

# A New Method for Determining Most Congest Decision Making Unit in Data Envelopment Analysis

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## ABSTRACT

One of the problems in data envelopment analysis (DEA) is congestion concept. This is the subject of debates for researchers since 2 decades. Congestion in data envelopment analysis means reducing one or more input and increasing one or more output without worsening any of the other inputs or outputs. There are several ways to calculate congestion that are investigated in different papers. The existent methods determine congestion decision making units. In this paper, we tried to gain most congest decision making unit (DMU) that with smaller calculations so we can have the most congest DMU very quickly. We explained the application of our new method by 2 numerical examples.

**KEY WORDS:** data envelopment analysis, congestion.

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## 1- INTRODUCTION

Data envelopment analysis (DEA) invented by Charnes, Cooper and Rhodes (1987) [3]. It can determine the relative efficiency of the decision making units (DMU) with multiple inputs and outputs. There are different subjects in DEA debates including some very important subjects that managers must consider; one of them is congestion.

Since two decades ago, one of the most important subjects for researchers is congestion. At first, investigating about congestion began by Fare and Svensson (1980) [9]. In the past, this subject was unknown. Then this subject was completed by Fare and Grosskopf (1983) [7]. They presented a model according to the concepts of data envelopment analysis. At that time it was the only existing method for evaluating congestion. Then FGL method was presented by Fare, Grosskopf and Lovell (1985) [8] which is the first method in DEA for evaluating congestion. In this method; you can solve two models of DEA models and calculate the congestion. After that the speed of investigation in congestion increased wonderfully. BCSW presented by Cooper and Thompson and Thrall (1996) [6]. Then in (1998), Brockett performed it on real data. Then this method was presented by Brockett, Cooper, Shin and Wang [2] in a paper that evaluated congestion by solving 2 models in DEA in two steps and also it calculated the source and the congestion amount. BCSW can determine the congestion for each input but it can't present a total amount for congestion. So Cooper, Seiford and Zhu (2000) [5], corrected this model and used other new models for measuring congestion and analyzing inefficiencies. All the methods before Cooper's single model were two-model. Cooper (2002) [4], used BCSW and combined the 2 steps and presented one new model. Jahanshahloo et al (2010) [10] presented a new model for calculating congestion for each DMU in which the calculations are reduced wonderfully.

It is said in Cooper's paper [6] that congestion is when increasing one or more input cause decreasing one or more outputs without improving other inputs or outputs or conversely; it will occur when decreasing one or more inputs lead to increasing one or more outputs without worsening other inputs or outputs.

For decision makers who don't pay attention to congestion amount and finding all the DMUs with congestion; but they are interested in finding the most congest DMU, this paper presents a method to calculate all the concepts that are mentioned in other papers but in less calculations for finding the most congest DMU.

The rest of the paper is organized as follows:

In section 2, we discuss a summary of congestion paper by Jahanshahloo et al [10]. Then in section 3, we present the new method for calculating the most congest DMU. In section 4 we analyze the data and in section 5, you will have the conclusion.

## 2- The method of Jahanshahloo et al

There are many methods and models for finding congestion and in this paper we present a summary of method of Jahanshahloo et al [10].

In this method for finding congestion, BCC model in output oriented will be solved and then optimal solutions for each DMU will be found. Then set "E" is define as  $E = \{j | \varphi_j^* = 1\}$  Among the DMUs in set "E"; the highest

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input amount for each input components of the vector will be found and then they show it as  $X^*$ . At the end, they define congestion as follows:

If at least one of the following conditions is present  $DMU_0$  will have congestion.

- a)  $\varphi^* > 1$  and at least one  $i$  ( $1 \leq i \leq m$ ) is present then  $x_{io} > x_i^*$
- b) There must be at least one  $r$  ( $1 \leq r \leq s$ ) that  $s_r^{+*} \geq 0$  and at least one  $i$  ( $1 \leq i \leq m$ ) is present then  $x_{io} > x_i^*$

Then, when we have  $x_{io} > x_i^*$ ; they define the “i” input congestion amount as  $S_i^C = x_{io} - x_i^*$ . If  $x_{io} \leq x_i^*$  then they say there are not congestion and  $S_i^C = 0$ . The sum of all  $S_i^C$ s shows the congestion amount in  $DMU_0$ .

