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Ranking DMUs in the Presence of Undesirable Data

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ABSTRACT

Data Envelopment Analysis (DEA) models clusters the DMUs into two groups, namely efficient DMUs and inefficient DMUs. All the DMUs in the efficient group have the same efficiency score, namely 1. DEA cannot provide enough information to rank the efficiency DMUs with the same measure 1. If one further wants to understand which the best is, he/she needs another to discriminate among the efficiency DMUs. There exist many different methods for ranking efficient DMU in DEA. Also, many production processes yield both desirable factors (inputs/outputs) and undesirable ones. Obviously, undesirable factors in production process should be reduced to improve the performance. In this paper we develop three performance indices for ranking DMUs that treat undesirable factors together with desirable factors. The proposed model uses the linear programming problem for efficiency evaluation of DMUs and ranking. The numerical example shows the reasonability of our methods.
KEYWORDS: DEA; Ranking; Undesirable data; Efficiency; L_1Norm method; AP method; MAJ method.

1. INTRODUCTION

An amazing variety of practical problems involving decision making (or system design, analysis, and operation) can be cast in the form of a mathematical optimization problem, or some variation such as a multi-criteria optimization problem. Indeed, mathematical optimization has become an important tool in many areas. It is widely used in engineering, in electronic design automation, automatic control systems, and optimal design problems arising in civil, chemical, mechanical, and aerospace engineering. Optimization is used for problems arising in network design and operation, finance, supply chain management, scheduling, and many other areas. There are some methods for obtaining the solution of optimizing problems (see for example Hasuike [11] and Moengin [27]). In real-world situation, because of incomplete or non-obtainable information, the data are often not so deterministic; therefore they usually are fuzzy/imprecise (see for example Hasuike et al. [12], Yano [38] and Uno et al. [35]). Data envelopment analysis (DEA)is a nonparametric method of measuring the efficiency of a decision-making unit (DMU)such as schools, hospital, or sales outlets. Data envelopment analysis (DEA) is a methodology that uses linear programming in the evaluation of the relative efficiency of decision making units (DMUs) with multiple inputs and outputs. Charnes, Cooper and Rhodes developed Farrell's ideas and the efficiency value that is obtained by dividing single output, to single input was extended to multiple output⁄input ratio and they proposed the CCR model in 1978 [3]. Banker, Charnes and Cooper also proposed a modified model, named BCC in 1984 [4].DEA measure the relative efficiency of decision making units (DMUs) with multiple performance factors which are grouped into outputs and inputs and deals with the ratio between weighted sum of outputs and the weighted sum of inputs. DEA discriminates DMUs into two categories: efficient DMUs and inefficient DMUs. Each DMUs in the efficient category is assigned a set of weights of indices so that its relative efficiency score is equal to one, the maximum. Although efficiency score can be a criterion for ranking inefficient DMUs, this criterion cannot rank efficient DMUs. Therefore selecting the best ranking method or the way of combining different ranking methods for ranking DMUs is an important point in ranking DMUs in DEA. Several authors have proposed methods for ranking the best performers ([13, 4, 30, 39, 26, 33]). For a review of ranking methods, readers are referred to Adler et al. [1]. In some cases, the models proposed by Andersen and Petersen [5] and Mehrabian et al. [26] can be infeasible. In addition to this difficulty, the Andersen and Petersen [5] model may be unstable because of extreme sensitivity to small variations in the data when some DMUs have relatively small values for some of their inputs. Jahanshahlooet al. [17] present a method for ranking extreme efficient decision making units in data envelopment analysis models with constant and variable returns to scale. In their method, they exploit the leave-one-out idea and L_1-norm, also, Jahanshahloo et al. [19] proposed a ranking system for extreme efficient DMUs based upon the omission of efficient DMUs from reference set of the inefficient DMUs. Li et al. [21] developed a super-efficiency model to overcome some deficiencies in the earlier models. Izadikhah [15] proposed a method for ranking decision making units with interval data by introducing two efficient and inefficient frontiers. Wang et al. [37] proposed a methodology for ranking decision making units. That methodology ranks DMUsby imposing an appropriate minimum weight

