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Comprehensive Investigation in Buckling and Free Vibration of Laminate Composite cylindrical Shell

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ABSTRACT

In this paper, the effect of compressive axial loading on natural frequency of the composite shell and the influences of geometrical parameter and the fiber orientation on the buckling load and natural frequency are studied. Bucking and free vibration of cylindrical laminate composite shell are inspected regarding to three approaches; classical theory of laminate, first order shear deformation, finite element method. The composite is taken into account as orthotropic laminate. Substituting strain-displacement based on love's first approximation into strain-stress equation lead to yield equilibrium equations respect to displacement components. The boundary conditions are considered clamp on one side and free on other. The components of displacement are considered as double Fourier's series which is more general and precise. A remarkable accuracy has been admitted by comparing the results with other available references.

KEYWORDS; Vibration, Buckling, Cylindrical Shell, First order of Shear Deformation, Classical Theory.

INTRODUCTION

Cylindrical shells have diversity applications in industrial. Some of prominent parameters in cylindrical shells subjected to axial loading are the magnitude of buckling load and the effect of axial loading on their vibration. Buckling is so important in some structures like missiles and aircrafts, because some of their components may buckle through the applied loading. As it is known, the applied loading on structure influences the natural frequency. Besides, the natural frequencies of the shell should be known so that it prevents the resonance phenomena. According to this, the reduction of weight in aero structures is so essential. Therefore, it is required that cylindrical shell would be made of composite material in order to increase the strength of structure in comparison to its weight. The structure is optimized economically. The other benefits of composite materials are the high stiffness and the anticorrosion properties. Buckling and vibration of the cylindrical shell attract researcher's regard in mechanical and other engineering sciences. Singer and et al. [1] studied the stability of the cylindrical shell under axial compressive loading and critical loading for bucking. Cheng and Ho [2] presented an analytical theory for buckling of cylindrical shell composite problem. Xavier, and Chew [3] investigated the buckling and vibration of orthotropic laminate composite cylindricalshell by using the first and higher order of layerwise's theories. Reddy and et al. [4] studied the dynamic stability of cross ply laminate composite of cylindrical shell under a combine axial static and periodic loading. Hawkes and Soldatos [5] studied the vibration of three dimensional axisymmetric hollow cylindrical shell by using the Navier solution method. Lam and Loy [6] examined the effect of the boundary conditions on rotating thin wall cylindrical shell by using the law's first order approximation theory and Galerkin solution method. Suzuki and et al. [7] examined the vibration of the composite cylindrical circular vessel by a power series and minimum Lagrangian. Hua and Lam [8] studied the frequency characteristics of a thin rotating cylindrical shell using the generalized differential quadrature method. Zhang [9] analyzes vibration of cross-ply laminated composite cylindrical shells using the wave propagation approach. Jack and Vinson [10] reviewed the mechanical behavior shells composed of isotropic and composite materials.

1. Theoretical formulation and equilibrium equations

Consider a laminate composite cylindrical shell with radius R , thickness t and length of L . For the following analysis, a Cartesian coordinates system (R,θ,z) is adopted with the origin at the center of cylinder. Based on shell classical theory, the equilibrium equation will be as follows [6,10];

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{\theta x}}{R\,\partial x}=I_1\frac{\partial^2 u}{\partial^2 x} \tag{1a}$$

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$$\frac{\partial N_{\theta}}{R\,\partial\theta}+\frac{\partial N_{x\theta}}{\partial x}-\frac{1}{R}\frac{\partial M_{x\theta}}{\partial x}-\frac{1}{R^{2}}\frac{\partial M_{\theta}}{\partial x}+N_{x}\frac{\partial^{2}v}{\partial x^{2}}=I_{1}\frac{\partial^{2}v}{\partial^{2}t} \quad (1b)$$

$$\frac{\partial^{2}M_{x}}{\partial x^{2}}+\frac{2}{R}\frac{\partial^{2}M_{x\theta}}{\partial\theta\,\partial x}+\frac{1}{R^{2}}\frac{\partial^{2}M_{\theta}}{\partial\theta^{2}}+RN_{\theta}+N_{x}\frac{\partial^{2}w}{\partial x^{2}}=I_{1}\frac{\partial^{2}w}{\partial^{2}t} \quad (1c)$$

