of the portfolio. Table 2 demonstrates these findings in the form of a matrix.

Table2. Covariance Matrix of the Daily Returns

| | kama | Faravar | Defra | Haptro | Track | Kgol | Dzheravi | Foka | Codema | Diran | Sabad | Daro | Fasmin | Floleh | Ghanghash |
|-----------|-------|---------|--------|--------|-------|-------|----------|--------|--------|-------|-------|-------|--------|--------|-----------|
| kama | 4.57 | -0.43 | -0.08 | -0.02 | -1.45 | 0.83 | 0.65 | 0.02 | 0.55 | 1.61 | -0.40 | 0.37 | 4.27 | -0.19 | -2.15 |
| Faravar | -0.43 | 87.94 | 0.85 | -2.36 | 0.82 | 4.86 | -0.20 | -0.001 | 2.31 | 1.33 | -0.65 | 0.21 | -11.63 | -1.99 | -3.85 |
| Defra | -0.08 | 0.85 | 2.78 | 0.28 | -0.20 | 0.48 | 0.19 | -0.001 | 0.80 | 0.52 | -0.01 | 0.03 | 0.10 | 0.01 | -0.22 |
| Haptro | -0.02 | -2.36 | 0.28 | 11.68 | -1.95 | -0.59 | 0.17 | -0.001 | -0.87 | 1.33 | 0.04 | 0.27 | 3580 | 0.17 | 0.83 |
| Track | -1.45 | 0.82 | -0.20 | -1.95 | 8.67 | 0.26 | -1.12 | -0.01 | -0.20 | -1.15 | -0.38 | 0.17 | -5.59 | -0.83 | 3.24 |
| Kgol | 0.83 | 4.86 | 0.48 | -0.59 | 0.26 | 4.16 | -0.88 | 0.01 | 1.55 | 0.72 | -0.41 | 0.47 | 2.38 | 0.04 | -1.79 |
| Dzheravi | 0.65 | -0.20 | 0.19 | 0.17 | -1.12 | -0.88 | 1.49 | -0.001 | -0.71 | -0.39 | -0.01 | -0.02 | -0.21 | 0.08 | -0.66 |
| Foka | 0.02 | -0.001 | -0.001 | -0.001 | -1.01 | 0.01 | -0.001 | 0.00 | -0.01 | 0.02 | 0.00 | 0.01 | -0.02 | -0.001 | -0.01 |
| Codema | 0.55 | 2.31 | 0.80 | -0.87 | -0.20 | 1.55 | 0.71 | -0.01 | 5.11 | 0.44 | 0.01 | 1.10 | -2.32 | -0.16 | -0.60 |
| Diran | 1.61 | 1.33 | 0.52 | 1.33 | -1.15 | 0.72 | -0.39 | 0.02 | 0.44 | 16.38 | -0.16 | -0.45 | -0.55 | 1.10 | -2.02 |
| Sabad | -0.40 | -0.65 | -0.01 | 0.04 | -0.38 | -0.41 | -0.01 | 0.00 | 0.01 | -0.16 | 0.43 | -0.01 | -1.19 | -0.01 | -0.02 |
| Daro | 0.27 | 0.21 | 0.03 | 0.27 | 0.17 | 0.47 | -0.02 | 0.01 | 1.10 | -0.45 | -0.01 | 2.82 | 0.10 | -0.10 | 0.85 |
| Fasmin | 4.27 | -11.63 | 0.10 | 3580 | -5.59 | 2.38 | -0.21 | -0.02 | -2.32 | -0.55 | -1.19 | 0.10 | 156.00 | -2.45 | -1.91 |
| Floleh | -0.19 | -1.99 | 0.01 | 0.17 | -0.83 | 0.04 | 0.08 | -0.001 | -0.16 | 1.10 | -0.01 | -0.10 | -2.45 | 9.12 | -0.55 |
| Ghanghash | -2.15 | -3.85 | -0.22 | 0.83 | 3.24 | -1.79 | -0.66 | -0.01 | -0.60 | -2.02 | -0.02 | 0.85 | -1.91 | -0.55 | 9.00 |

5-3- Calculating the Return of the Total Portfolio

Return of the portfolio (R_p) is measured by using the average return of each stock and the primary weights (the proposed weight of the optimization). Table 3 exhibits the calculation process. Total number of the investment is supposed to be 15000.