In the paper of Jahanshahloo et al there are theorems for show that their method is equal with cooper method.

Theorem 1: if  $DMU_o^* = (x_1^*, x_2^*, \dots, x_m^*, \varphi^* y_{1o} + s_1^{+*}, \varphi^* y_{2o} + s_2^{+*}, \dots, \varphi^* y_{so} + s_s^{+*})$  then  $DMU_o^* \in PPS_{T_v}$ .

Proof: refer to [10].

Theorem 2: when  $x_{io} \geq x_i^*$  then  $S_i^{C*} = S_i^{\hat{C}}$  that  $S_i^{\hat{C}}$  determine with solve under model in the cooper [4] method.

Proof: refer to [10].

$$\begin{aligned}
 & \text{Min} \quad \sum_{i=1}^m s_i^{c'} \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} + s_i^{c'} = x_{io} \quad i = 1, \dots, m \\
 & \sum_{j=1}^n \lambda_j y_{rj} = \hat{y}_r = \varphi^* y_r + s_r^{+*} \quad r = 1, \dots, s \\
 & \sum_{j=1}^n \lambda_j = 1 \\
 & \lambda_j \geq 0 \\
 & s_i^{c'} \geq 0
 \end{aligned}$$

Theorem 3: if  $x_{io} - x_i^* \leq 0 \quad \forall i$  ( $1 \leq i \leq m$ ) then  $DMU_o$  is not congestion.

Proof: refer to [10].

### 3- The new method for finding the most congest DMU:

Now we present a method in this paper to determine the most congest decision making unit with less calculations in compare with the method presented by Jahanshahloo et al [10].

**Definition 1:** the most congest DMU in a selected component is the DMU that have the highest amount of congestion in that component.

First, all the DMUs that have the most inputs for each input component must be determined. There is always a DMU among the others that have the highest usage for its first inputs. We show this DMU as  $DMU_1$

$$\exists l: \forall j (x_{1l} \geq x_{1j})$$

We show  $x_{1l}$  with  $x_1^*$ . Then we must a DMU among the other DMUs that have the most usage for its second input and then we show it as  $DMU_t$ . It means:

$$\exists t: \forall j (x_{2t} \geq x_{2j})$$

Then we show  $x_{2t}$  with  $x_2^*$ . Then the same way must be performed to find all the input components ( $i=1, \dots, m$ ) and then we find a DMU that its “i” input usage is more than the other DMUs. This input is shown by  $x_i^*$  ( $i=1, \dots, m$ ).

**Note 1:** there is no need to select  $x_1^*, x_2^*, \dots, x_m^*$  of one DMU.

Then we form the set of  $M_i = \{DMU_t | x_{it} \geq x_{ij} \quad j = 1, \dots, n\}$  for  $i=1, \dots, m$ .

Then we solve the model (1) for all the DMUs that are present in  $M_i$  set for  $i=1, \dots, m$ .

$$\begin{aligned}
 & \text{Max} \quad \varphi + \varepsilon \left( \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = x_{io} \quad i = 1, \dots, m \quad (1) \\
 & \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = \varphi y_r \quad r = 1, \dots, s \\
 & \sum_{j=1}^n \lambda_j = 1 \\
 & s_i^- \geq 0 \quad i = 1, \dots, m \\
 & s_r^+ \geq 0 \quad r = 1, \dots, s \\
 & \lambda_j \geq 0 \quad j = 1, \dots, n
 \end{aligned}$$

There will be 2 situations:

- a) If a unit with the most input component is efficient, it means:  $\varphi^* = 1$ ; so we say there is no congestion. We compare it with the rest units with the most related input components and then we say there is no congestion.
- b) If a unit with the most input component is inefficient, it means:  $\varphi^* > 1$ ; so we say there is congestion. In this case we say there is congestion because in compare with the rest units with the most related input components so it is called the most congest DMU in related input component.

We do this for all the DMUs with the most inputs for related component. By this we can find the most congest DMU in all the components of the input vector (if exist).

One of the advantages of this method is we done need to solve “n” model because we can find the solution in the initial steps. This is the merit of this method because can reduce the calculations. It means by solving a model you can determine that the related input component has congestion or not. In fact, solving one model can show us the congestion in input components.

**Note 2:** if more than one inefficient DMU have the most input components, each of the DMUs can be chosen as the most congest DMU for this component.