Based on shear deformation theory, the equilibrium equations will be as follows [11];

$$\frac{\partial N_{x}}{\partial x}+\frac{\partial N_{x\varphi}}{R\,\partial\varphi}=I_{1}\frac{\partial^{2}u}{\partial t^{2}}+I_{2}\frac{\partial^{2}\beta_{x}}{\partial t^{2}} \quad (2a)$$

$$\frac{\partial N_{x\varphi}}{\partial x}+\frac{\partial N_{\varphi}}{R\,\partial\varphi}+\frac{Q_{\varphi}}{R}+N_{x}\frac{\partial^{2}v}{\partial x^{2}}=(I_{1}+\frac{2I_{2}}{R})\frac{\partial^{2}v}{\partial t^{2}}+(I_{2}+\frac{I_{3}}{R})\frac{\partial^{2}\beta_{\varphi}}{\partial t^{2}} \quad (2b)$$

$$\frac{\partial Q_{x}}{\partial x}+\frac{\partial Q_{\varphi}}{R\,\partial\varphi}-\frac{N_{\varphi}}{R}+N_{x}\frac{\partial^{2}w}{\partial x^{2}}=I_{1}\frac{\partial^{2}w}{\partial x^{2}} \quad (2c)$$

$$\frac{\partial M_{x}}{\partial x}+\frac{\partial M_{x\varphi}}{R\,\partial\varphi}-Q_{x}=I_{3}\frac{\partial^{2}\beta_{x}}{\partial t^{2}}+I_{2}\frac{\partial^{2}u}{\partial t^{2}} \quad (2d)$$

$$\frac{\partial M_{x\varphi}}{\partial x}+\frac{\partial M_{\varphi}}{R\,\partial\varphi}-Q_{\varphi}=I_{3}\frac{\partial^{2}\beta_{\varphi}}{\partial t^{2}}+(I_{2}+\frac{I_{3}}{R})\frac{\partial^{2}v}{\partial t^{2}} \quad (2f)$$

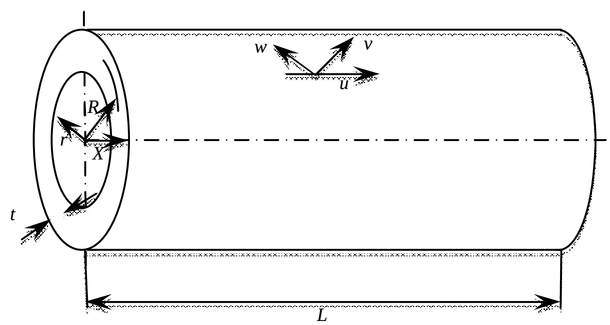

Fig.1 Cylindrical shell and reference coordinates

Which β_x and β_φ are the slops in $x-z$ and $\varphi-z$. I_1, I_2 and I_3 are defined as follows [10,11];

$$(I_{1},I_{2},I_{3})=\int_{-\frac{h}{2}}^{\frac{h}{2}}(1,z,z^{2})\rho_{k}\,dz \quad (3)$$

ρ_k is described as density of each layer. The equations of composite shells based on classical theory of laminate are defined as follows [10, 11];

$$\begin{Bmatrix} N_{x} \\ N_{\theta} \\ N_{x\theta} \end{Bmatrix}=\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_{x}^{o} \\ \varepsilon_{\theta}^{o} \\ \gamma_{x\theta}^{o} \end{Bmatrix}+\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}\begin{Bmatrix} k_{x}^{o} \\ k_{\theta}^{o} \\ k_{x\theta}^{o} \end{Bmatrix} \quad (4a)$$

$$\begin{Bmatrix} M_{x} \\ M_{\theta} \\ M_{x\theta} \end{Bmatrix}=\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_{x}^{o} \\ \varepsilon_{\theta}^{o} \\ \gamma_{x\theta}^{o} \end{Bmatrix}+\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}\begin{Bmatrix} k_{x}^{o} \\ k_{\theta}^{o} \\ k_{x\theta}^{o} \end{Bmatrix} \quad (4b)$$