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restriction on all inputs and outputs, which is decided by a decision maker (DM) or an assessor in terms of the solutions to a series of linear programming (LP) models that are specially constructed to determine a maximum weight for each DEA efficient unit. Liu and Peng [24] proposed a methodology to determine one common set of weights for the performance indices of only DEA efficient DMUs. Then, these DMUs are ranked according to the efficiency score weighted by the common set of weights. For the decision maker, this ranking is based on the optimization of the group's efficiency. Jahanshahloo et al. [18] proposed two ranking methods. In the first method, an ideal line was defined and determined a common set of weights for efficient DMUs then a new efficiency score obtained and ranked them with it. In the second method, a special line was defined then compared all efficient DMUs with it and ranked them. Wang et al. [36] proposed a new methodology based on regression analysis to seek a common set of weights that are easy to estimate and can produce a full ranking for DMUs. Chen and Deng [7] proposed a new method for ranking units. Their method develop a new ranking system under the condition of variable returns to scale (VRS) based on a measure of cross-dependence efficiency, where the evaluation for an efficient DMU is dependent of the efficiency changes of all inefficient units due to its absence in the reference set, while the appraisal of inefficient DMUs depends on the inuence of the exclusion of each efficient unit from the reference set. Recently, RezaiBalf et al. [29] proposed a method for ranking extreme efficient decision making units (DMUs). Their method uses L1(or Tchebycheff) Norm, and it seems to have some superiority over other existing methods, because this method is able to remove the existing difficulties in some methods, such as Andersen and Petersen (AP) that it is sometimes infeasible. Hosseinzadeh Lotfi et al. [14] proposed a method for ranking DMUs. They consider some CCR efficient DMUs, and then rank them by using some ranking methods, each of which is important and significant. Afterwards, by using TOPSIS method, they suggested the ranks of efficient DMUs. Jahanshahloo et al. [16] proposed some different methods and compared them. These DEA models were all formulated for desirable inputs and outputs. In some situation, some of inputs and outputs are undesirable. However, the production of undesirable outputs is unavoidable in certain industries. Certain by-products such as pollution from factories, high default ratio for banking, etc. are examples of undesirable outputs (undesirable factors). Some of the existing approaches are briefly summarized as follows: Pittman [28] evaluated the production efficiency of the paper industry, and considered adding pollutants such as suspended solids, particulate and sulfur oxides as undesirable linear BCC model for the DEA model. Specifically, Banker et al.[4] designed a linear and convexity model capable of simultaneously implementing desirable and undesirable problems. It was mentioned already in Koopmans [20] that the production or waste. Main idea of his approach is to apply some transformations on data. Then the undesirable inputs or outputs will become desirable after these transformations. A symmetric case of input, which should be maximized, may occur, see Allen [2]. Scheel [32] presented some radial measures which assume that any change of output level will involve both desirable and undesirable outputs. Seiford and Zhu [31] proposed a DEA model, in the presence of undesirable outputs, to improve the performance via increasing the desirable outputs and decreasing the undesirable outputs. Lu and Lo [25] classified the alternatives for dealing with undesirable outputs in the DEA. Excellent literature surveys can be found in, for instance Smith [34], Fare, Grosskopf and Lovell [10]. Classical Data Envelopment Analysis (DEA) models rely on the assumption that inputs have to be minimized and output have to be maximized. Fare et al. [9] developed a non-linear DEA model where the desirable outputs are increased and undesirable outputs are decreased. In this current paper, we propose a ranking method with L1-norm in the presence of undesirable data in Data Envelopment Analysis. For this purpose, we first review L_1norm for ranking of DMUs, second, we introduce DEA model with undesirable data finally we propose a method for ranking DMUs using L_1norm in the presence of undesirable data. The methods are illustrated by solving a numerical example. The rest of paper is as follows: In section 2, first we review the standard DEA model, DEA model with undesirable data and Ranking methods. In section 3, we will discuss the proposed models with ranking methods in the presence of undesirable data. By solving these models, we achieve the exact solution. In section 4, we obtain the efficiency score and ranking score of DMUs through numerical example with undesirable data. Conclusions are given in section 5.