We repeat under process for finding most congests DMU:

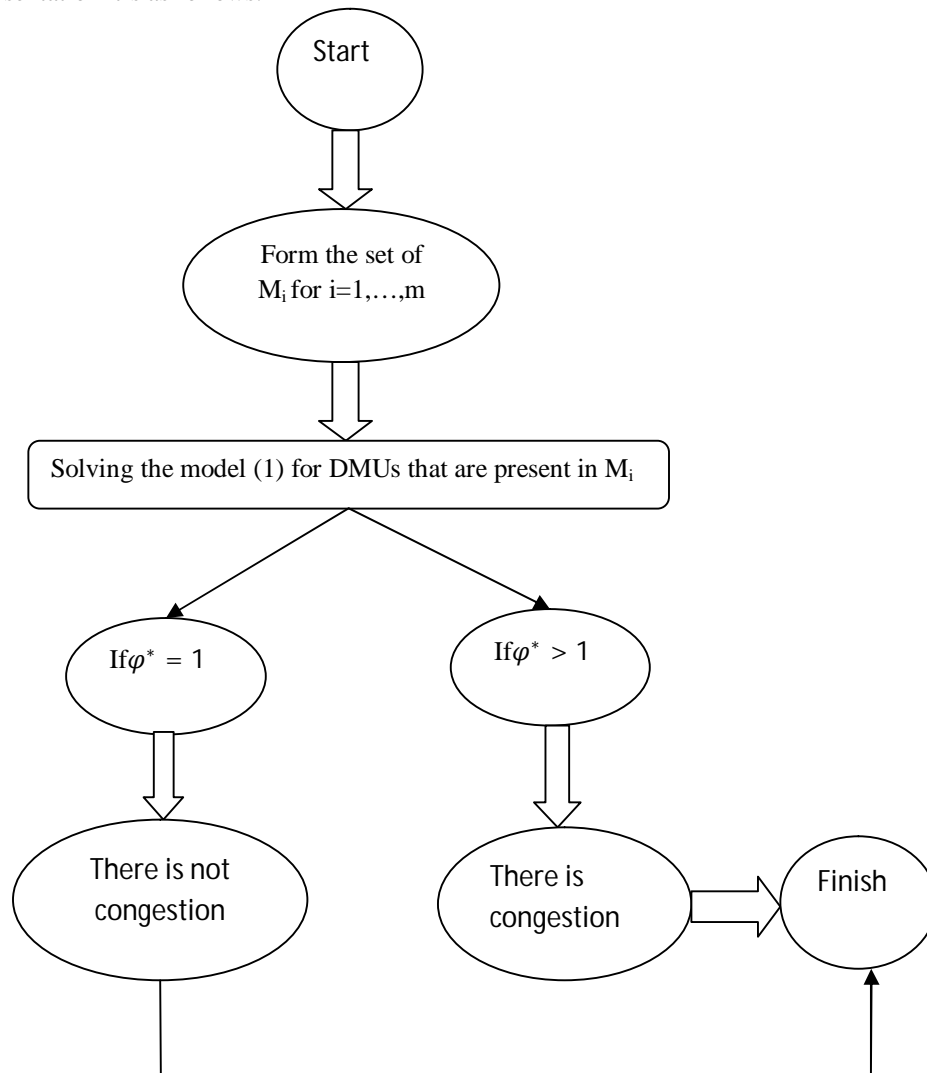
Step1: form the set of  $M_i$  for  $i=1, \dots, m$ .

Step2: solving the model (1) for DMUs that are present in  $M_i$ .

Step 2-1: if  $\varphi^* = 1$  then, we say there is no congestion in related input component.

Step2-2: if  $\varphi^* > 1$  then, we say there is congestion and we say DMU<sub>j</sub> is most congest in related input component.

That flowchart presentation it's as follows:



### 3- Numeric examples

**Example 1:** we discuss our new method for calculating most congest case on actual data of 42 university departments of Islamic Azad University, Karaj Branch (KazemiMatin&Kuosmanen [11]) for 3 inputs and 4 outputs. These data are shown in table 1.