(4c)

$$\begin{Bmatrix} Q_{x} \\ Q_{\varphi} \end{Bmatrix}=\begin{bmatrix} H_{55} & H_{45} \\ H_{45} & H_{44} \end{bmatrix}$$

Which A, B, D and H are respectively extensional stiffness matrix, coupling stiffness matrix, bending stiffness matrix and shear stiffness matrix which are defined as bellow [6,10,11];

$$A_{ij} = \sum_{k=1}^{N} \left(\bar{Q}_{ij}\right)\left(h_k - h_{k-1}\right) \qquad \left(i, j = 1,2,6\right) \qquad (5a)$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N} \left(\bar{Q}_{ij}\right)\left(h^2{}_k - h^2{}_{k-1}\right) \qquad \left(i, j = 1,2,6\right) \qquad (5b)$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} \left(\bar{Q}_{ij}\right)\left(h^3{}_k - h^3{}_{k-1}\right) \qquad \left(i, j = 1,2,6\right) \qquad (5c)$$

$$H_{ij} = k_o \sum_{k=1}^{N} \left(\bar{Q}_{ij}\right)\left(h_k - h_{k-1}\right) \qquad \left(i, j = 4,5\right) \qquad (5d)$$

Fig. 2 shows configuration of layers in composite. u,v and$w$ are the components of displacement in axial, circumferential and radial direction.

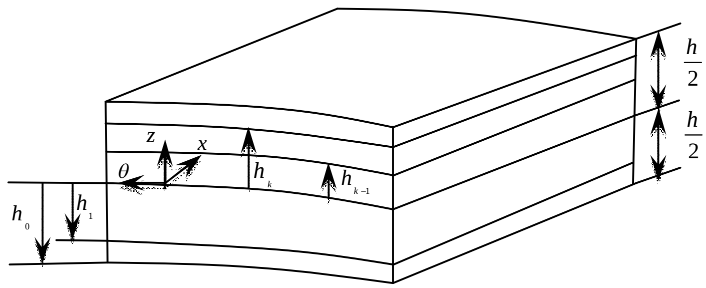

Fig.2 Configuration of layers in cylindrical laminate composite

k_o is considered as correction shear factor with magnitude of $\frac{\pi^2}{12}$ [11]. h is the shell thickness and $\bar{Q}_{ij}$ is the reduced stiffness matrix $\varepsilon^o{}_{x\varphi}, \varepsilon^o{}_{\varphi}, \varepsilon^o{}_{x}$ are the mid-surface strain. $k^o{}_{x\varphi}, k^o{}_{\varphi}, k^o{}_{x}$ are the mid-surface curvature and $\gamma^o{}_{\varphi z}, \gamma^o{}_{xz}$ are the shear strains and defined as follows [5,11];

$$\begin{Bmatrix} k_x^o \\ k_\theta^o \\ k_{x\theta}^o \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \beta_x}{\partial x} \\ \dfrac{1}{R}\dfrac{\partial \beta_\varphi}{\partial \varphi} \\ \dfrac{1}{R}\dfrac{\partial \beta_x}{\partial \varphi} + \dfrac{\partial \beta_\varphi}{\partial x} \end{Bmatrix} \qquad (6a)$$