2 Preliminaries

In this section we review some required basic concepts as follows:

- **Classic DEA models**
- **Ranking methods**
- **Classic DEA model with undesirable data**
-

Assume that there are n DMUs to be evaluated, indexed by j = 1,…,n. And each DMU is assumed to produce s different outputs from m different inputs. Let the observe input and output vectors of DMUj be Xj = (x_{1j},…,x_{mj})

and Y_j= (y_{1j},…,y_{sj}) respectively, that all component of vectors Xj and Yj for all DMUs are non-negative and each DMU has at least one strictly positive input and output. Now we review the DEA models.

2.1 Classic DEA models

One the basic model used to evaluate DMUs is the input-oriented CCR model introduced by Charnes et al. [3]. The CCR efficiency is obtained by calculating following model:

$$\begin{aligned} & min\theta_o \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \qquad (2\text{-}1) \\ & \sum_{j=1}^{n} \lambda_j y_{ij} \ge y_{ro} \\ & \lambda_j \ge 0 \quad , \qquad j = 1, \dots, n \end{aligned}$$

Definition 2.1. (CCR-efficient) DMUo is CCR – efficient if:

1. θ_o^*= 0
2. All slack variables are zero in alternative optimal solution.

2.2 Ranking methods

In this section, we review some of the existence ranking methods.

2.2.1 AP method

One of the well-known methods for ranking the efficient DMUs is AP method which is introduced by Andersen and Petersen [5] and called super-efficiency method. Super efficiency model proposed for ranking efficient units and is defined as follows:

$$\begin{aligned} & \min \theta_o \\ s.t. \quad & \sum_{\substack{j=1 \\ j\neq O}}^{n} \lambda_j x_{ij} \le \theta_o x_{iO} \qquad (2.2) \\ & \sum_{\substack{j=1 \\ j\neq O}}^{n} \lambda_j y_{rj} \ge y_{rO} \\ & \lambda_j \ge 0 \,, j = 1,2, \dots, n \end{aligned}$$

Now, without solving the CCR model, one can rank efficient DMUs by simply solving the super-efficiency model. If we denote the optimal solution of model (2.2) by θ_o^*then we can rank DMUs according to decreasing order of θ^*.

2.2.2 Norm L1 method

Assume that there are n DMUs to be evaluated, indexed by j = 1,…, n, and each DMU_j is assumed to produces different outputs yrj from m different inputs xij. The production possibility set $T_c$is defined as

$$T_c = \left\{ \binom{X}{Y} \middle| \; X \ge \sum_{j=1}^{n} \lambda_j x_j \;, \qquad Y \le \sum_{j=1}^{n} \lambda_j y_j \;, \lambda_j \ge 0 \,, \qquad j = 1,2, \dots, n \right\}.$$

Jahanshahloo et al. [17] introduced ranking model L_1-norm in data envelopment analysis. They assumed that the DMUo is extreme efficient. By omitting (X_O, Y_O)fromT_c(PPS of CCR model), they defined the production possibility$T' = \left\{ \binom{X}{Y} \middle| \; X \ge \sum_{\substack{j=1 \\ j\neq O}}^{n} \lambda_j X_j \;, \;\; Y \le \sum_{\substack{j=1 \\ j\neq O}}^{n} \lambda_j Y_j \;, \; \lambda_j \ge 0 \,, \; j = 1,2, \dots, n \;, \; j \neq o \right\}$.