Input variables are: the number of post graduate students: ( $X_1$ ), the number bachelor students: ( $X_2$ ) and the number of master students ( $X_3$ )

Output variables are: the number of graduations: ( $y_1$ ), the number of scholarships: ( $y_2$ ), the number of research products: ( $y_3$ ) and the level of manager satisfaction: ( $y_4$ )

Table 1: data of university departments of Islamic Azad University, Karaj Branch

DMU	$X_1$	$X_2$	$X_3$	$Y_1$	$Y_2$	$Y_3$	$Y_4$
1	0	261	0	225	1	1	3
2	0	170	56	213	2	0	3
3	0	281	70	326	2	0	3
4	0	138	33	159	1	0	2
5	164	0	0	52	1	0	3
6	291	815	0	1014	2	2	2
7	0	0	61	50	0	0	4
8	113	95	0	73	0	0	2
9	0	727	0	675	3	0	3
10	0	773	0	697	2	0	3
11	0	0	66	46	0	0	3
12	346	197	0	132	0	0	1
13	0	988	0	812	8	10	2
14	0	0	34	32	0	0	2
15	0	795	0	601	6	2	2
16	0	672	0	591	6	12	2
17	0	166	0	166	7	0	4
18	0	761	0	761	0	3	2
19	193	124	0	293	0	0	3
20	484	0	0	361	0	0	1
21	0	517	0	434	0	4	2
22	0	548	0	492	1	4	2
23	0	682	0	565	2	3	2
24	0	565	0	423	1	2	2
25	0	603	0	433	1	3	2
26	0	373	0	332	1	1	1
27	0	348	0	328	2	3	3
28	0	0	70	51	0	3	4
29	0	328	0	170	0	1	3
30	0	267	0	123	0	0	3
31	262	0	0	219	3	0	3
32	0	1023	0	794	2	0	4
33	366	995	0	1111	2	2	3
34	0	266	15	238	3	4	3
35	172	375	0	547	4	3	3
36	0	460	0	385	4	8	3
37	223	0	535	232	14	6	4
38	0	1202	58	1158	12	0	3
39	0	1025	61	394	4	1	3
40	0	0	69	50	0	2	4
41	314	0	0	204	0	0	1
42	371	0	0	226	0	0	1

Now for performing our method with the data in table 1; first we must find the most inputs of each component.

In first input component ( $X_1$ ), the highest numerical amount is 484 which is related to  $DMU_{20}$ . Then solving model (1) for  $DMU_{20}$  is  $\varphi_{20}^* = 1$ . So we don't consider  $DMU_{20}$  as the most congest unit. It means the first input component doesn't have congestion (situation "a").

In second input component ( $X_2$ ), the highest numerical amount is 1202 which is related to  $DMU_{38}$ . Then solving model (1) for decision making unit 38 under evaluation we find  $\varphi_{38}^* = 1$ . So we don't consider  $DMU_{38}$  as the most congest unit. It means the second input component doesn't have congestion, too (situation "a").

In third input component ( $X_3$ ), the highest numerical amount is 535 which is related to  $DMU_{37}$ . By solving optimized model (1) we find:  $\varphi_{37}^* = 1$ . So we don't consider  $DMU_{37}$  as the most congest unit. It means the first input component doesn't have congestion (situation "a")

Table 2: finding the most congestion DMU with method jahanshahloo et al for data of university departments of Islamic Azad University, Karaj Branch