Table3. Total Return of Daily Portfolio pre-optimization

| Company | Weight of the stock pre-optimization | The number of the stock pre-optimization | Average of daily returns | $W_i * R_i$ |
|---|--------------------------------------|--|--------------------------|--------------|
| kama | $W_1=1/15$ | 1000 | 0.000751 | 0.000050317 |
| Faravar | $W_2=1/15$ | 1000 | 0.000535 | 0.000035845 |
| Defra | $W_3=1/15$ | 1000 | 0.000433 | 0.000029011 |
| Haptro | $W_4=1/15$ | 1000 | 0.00372 | 0.00024924 |
| Track | $W_5=1/15$ | 1000 | -0.002582 | -0.000172994 |
| Kgol | $W_6=1/15$ | 1000 | -0.001330 | -0.00008911 |
| Dzheravi | $W_7=1/15$ | 1000 | 0.001338 | 0.000089646 |
| Foka | $W_8=1/15$ | 1000 | 0.000004 | 0.000000268 |
| Codema | $W_9=1/15$ | 1000 | -0.000409 | -0.000027403 |
| Diran | $W_{10}=1/15$ | 1000 | 0.001129 | -0.000075643 |
| Sabad | $W_{11}=1/15$ | 1000 | -0.002620 | -0.00017554 |
| Daro | $W_{12}=1/15$ | 1000 | 0.000173 | 0.000011591 |
| Fasmin | $W_{13}=1/15$ | 1000 | -0.001377 | -0.000092259 |
| Floleh | $W_{14}=1/15$ | 1000 | -0.003298 | -0.000220966 |
| Ghanghash | $W_{15}=1/15$ | 1000 | 0.004429 | 0.000296743 |
| Total Return of Daily Portfolio pre-optimization | | | | 0.00006 |

5-4- Risk Determination and the Optimal Weights of the Stocks

Using a linear programming, the optimal risk level (portfolio variance) and optimal weights are determined. The general format of calculating the optimal value σ_p^2 (risk) and W_i (weight) for 15 selected companies is in accordance with model 1:

$$\begin{aligned}
 &M1: \\
 &\text{Min } \sigma_p^2 W_i \\
 &s.t: \\
 &\sum_{i=1}^{15} W_i = 1 \\
 &\sum_{i=1}^{15} W_i R_i \geq R_p \\
 &W_i \geq 0, \quad i = 1, \dots, 15
 \end{aligned}$$

M2:

$$\begin{aligned} \text{Min } \sigma_p^2 = & 0.000457 W_1^2 + 0.008794 W_2^2 + 0.000278 W_3^2 + 0.001168 W_4^2 + 0.000867 W_5^2 + 0.000416 W_6^2 + \\ & 0.000449 W_7^2 + 0.000000 W_8^2 + 0.000511 W_9^2 + 0.001638 W_{10}^2 + 0.000043 W_{11}^2 + 0.000282 W_{12}^2 + 0.015600 W_{13}^2 \\ & + 0.000912 W_{14}^2 + 0.00900 W_{15}^2 - 0.000086 W_1 W_2 - 0.000016 W_1 W_3 - 0.000000 W_1 W_4 - 0.00029 W_1 W_5 + \\ & 0.000166 W_1 W_6 + 0.00013 W_1 W_7 + 0.000004 W_1 W_8 + 0.00011 W_1 W_9 + 0.00322 W_1 W_{10} - 0.00008 W_1 W_{11} + \\ & 0.000054 W_1 W_{12} + 0.000854 W_1 W_{13} - 0.000038 W_1 W_{14} - 0.00043 W_1 W_{15} + 0.00017 W_2 W_3 - 0.000472 W_2 W_4 + \\ & 0.000164 W_2 W_5 + 0.000972 W_2 W_6 - 0.00004 W_2 W_7 - 0.0000002 W_2 W_8 + 0.000462 W_2 W_9 + 0.000266 W_2 W_{10} - \\ & 0.00013 W_2 W_{11} + 0.000042 W_2 W_{12} - 0.002326 W_2 W_{13} - 0.000398 W_2 W_{14} - 0.0077 W_2 W_{15} + 0.000056 W_3 W_4 - \\ & 0.00004 W_3 W_5 + 0.000096 W_3 W_6 + 0.000038 W_3 W_7 - 0.0000002 W_3 W_8 + 0.00016 W_3 W_9 + 0.000154 W_3 W_{10} - \\ & 0.000002 W_3 W_{11} + 0.000006 W_3 W_{12} + 0.00002 W_3 W_{13} + 0.000002 W_3 W_{14} - 0.000044 W_3 W_{15} - 0.00039 W_4 W_5 - \\ & 0.000118 W_4 W_6 + 0.000034 W_4 W_7 - 0.0000002 W_4 W_8 - 0.000174 W_4 W_9 + 0.000266 W_4 W_{10} + 0.000008 W_4 W_{11} + \\ & 0.000054 W_4 W_{12} + 0.00716 W_4 W_{13} + 0.000034 W_4 W_{14} + 0.000166 W_4 W_{15} + 0.000052 W_5 W_6 - 0.000224 W_5 W_7 - \\ & 0.000002 W_5 W_8 - 0.00004 W_5 W_9 - 0.00023 W_5 W_{10} - 0.000076 W_5 W_{11} + 0.000034 W_5 W_{12} - 0.001118 W_5 W_{13} - \\ & 0.000166 W_5 W_{14} + 0.000648 W_5 W_{15} - 0.000176 W_6 W_7 + 0.000002 W_6 W_8 + 0.00031 W_6 W_9 + 0.000144 W_6 W_{10} - \\ & 0.000082 W_6 W_{11} + 0.000094 W_6 W_{12} + 0.000476 W_6 W_{13} + 0.000008 W_6 W_{14} - 0.000358 W_6 W_{15} - 0.0000002 W_7 W_8 \\ & + 0.000142 W_7 W_9 - 0.000078 W_7 W_{10} - 0.000002 W_7 W_{11} - 0.000004 W_7 W_{12} - 0.000042 W_7 W_{13} + 0.000016 W_7 W_{14} \\ & - 0.000132 W_7 W_{15} - 0.000002 W_8 W_9 + 0.000004 W_8 W_{10} + 0.000000 W_8 W_{11} + 0.000002 W_8 W_{12} - 0.000004 W_8 W_{13} \\ & - 0.0000002 W_8 W_{14} - 0.000002 W_8 W_{15} + 0.000088 W_9 W_{10} + 0.00002 W_9 W_{11} + 0.00022 W_9 W_{12} - 0.000264 W_9 W_{13} \\ & - 0.000032 W_9 W_{14} - 0.00012 W_9 W_{15} - 0.000032 W_{10} W_{11} - 0.00009 W_{10} W_{12} - 0.00011 W_{10} W_{13} + 0.00022 W_{10} W_{14} \\ & - 0.000404 W_{10} W_{15} - 0.000002 W_{11} W_{12} - 0.000238 W_{11} W_{13} - 0.000002 W_{11} W_{14} - 0.000004 W_{11} W_{15} + 0.00002 \\ & W_{12} W_{13} - 0.00002 W_{12} W_{14} + 0.00017 W_{12} W_{15} - 0.00049 W_{13} W_{14} - 0.000382 W_{13} W_{15} - 0.00011 W_{14} W_{15} \end{aligned}$$

S.t:

$$W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10} + W_{11} + W_{12} + W_{13} + W_{14} + W_{15} = 1$$

$$0.000751 W_1 + 0.000535 W_2 + 0.000433 W_3 + 0.00372 W_4 - 0.00258 W_5 - 0.00133 W_6 + 0.00134 W_7 + 0.000004 W_8 - 0.000409 W_9 + 0.001129 W_{10} - 0.002620 W_{11} + 0.000173 W_{12} - 0.00138 W_{13} - 0.00330 W_{14} + 0.00443 W_{15} \geq 0.00006$$

$$W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10}, W_{11}, W_{12}, W_{13}, W_{14}, W_{15} \geq 0$$

The first linear programming is calculated to determine the weight of each stock for the purpose of minimizing the total investment risk of a portfolio with a constant return level. This is achieved by considering the weight of the stocks and total return of a portfolio. The risk level of investment in a portfolio can be minimized by solving model 2.

The findings related to the model solution are as follows:

$$\sigma_p^{2*} = 0$$

$$W_1^* = 0$$

$$W_6^* = 0$$

$$W_{11}^* = 14.6$$

$$W_2^* = 0$$

$$W_7^* = 9$$

$$W_{12}^* = 10.8$$

$$W_3^* = 11$$

$$W_8^* = 37.5$$

$$W_{13}^* = 0$$

$$W_4^* = 8.4$$

$$W_9^* = 0$$

$$W_{14}^* = 0$$

$$W_5^* = 5.1$$

$$W_{10}^* = 2.6$$

$$W_{15}^* = 1$$

As it can be seen, the outputs of the model show that there should be at least 9 investments and as much as the determined percentages of the investment model, in order to minimize the portfolio risk. Additionally, the percentage assigned to 6 stocks in the optimum portfolio is zero. The results demonstrated in table 4 shows that the portfolio return after optimization is as much as the return before the optimization. They are different in terms of portfolio risk which is zero in the optimum portfolio. This risk is measured through standard deviation of the portfolio.