To obtain the ranking score of DMU_O,they considered the following model:

$$min\, \Gamma_c^o(X,Y) = \sum_{i=1}^{m} |x_i - x_{iO}| + \sum_{r=1}^{s} |y_r - y_{rO}| \qquad (2.3)$$

$$
\begin{aligned}
s.t. \quad & \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} \leq x_i \quad , \quad i = 1,2,\dots,m \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y_{rj} \geq y_r \quad , \quad r = 1,2,\dots,s \\
& x_i \geq 0 \quad , \quad i = 1,2,\dots,m \\
& y_r \geq 0 \quad , \quad r = 1,2,\dots,s \\
& \lambda_j \geq 0 \quad , \quad j = 1,2,\dots,n
\end{aligned}
$$

Where$X = (X_1,\dots,X_m)$, $Y = (Y_1,\dots,Y_s)$and $\lambda = (\lambda_1,\dots,\lambda_{O-1},\lambda_{O+},\dots,\lambda_n)$are the variables of the model (2.3) and Γ_c^o is the distance (X_O, Y_O) from (X ,Y). For converting the non-linear model (2.3) into the linear from, the set $T''$is defined as:T''=$T' \cap \left\{ \binom{X}{Y} \middle| X \geq X_O , Y \leq Y_O \right\}$ .Therefore we can convert the model (2.3) into the following linear model:

$$
\begin{aligned}
min \ \ & \Gamma_c^o(X,Y) = \sum_{i=1}^{m}(x_i - x_{iO}) - \sum_{r=1}^{s}(y_r - y_{rO}) \\
s.t. \quad & \sum_{\substack{j=1 \\ j\neq O}}^{n} \lambda_j x_{ij} \leq x_i \quad , \quad i = 1,2,\dots,m \\
& \sum_{\substack{j=1 \\ j\neq O}}^{n} \lambda_j y_{rj} \geq y_r \quad , \quad r = 1,\dots,s \\
& x_i \geq x_{iO} \quad , \quad i = 1,2,\dots,m \\
& 0 \leq y_r \leq y_{rO} \quad , \quad r = 1,2,\dots,s \\
& \lambda_j \geq 0 \quad , \quad j = 1,2,\dots,n , j \neq O
\end{aligned}
\tag{2.4}
$$

The model (2.4) ranks only the extreme efficient DMUs, and for full ranking we can use the efficiency score for inefficient DMUs.

2.2.3 MAJ method

To solve the important drawbacks of AP models, Mehrabian et al. [26], proposed another model for ranking efficient units. Their proposed model is:

$$
\begin{aligned}
& min \ w_O + 1 \\
s.t. \quad & \sum_{\substack{j=1 \\ j\neq O}}^{n} \lambda_j x_{ij} \leq x_{iO} + w_O 1 \\
& \sum_{\substack{j=1 \\ j\neq O}}^{n} \lambda_j y_{rj} \geq y_{rO} \\
& \lambda_j \geq 0 , j = 1,2,\dots,n , j \neq o
\end{aligned}
\tag{2.5}
$$

The necessary and sufficient conditions for feasibility of MAJ model is that in evaluating of DMU_O ,ory_{rO} = 0; r = 1,…,s or there exists DMU_j , $j \neq o$such that y_{rj}= 0.

2.3 Classic DEA model with undesirable data

Since in real-world situations the undesirable inputs and outputs can be arisen, therefore we must extend the classic DEA models to deal with undesirable data. DEA models of radial type for undesirable inputs and outputs can be found by solving the following input oriented CCR model [22]:

$$min\ \theta - \varepsilon\left(\sum_{i\epsilon\{DI\}} s_i^- + \sum_{i\epsilon\{UI\}} s_i^- + \sum_{r\epsilon\{DO\}} s_r^+ + \sum_{r\epsilon\{UO\}} s_r^+\right)$$

$$s.t.\quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io} \quad ,i\epsilon\{DI\}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} - s_i^- = \theta x_{io} \quad ,i\epsilon\{UI\} \tag{2-5}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad ,r\epsilon\{DO\}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = y_{ro} \quad ,r\epsilon\{UO\}$$

$$\lambda_j \geq 0\ ,j = 1,2,\dots,n\ ,\ s_i^- \geq 0\ ,i = 1,2,\dots,m,\ s_r^+ \geq 0\ ,r = 1,2,\dots,s$$

Where DI ,DO are index set show the desirable inputs and outputs and UI, UO are index set show the undesirable inputs and outputs.