DMU	$\varphi^*$	$s_1^{+*}$	$s_2^{+*}$	$s_3^{+*}$	$s_4^{+*}$	$x_1 - x_1^*$	$x_2 - x_2^*$	$x_3 - x_3^*$
1	1.14	0	5.09	0	0.25	-484	-941	-535
2	1	0	0	0	0	-484	-1032	-479
3	1	0	0	0	0	-484	-921	-465
4	1.05	0	0	0.33	0.31	-484	-1064	-502
5	1	0	0	0	0	-320	-1202	-535
6	1	0	0	0	0	-193	-387	-535
7	1	0	0	0	0	-484	-1202	-474
8	1.79	58.28	5.29	0	0	-371	-1107	-535
9	1	0	0	0	0	-484	-475	-535
10	1.01	0	0.48	0.36	0	-484	-429	-535
11	1.10	0	0	1.67	0.70	-484	-1202	-469
12	3	0	2.80	0.40	0	-138	-1005	-535
13	1	0	0	0	0	-484	-214	-535
14	1	0	0	0	0	-484	-1202	-501
15	1.13	0	0	6.34	0	-484	-407	-535
16	1	0	0	0	0	-484	-530	-535
17	1	0	0	0	0	-484	-1036	-535
18	1	0	0	0	0	-484	-441	-535
19	1	0	0	0	0	-291	-1078	-535
20	1	0	0	0	0	0	-1202	-535
21	1.14	0	4.33	0	0.46	-484	-685	-535
22	1.14	0	2.23	0	0.23	-484	-618	-535
23	1.17	0	0	0.22	0	-484	-520	-535
24	1.32	0	1.34	0	0	-484	-637	-535
25	1.32	0	2.31	0	0	-484	-599	-535
26	1.12	0	3.48	0	2.18	-484	-829	-535
27	1	0	3.96	0	0.32	-484	-855	-535
28	1	0	0	0	0	-484	-1202	-465
29	1.28	0	6.39	0	0	-484	-874	-535
30	1.33	2	7	0	0	-484	-935	-535
31	1	0	0	0	0	-222	-1202	-535
32	1	0	0	0	0	-484	-179	-535
33	1	0	0	0	0	-118	-207	-535
34	1.04	0	0.98	0	0	-484	-936	-520
35	1	0	0	0	0	-312	-827	-535
36	1	0	0	0	0	-484	-742	-535
37	1	0	0	0	0	-261	-1202	0
38	1	0	0	0	0	-484	0	-477
39	1.26	0	0	0	0	-484	-177	-474
40	1	0	0	0	0	-484	-1202	-466
41	1.24	0	2.30	0	1.29	-170	-1202	-535
42	1.28	0	1.53	0	0.74	-113	-1202	-535

For investigating the concept of congestion in the paper of Jahanshahloo et al; first we must solve the model of BCC in output oriented (model (1)) and then we consider  $E = \{j|\varphi_j^* = 1\}$ . The amounts for all  $\varphi^*$  are shown in table 2. So the set "E" is as follows:

$E = \{2,3,5,6,7,9,13,14,16,17,18,19,20,27,28,31,32,33,35,36,37,38,40\}$

For next step we must find the highest maximum in set "E" related to "X" components.

The most first input component in "E" set is  $x_1^* = 484$  that is related to  $DMU_{20}$ .

The most second input component in "E" set is  $x_2^* = 1202$  that is related to  $DMU_{38}$ .

The most third input component in "E" set is  $x_3^* = 535$  that is related to  $DMU_{37}$ .

As you can see in table 2; we have  $x_{i0} - x_i^* \leq 0$  for  $i=1,2,3$  so we see no congestion in each inputs. We have seen in our method that there is no congestion in inputs to find the most congest unit.

Example 2: in this example we investigate the mostcongest country among 30 countries of OECD (Bai, Orkcu and Chelebiuglu).

30 countries of OECD with 3 inputs and 5 outputs are considered for calculating the most congest DMU. The data are shown in table 3:

Input 1 ( $X_1$ ): unemployment ratio,

Input 2 ( $X_2$ ): rate of inflation,

Input 3 ( $X_3$ ): baby death rate,  
 Output 1 ( $Y_1$ ): national income per capita (USA dollars),  
 Output 2 ( $Y_2$ ): human development index: life expectancy from birth,  
 Output 3 ( $Y_3$ ): human development index: education index,  
 Output 4 ( $Y_4$ ): contribution rate to labor force of woman population,  
 Output 5 ( $Y_5$ ): health expenditure per capita (USA dollars).