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_\theta^o \\ \gamma_{x\theta}^o \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{1}{R}\dfrac{\partial v}{\partial \varphi} + \dfrac{w}{R} \\ \dfrac{1}{R}\dfrac{\partial u}{\partial \varphi} + \dfrac{\partial v}{\partial x} \end{Bmatrix} \qquad (6b)$$

$$\begin{Bmatrix} \gamma_{xz}^o \\ \gamma_{\varphi z}^o \\ k_{x\theta}^o \end{Bmatrix} = \begin{Bmatrix} \beta_x + \dfrac{\partial w}{\partial x} \\ \beta_\varphi + \dfrac{1}{R}\dfrac{\partial w}{\partial \varphi} - \dfrac{v}{R} \end{Bmatrix} \qquad (6c)$$

PROBLEM RESPONS AND SOLUTION METHOD

There are differentmethods to compute the response of the problem, but each method has its own benefits and disadvantage. Among various methods, Double Fourier Series is more efficientand accurate. This is more general than the single series. However the heavy mathematical calculations associated with this type of method have restricted its wide application, due to the invention of advanced computing machines and methods, this method of reactance calculation is now widely adopted. Now, by applying the boundary conditionsforthe cylindrical shell which is free and;

For the free boundary condition on $x = 0$;

$$N_x = N_{x\varphi} = Q_x = M_x = 0 \tag{7}$$

For the clamped boundary condition clamped on $x = L$;

$$u = v = w = \beta_x = \beta_\varphi = \frac{\partial w}{\partial x} = \frac{\partial w}{\partial \varphi} = 0 \tag{8}$$

Substituting equations 4 and 6 in the equilibrium equations, classical shell theory will give the following equations;

$$[L]\{U\} = \{0\} \tag{9a}$$

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{16} \\ L_{12} & L_{22} & L_{26} \\ L_{16} & L_{26} & L_{66} \end{bmatrix} \tag{9b}$$

$$\{U\} = \begin{Bmatrix} u(x,\theta,t) \\ v(x,\theta,t) \\ w(x,\theta,t) \end{Bmatrix} \tag{9c}$$

For the first order shear deformation theory, the equations will be;

$$[L]\{U\} = \{0\} \tag{10a}$$

$$\{U\} = \begin{Bmatrix} u(x,\varphi,t) \\ v(x,\varphi,t) \\ w(x,\varphi,t) \\ \beta_x(x,\varphi,t) \\ \beta_\varphi(x,\varphi,t) \end{Bmatrix} \tag{10b}$$

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ L_{21} & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\ L_{41} & L_{42} & L_{43} & L_{44} & L_{45} \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix} \tag{11}$$

L_{ij} isthe differential operator which its equations came in appendix . In order to satisfy the boundary conditions, $u, v, w, \beta_x, \beta_\varphi$ are described as double Fourier series;

$$u = \sum_m \sum_n \bar{A}_{mn} T_{mn}(t) = \sum_m \sum_n A_{mn} \frac{d\eta_u(x)}{dx} \cos n\varphi T_{mn}(t) \tag{12a}$$

$$v = \sum_m \sum_n \bar{B}_{mn} T_{mn}(t) = \sum_m \sum_n B_{mn} \eta_v(x) \sin n\varphi T_{mn}(t) \tag{12b}$$

$$w = \sum_m \sum_n \bar{C}_{mn} T_{mn}(t) = \sum_m \sum_n C_{mn} \eta_w(x) \cos n\varphi T_{mn}(t) \tag{12c}$$

$$\beta_x = \sum_m \sum_n \bar{D}_{mn} T_{mn}(t) = \sum_m \sum_n D_{mn} \eta_{\beta_x}(x) \cos n\varphi T_{mn}(t) \qquad (12d)$$

$$\beta_\varphi = \sum_m \sum_n \bar{E}_{mn} T_{mn}(t) = \sum_m \sum_n E_{mn} \eta_{\beta_\varphi}(x) \sin n\varphi T_{mn}(t) \qquad (12f)$$

$$\eta_i(x) = \alpha_1 \cosh \frac{\lambda_m x}{L} + \alpha_2 \cos \frac{\lambda_m x}{L} - \sigma_m \left(\alpha_3 \sinh \frac{\lambda_m x}{L} - \alpha_4 \sin \frac{\lambda_m x}{L} \right) \quad \left(i = u, v, w, \beta_x, \beta_\varphi \right) \qquad (13)$$