3 Ranking methods in the presence of undesirable data

In this section we extend the existence ranking methods for ranking DMUs in the presence of undesirable data.

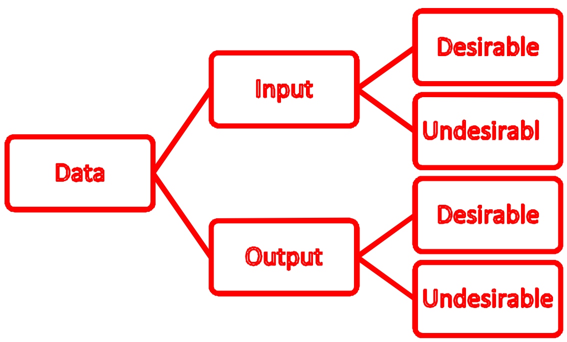

Fig. 1: Undesirable data

3.1 Ranking with L1-norm in the presence of undesirable data

We propose a ranking model based on L1-norm in data envelopment analysis in the presence of undesirable data. Suppose we have n observation on n DMUs. We assume that the DMUo is efficient. In order to find efficient DMUs first we must solve CCR model. By omitting (X_O, Y_O) from Tc (PPS of CCR model), we define the production possibility set T' as follows:

$T' = \left\{\begin{pmatrix} X \\ Y \end{pmatrix} \middle| X \geq \sum_{j=1, j\neq O}^{n} \lambda_j X_j\ ,\ \ Y \leq \sum_{j=1, j\neq O}^{n} \lambda_j Y_j\ ,\ \lambda_j \geq 0\ ,\ j = 1,2,\dots,n\ ,\ j \neq o\right\}$ and the set T'' is defined as:

$T'' = T' \cap \left\{\begin{pmatrix} X \\ Y \end{pmatrix} \middle| X \geq X_O\ , Y \leq Y_O\right\}.$

A classic DEA data domain can be characterized by a data matrix $P = \begin{bmatrix} Y \\ -X \end{bmatrix} = (P_1, P_2, \dots, P_n)$ White s + m rows and n columns. Each column corresponds to one of the DMUs. The jth column $P_j = \begin{pmatrix} Y_j \\ -X_j \end{pmatrix}$ is composed of an input vectors X_j whose ith component xij is the amount of input i used by DMU_j and an output vector Y_j whose rth component y_{rj} is the amount of output r produced by DMU_j. Now suppose that some inputs and outputs are undesirable, so the DEA data domain is expressed as

$$P = \begin{bmatrix} Y^{DO} \\ Y^{UO} \\ -X^{DI} \\ -X^{UI} \end{bmatrix}$$

where DI and DO are index set show the desirable inputs and outputs and UI and UO are index set show the undesirable inputs and outputs. Y^{DO} and X^{DI} represent the desirable (good)outputs and inputs and Y^{DO} and X^{DI} represent the undesirable (bad)outputs and inputs. We consider desirable inputs and outputs as desirable outputs and inputs respectively. Therefore we increase Y^{DO} and X^{DI} and decrease Y^{DO} and X^{DI} , see [31]. Therefore model (2.4) in the presence of undesirable data is changed as following model:

$$
\begin{aligned}
min\ \Gamma_c^O(X,Y) = &\sum_{i\epsilon\{DI\}}(x_i - x_{iO}) + \sum_{r\epsilon\{UO\}}(y_r - y_{rO}) - \sum_{r\epsilon\{DO\}}(y_r - y_{rO}) - \sum_{i\epsilon\{UI\}}(x_i - x_{iO}) \\
s.t. \quad &\sum_{\substack{j=1\\ j\neq O}}^{n} \lambda_j x_{ij} \le x_i \quad , \quad i\epsilon\{DI\} \\
&\sum_{\substack{j=1\\ j\neq O}}^{n} \lambda_j y_{rj} \le y_r \quad , \quad r\epsilon\{UO\} \\
&\sum_{\substack{j=1\\ j\neq O}}^{n} \lambda_j y_{rj} \ge y_r \quad , \quad r\epsilon\{DO\} \qquad (3.1) \\
&\sum_{\substack{j=1\\ j\neq O}}^{n} \lambda_j x_{ij} \ge x_i \quad , \quad i\epsilon\{UI\} \\
&x_i \ge x_{iO} \quad , \quad i\epsilon\{DI\} \\
&0 \le y_r \le y_{rO} \quad , \quad r\epsilon\{DO\} \\
&y_r \ge y_{rO} \quad , \quad r\epsilon\{UO\} \\
&0 \le x_i \le x_{iO} \quad , \quad i\epsilon\{UI\} \\
&\lambda_j \ge 0 \quad , \quad j = 1,2,\dots,n\ , j \neq O
\end{aligned}
$$