Table 3: data of countries of OECD

DMU	countries	$X_1$	$X_2$	$X_3$	$Y_1$	$Y_2$	$Y_3$	$Y_4$	$Y_5$
1	Australia	5.1	3	6	34740	80.9	0.993	67.4	2036
2	Austria	7.2	1.8	5	37117	79.4	0.966	63.8	1968
3	Belgium	12.1	1.6	6	35712	78.8	0.977	57.3	2081
4	Canada	6.8	2.2	6	35133	80.3	0.991	72.8	2312
5	Czech Republic	8.9	1.8	5	12152	75.9	0.936	64	930
6	Denmark	5.6	2.4	4	47984	77.9	0.993	74.2	2133
7	Finland	8.4	1.7	4	37504	78.9	0.993	72.8	1502
8	France	9.1	1.9	4	33918	80.2	0.982	62.4	205
9	Germany	9.2	2.3	5	33854	79.1	0.953	67.4	2424
10	Greece	9.9	4.6	5	20327	78.9	0.97	56	1167
11	Hungary	7.2	5.3	8	10814	72.9	0.958	53.5	705
12	Iceland	1.8	4.8	4	52764	81.5	0.978	82.9	2103
13	Ireland	4.3	4.7	6	48604	78.4	0.993	62.2	1436
14	Italy	7.7	2.5	6	30200	80.3	0.958	50.1	1783
15	Japan	4.4	1	4	35757	82.3	0.946	60.5	1822
16	South Korea	3.7	2.8	5	16308	79	0.904	49.9	730
17	Luxembourg	4.2	1.1	5	80288	78.4	0.942	55.7	2215
18	Mexico	3.6	5	25	7298	75.6	0.863	42.6	356
19	Netherland	4.3	3.5	5	38618	79.2	0.988	69.5	2070
20	New Zealand	3.7	2.7	6	26464	79.8	0.993	71.2	1424
21	Norway	3.5	1.3	4	64193	79.8	0.991	77.3	2330
22	Poland	18.2	1.9	9	7946	75.2	0.951	57.6	496
23	Portugal	7.6	3.5	6	17456	77.7	0.925	67.8	1237
24	Slovak Republic	11.7	3.3	8	8775	74.2	0.921	62.4	930
25	Spain	9.2	3.1	5	27226	80.5	0.987	57.2	1218
26	Sweden	5.8	2.2	3	39694	80.5	0.978	74.9	1746
27	Switzerland	3.8	0.9	3	50532	81.3	0.946	75.3	2794
28	Turkey	10.3	13.7	38	5816	71.4	0.812	26.5	255
29	England	2.8	1.6	6	37023	79	0.97	69.3	1461
30	USA	5.1	1.6	7	42000	77.9	0.971	70.1	4178

For performing our method; we must find the most congest DMU by data in table 3 so we first must highest input for each component.

In first input component( $X_1$ ), the highest numerical amount is 18.2 which is related to  $DMU_{22}$ . Then by solving model (1) for  $DMU_{22}$  we find that:  $\varphi_{22}^* = 1.04$ . So consider  $DMU_{22}$ ; which is Poland as the most congest unit in first input component (unemployment ratio) (situation "b")

In second input component( $X_2$ ), the highest numerical amount is 13.7 which is related to  $DMU_{28}$ . Then by solving model (1) we find  $\varphi_{28}^* = 1.15$ . So we consider  $DMU_{28}$  which is Turkey as the most congest DMU as the most congest unit in second input component (inflation ratio)(situation "b").

In third input component( $X_3$ ), the highest numerical amount is 38 which is related to  $DMU_{28}$ . By solving optimized model (1) we find:  $\varphi_{28}^* = 1.15$ . So we consider  $DMU_{28}$  which is related to Turkey again as the most congest DMU in third component (baby death ratio) (situation "b")

As you can see, Turkey is chosen as the most congest unit in 2 input components (second and third components). Note 2 is that occur. This has been investigated in section 3.

Table 4: finding the most congestion DMU with method jahanshahloo et al for data of countries of OECD