In addition, the findings indicate that holding an optimum portfolio and investment in a daily time horizon would have no loss more than 132818.95 Rials (the amount of VaR) with the probability of 95 percent. This maximum amount is achieved through the following relationship, so that the figure of 80741 shows portfolio value, $\sqrt{0}$ is the standard deviation of the portfolio and the figure of 1.645 and $\sqrt{1}$ are indicators of Z_α and the effect of holding portfolio, respectively.

$$80741 * \sqrt{0} * 1.645 * \sqrt{1} = 132818.95$$

6- Conclusion

As stated previously, model 2 is solved by linear programming and Lingo software package for which the outputs include the weight of each stock in optimum portfolio and portfolio risk (standard deviation of the portfolio). The output of the model demonstrates that the weight of 6 stocks in the final model is zero and only 9 stocks are selected for the investment for the purpose of risk minimization. Investments in these 9 stocks can be accompanied by using the advantages of investment diversification in 15 stocks (i.e., the return on portfolio when investing in 15

stocks) along with risk minimization (i.e. when the standard deviation of the portfolio is zero). In other words, if 1000 units are invested before optimization and the weight of each stock in the portfolio is 6.7%, then some points are found. These findings show that for the first and the second stocks, there should be an investment equal to zero and this amounts to $15000 * 11\% = 1650$ units for the third stock. Table 4 depicts the total return of the portfolio after optimization. In this table, the number and the weight of each stock can be observed in the optimum portfolio. Since the linear programming mainly aims to minimize the risk with a constant return, then the calculated return of the portfolio after optimization is also equal to the return before optimization and is as much as 0.00006 Rials. Therefore, it can be concluded that the main question of this study is responded. This question was related to examining the possibility of optimizing the investment by using a linear programming model in the form of VaR.

Table4. Total Return of Daily Portfolio post-optimization

| Company | Weight of the stock post-optimization | The number of the stock post-optimization | Average of daily returns | $W_i * R_i$ |
|--|---------------------------------------|---|--------------------------|-------------|
| kama | W1= 0 | 0 | 0.000751 | 0 |
| Faravar | W2= 0 | 0 | 0.000535 | 0 |
| Defra | W3= 0.11 | 1650 | 0.000433 | 0.00004763 |
| Haptro | W4= 0.084 | 1260 | 0.00372 | 0.00031248 |
| Track | W5= 0.051 | 765 | -0.002582 | -0.00013682 |
| Kgol | W6= 0 | 0 | -0.001330 | 0 |
| Dzheravi | W7= 0.09 | 1350 | 0.001338 | 0.00012042 |
| Foka | W8= 0.375 | 5625 | 0.000004 | 0.0000015 |
| Codema | W9= 0 | 0 | -0.000409 | 0 |
| Diran | W10= 0.026 | 390 | 0.001129 | 0.000029354 |
| Sabad | W11= 0.146 | 2190 | -0.002620 | -0.00038252 |
| Daro | W12= 0.108 | 1620 | 0.000173 | 0.000018684 |
| Fasmin | W13= 0 | 0 | -0.001377 | 0 |
| Fleleh | W14= 0 | 0 | -0.003298 | 0 |
| Ghanghash | W15= 0.01 | 150 | 0.004429 | 0.0004429 |
| Total Return of Daily Portfolio post-optimization | | | | 0.00006 |

7- Suggestions

The optimal stock portfolio can be determined by ascertaining the value at risk of the portfolio with the other stocks through covariance calculations. According to the findings of optimization and the optimal portfolio resulted from the linear programming, the accuracy of this model in determining the risk and optimal weights is indicated. Therefore, the investors and portfolio managers are offered to use the advantages of value at risk technique and the high accuracy of the programming model in optimization. Comparing the classic optimization models (describing the risk of the total portfolio) with the modern models (concentrated on the undesired risks), it is suggested to calculate and measure the optimal portfolio by considering these models. It is also suggested to calculate VaR in different models and for long time horizons such as monthly, quarterly and annual periods.

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