3.2 Super-efficiency measure with undesirable data

By using model(2.2) and(2.6) we can obtain the super-efficiency measure in the presence of undesirable data as follows:

$$
\begin{aligned}
&\min \theta_o \\
&\text{s.t.} \quad \sum_{j=1,\ j\neq O}^{n} \lambda_j x_{ij} \le \theta x_{iO} \quad , i\epsilon\{DI\} \\
&\sum_{j=1,\ j\neq O}^{n} \lambda_j x_{ij} \ge \theta x_{iO} \quad , i\epsilon\{UI\} \\
&\sum_{j=1, j\neq O}^{n} \lambda_j y_{rj} \le \theta y_{rO} \quad , r\epsilon\{UO\} \\
&\sum_{j=1, j\neq O}^{n} \lambda_j y_{rj} \ge y_{rO} \quad , r\epsilon\{DO\} \\
&\lambda_j \ge 0\ , j = 1,2,\dots,n
\end{aligned}
\qquad (3.2)
$$

Efficient DMUs have super-efficiency score greater than or equal to 1, while inefficient DMUs have super efficiency score less than 1. The super-efficiency score of the DMUs obtained by the above super-efficiency model can then be ranked in a descending order.

3.3 MAJ model in the presence of undesirable data

By using model(2.5) and(2.6) we can obtain the extension of MAJ model in the presence of undesirable data as follows:

$$
\begin{aligned}
&min\ w_O + 1 \\
&s.t. \quad \sum_{j=1, j\neq O}^{n} \lambda_j x_{ij} \le x_{iO} + w_O 1 \quad , i\epsilon\{DI\} \\
&\sum_{j=1, j\neq O}^{n} \lambda_j x_{ij} \ge x_{iO} \quad , i\epsilon\{UI\}
\end{aligned}
\qquad (3\text{-}3)
$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \le y_{rO} + w_O \qquad , r\epsilon\{UO\}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rO} \qquad , r\epsilon\{DO\}$$

$$\lambda_j \ge 0 , j = 1,2, \dots , n , j \ne o$$

4. Numerical example

In this section, we show the ability of the provided approach using a numerical example. In the last decade, ranking units in data envelopment analysis has become the interests of many DEA researchers and a variety of models were developed to rank units with multiple inputs and multiple outputs. These performance factors (inputs and outputs) are classified into two groups: desirable and undesirable. Obviously, undesirable factors in production process should be reduced to improve the performance. We applied the proposed method for evaluating 17 units, which each unit uses two inputs to produce three outputs. The input factors are, x_1 : Total Cost and x_2: Unit Reputation and The output factors are, y_1 : Number of deliveries on time, y_2 : Number of Bills received without errors, and y3 : Average number of Bills received with errors which is included as an undesirable output. The labels of inputs and outputs are presented in Table 1.

Table 1. The labels of inputs and outputs.

Input	Output
Total Cost	Number of deliveries on time
Unit Reputation	Number of Bills received without errors
	Average number of Bills received with errors

The data set for these 17 units is given in Table 2. Note that, the measures selected in this paper are not exhaustive by any means, but are some general measures that can be utilized to evaluate these DMUs. In an application of this methodology, decision makers must carefully identify appropriate inputs and outputs to be used in the decision making process. Table 2 depicts the DMU's data (inputs and outputs) and theirs CCR efficiencies.