DMU	$\varphi^*$	$S_1^{+*}$	$S_2^{+*}$	$S_3^{+*}$	$S_4^{+*}$	$S_5^{+*}$	$x_1 - x_1^*$	$x_2 - x_2^*$	$x_3 - x_3^*$	congestion <sub>in</sub> $x_1$	congestion <sub>in</sub> $x_2$	congestion <sub>in</sub> $x_3$
1	1	0	0	0	0	0	-4.1	-1.8	0			
2	1.02	10510.32	0	0	5.52	121.74	-2	-3	-1			
3	1.01	22195.01	0	0	17.35	144.86	2.9	-3.2	0	*		
4	1	0	0	0	0	0	-2.4	-2.6	0			
5	1.06	41169.48	0	0	5.89	1235.10	-0.3	-3	-1			
6	1	0	0	0	0	0	-3.6	-2.4	-2			
7	1	0	0	0	0	0	-0.8	-3.1	-2			
8	1	14750.27	0	0	11.22	0	-0.1	-2.9	-2			
9	1.03	7773.50	0	0	0	0	0	-2.5	-1			
10	1.02	27304.92	0	0	15.86	963.51	0.7	-0.2	-1	*		
11	1.04	23530.92	5.34	0	11.95	1305.24	-2	0.5	2		*	*
12	1	0	0	0	0	0	-7.4	0	-2			
13	1	0	0	0	0	0	-4.9	-0.1	0			
14	1.02	7171.26	0	0	16.20	153.39	-1.5	-2.3	0			
15	1	0	0	0	0	0	-4.8	-3.8	-2			
16	1.04	23391.05	0	0.02	14.68	1139.15	-5.5	-2	-1			
17	1	0	0	0	0	0	-5	-3.7	-1			
18	1.09	33068.90	0	0.02	21.16	1522.07	-5.6	0.2	19		*	*
19	1	4310.17	0	0	10.22	0	-4.9	-1.3	-1			
20	1	0	0	0	0	0	-5.5	-2.1	0			
21	1	0	0	0	0	0	-5.7	-3.5	-2			
22	1.04	28781.84	0.69	0	11.83	1066.25	9	-2.9	3	*		*
23	1.05	27072.35	0	0	3.26	717.60	-1.6	-1.3	0			
24	1.08	18512.64	0	0	3.23	532.93	2.5	-1.5	2	*		*
25	1	19865.46	0	0	16.14	916.20	0	-1.7	-1			
26	1	0	0	0	0	0	-3.4	-2.6	-3			
27	1	0	0	0	0	0	-5.4	-3.9	-3			
28	1.15	29053.12	0	0.01	29.95	1528.07	1.1	8.9	32	*	*	*
29	1	0	0	0	0	0	-6.4	-3.2	0			
30	1.02	8786.89	0.09	0	3.67	0	-4.1	-3.2	1			*

For investigating the concept of congestion in the paper of Jahanshahloo et al; first we must solve the model of BCC in output oriented (model (1)) and then we consider set E. The amounts for all  $\varphi^*$  are shown in table 4. So the set “E” is as follows:

$$E = \{1, 4, 6, 7, 8, 12, 13, 15, 17, 19, 20, 21, 25, 26, 27, 29\}$$

For next step we must find the highest maximum in set “E” related to “X” components.

The most first input component in “E” set is  $x_1^* = 9.2$  that is related to  $DMU_{25}$

The most second input component in “E” set is  $x_2^* = 4.8$  that is related to  $DMU_{12}$

The most third input component in “E” set is  $x_3^* = 6$  that is related to  $DMU_1, DMU_4, DMU_{13}, DMU_{20}, DMU_{29}$

In table 4; there are DMUs that have  $x_{i0} - x_i^* > 0$  for  $i=1,2,3$ . And also you can see the amount of congestion.

DMUs that have congestion in the first input component are:  $DMU_3, DMU_{10}, DMU_{24}, DMU_{28}, DMU_{22}$ . And the amount of congestion in them for first input component is 1.1, 2.5, 3, 0.7, 2.9; respectively. The highest amount is for  $DMU_{22}$ . We had considered  $DMU_{22}$  as the most congest unit in our method, too.

DMUs that have congestion in the second input component are:  $DMU_{11}, DMU_{18}, DMU_{28}$ . And the amount of congestion in them for second input component is 0.5, 0.2, 8.9; respectively. The highest amount is for  $DMU_{28}$ . We had considered  $DMU_{28}$  as the congest unit in our method, too.

DMUs that have congestion in the third input component are:  $DMU_{11}, DMU_{18}, DMU_{22}, DMU_{24}, DMU_{28}, DMU_{30}$ . And the amount of congestion in them for third input component is 2, 19, 3, 2, 32, 1; respectively. The highest amount is for  $DMU_{28}$ . We had considered  $DMU_{28}$  as the congest unit in our method, too.

### 5-Conclusion

In this paper, we present a new method by using congestion paper of Jahanshahloo et al. Now, we can not only determine the congestion amount by solving just one model but we can determine the most congest DMU. So our speed of progress in this method with solving just one model is much more than Jahanshahloo et al method with n models. We discussed the suggested model on real data from 42 university departments of Islamic Azad University, Karaj Branch (Kazemi Matin & Kuosmanen) and for data related to 30 countries of OECD (Bai, Orkcu and Chelebiuglu). As we mentioned in previous section, in this method the most congest DMU is calculated by less

calculations in compare with method of Jahanshahloo et al but we can find the same most congest DMU as they have found in their method.

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