The last two columns of Table 2 is obtained by solving model (2-6). It can be seen that 5 of 17 DMUs have efficiency score 1 with respect to CCR model, implying that they are operating efficiency. For example, the efficiency score for factory 6 is 1 in CCR model. Similarly for DMUs set 1, 3, 4, 6,16. Also, the efficiency score of the rest of 12 DMUs are less than 1, implying that they are operating inefficiency. For example the CCR efficiency score of DMU2 is 0.8516671532 that is unit 2 is operating as an inefficient DMU. We can easily rank the inefficient DMUs by using their CCR efficiency scores but the CCR model can't rank the efficient DMUs (see two last columns of Table 2). In order to rank the inefficient DMUs we use the proposed ranking methods. Results of ranking methods are shown in Table 3 for the purpose of comparison.

Table 2. The data set and related CCR efficiency and their ranking.

DMU_s	Inputs		Outputs			CCR Efficiency	Ranking Using CCR Efficiency
			Desirable outputs		Undesirable output		
	x_{1j}	x_{2j}	y_{1j}	y_{2j}	y_{3j}		
1	253	5	0.055589	0.035986	1	1.0000000000	1
2	268	10	0.057669	0.051979	5.3	0.8516671532	8
3	259	3	0.065398	0.079968	4.6	1.0000000000	1
4	180	6	0.047562	0.03998	30	1.0000000000	1
5	257	4	0.060642	0.06917	30	0.9266121413	7
6	248	2	0.057075	0.067973	30	1.0000000000	1
7	272	8	0.057669	0.02399	30	0.8161079435	10
8	330	11	0.057967	0.057977	13.8	0.6904852752	15
9	327	9	0.059453	0.059976	4	0.7595274305	12
10	330	7	0.050832	0.035986	30	0.5977455247	17
11	321	16	0.051724	0.039984	26.4	0.6255634741	16
12	329	14	0.062128	0.079968	25.8	0.7872340426	11
13	281	15	0.049049	0.065174	25.8	0.7511930929	14
14	309	13	0.059156	0.067973	21.9	0.7516895233	13
15	291	12	0.055886	0.07397	9	0.8232773350	9
16	249	1	0.052616	0.051979	6.3	1.0000000000	1
17	216	18	0.049643	0.063974	28.8	0.9592532615	6

Table 3. The results of using different models for ranking of extreme efficient DMUs in the presence of undesirable data.

DMUs		1	3	4	6	16
AP method	Value:	3.7400299396	1.6468686984	1.0464606461	1.0451040847	2.1756193664
	Rank:	1	3	4	5	2
L_1-Norm method	Value:	0.0599736957	0.0417494340	0.0021116525	0.0029335451	0.0511599158
	Rank:	1	3	5	4	2
MAJ	Value:	3.8391642751	2.8877199862	9.3629162971	1.2193829075	2.3204066043
	Rank:	2	3	1	5	4

It is noteworthy that, we observe that DMUs 1 has rank one with respect to both AP and L_1-Norm methods and clearly we can see the results of two methods AP and L1-Norm are very closely (see Fig. 2). This empirical example which we used to illustrate the nonparametric approach to multilateral productivity demonstrated its potential value in environments in which undesirable factors are existing together with desirable factors.

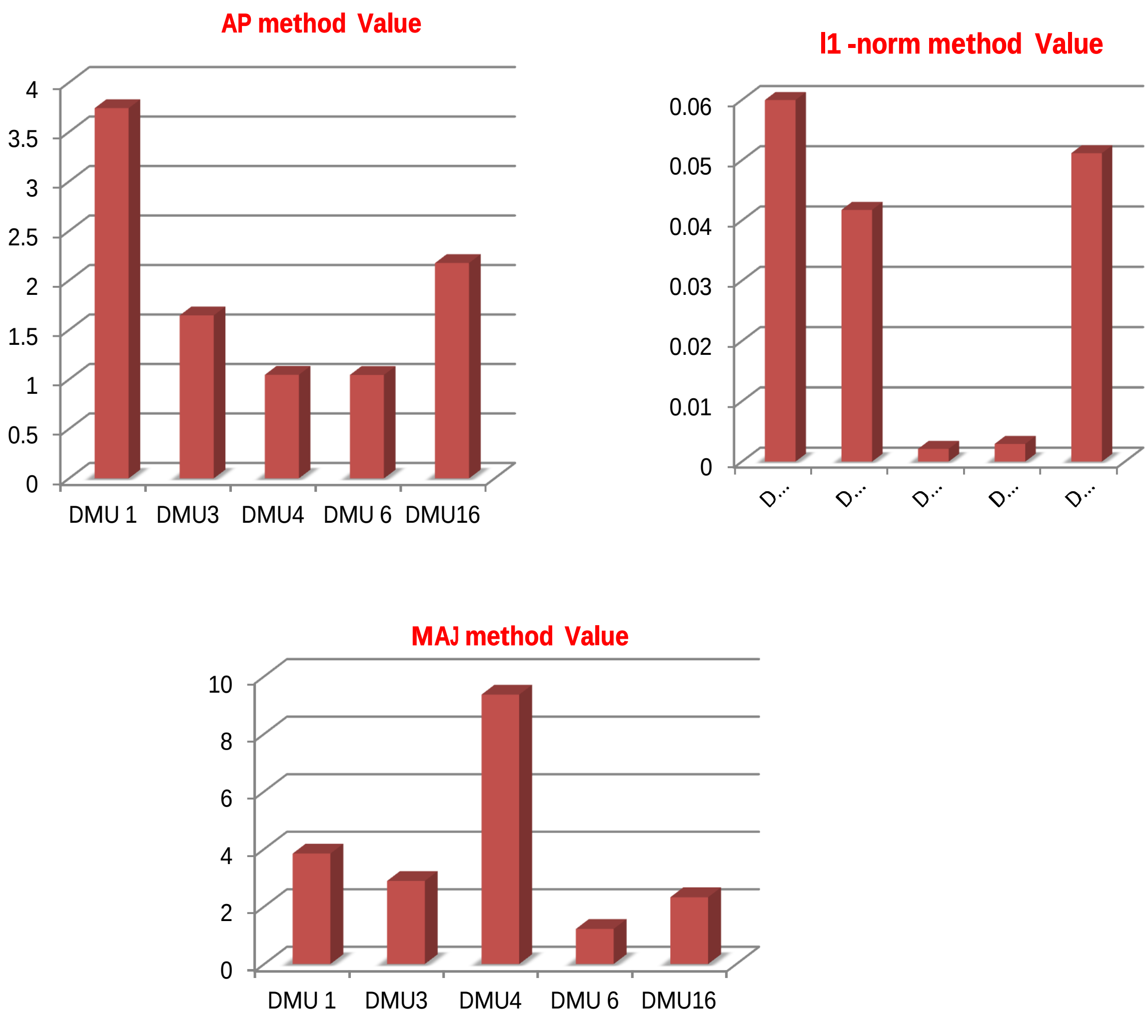

Fig. 2: Illustrations of the methods

5 Conclusions

Measurement of efficiencies of DMUs is a complicated yet important decision-making problem that requires consideration of multiple quantitative and qualitative selection criteria. Lack of discrimination power is a significant drawback that DEA suffers from and has aroused considerable research interest in the DEA literature. Therefore selecting a method for ranking DMUs is an important point in DEA. However, it was mentioned already in [20] that the production process may also generate undesirable outputs like smoke pollution or waste. However, both desirable (good) and undesirable (bad) output and input factors may be present. In this study we have developed and implemented three performance indices for ranking DMUs that treat undesirable factors together with desirable factors. The proposed model uses the linear programming problem for efficiency evaluation of DMUs and ranking. The numeral example shows the reasonability of our methods. It is shown that, the presented models are very sensitive to whether or not undesirable data were included or whether or not non-discretionary data were treated as discretionary. This suggests that conventional multilateral productivity comparison, across firms, industries, countries or whatever, may be seriously misleading if they ignore undesirable factors and those undesirable factors are subject to different degrees of regulatory constraint across the sample.

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