T_{mn} is the time function of displacement component and $A_{mn}, B_{mn}, C_{mn}, D_{mn}, E_{mn}$ are shape function of the natural modes which are calculated by solving the free vibration. m is the number of longitudinal half wave and n is the number of circumferential half wave. The values of α_i , σ_m and λ_m are calculated regarding to the considering boundary conditions as follows;

$$\alpha_1 = \alpha_3 = \alpha_4 = 1, \alpha_2 = -1 \qquad (14a)$$

$$\sigma_m = \frac{\sinh \lambda_m - \sin \lambda_m}{\cosh \lambda_m + \cos \lambda_m} \qquad (14b)$$

$$\cosh \lambda_m \cos \lambda_m = -1 \qquad (14c)$$

For solving the equation of (9) and (10), the Galerkin method is applied.

a. Buckling Analysis

In science, buckling is a mathematical instability, leading to a failure mode. Theoretically, buckling is caused by a bifurcation in the solution to the equations of static equilibrium.In practice, buckling is characterized by a sudden failure of a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. Mathematical analysis of buckling makes use of an axial load eccentricity that introduces a moment, which does not form part of the primary forces to which the member is subjected. When load is constantly being applied on a member, such as column, it will ultimately become large enough to cause the member to become unstable. Further load will cause significant and somewhat unpredictable deformations, possibly leading to complete loss of load-carrying capacity. In order to calculate the buckling load, static solution is done (i.e. the time terms will be neglected). Substituting (12) and (13)in the (9) and (10) and simplifying them will yield the bellow system of equations.Forthe classical shell theory;

$$\left[C_{ij} \right] \left\{ A_{mn} B_{mn} C_{mn} \right\}^T = 0 \; (i, j = 1, \dots, 3) \qquad (15)$$

For the first order shear deformation shell theory;

$$\left[C_{ij} \right] \left\{ A_{mn} B_{mn} C_{mn} D_{mn} E_{mn} \right\}^T = 0 \quad (i, j = 1, \dots, 5) \qquad (16)$$

The determinant of the coefficient $\left[C_{ij} \right]$ should be zero. So the buckling load equation will be obtained;

$$\gamma_1 N^2 + \gamma_2 N + \gamma_3 = 0 \qquad (17)$$

$\gamma_i$isconstant coefficient and N is buckling load. By solving the equation, shell buckling load corresponding to the different m and n will be obtained. The lowest value of the loadis the critical buckling load(N).

b. Free Vibration

In order to find solution for free vibrations,$T_{mn}(t) = e^{i\omega_{mn}t}$is considered which $\omega_{mn}$is natural frequency. Considering axial compressive loading (which is a fraction of the buckling load), the natural frequency and the mode shapes will be obtained. Applying similar procedure to the buckling analysis, the system of equation will be resulted for the classical shell theory;

$$\left[\left[K_{ij} \right] - \omega_{mn}^2 \left[M_{ij} \right] \right] \left\{ A_{mn} B_{mn} C_{mn} \right\}^T = 0 \; (i, j = 1, \dots, 3) \qquad (18)$$

For the first order shear deformation theory the equation will be as follows;

$$\left[\left[K_{ij} \right] - \omega_{mn}^2 \left[M_{ij} \right] \right] \left\{ A_{mn} B_{mn} D_{mn} C_{mn} E_{mn} \right\}^T = 0 \; (i, j = 1, \dots, 5) \qquad (19)$$

$[K_{ij}]$and$[M_{ij}]$are respectively the stiffness and mass matrix of the structure. The determinate of the coefficient will be zero. So the frequency equation for the shell classical theory will be obtained;

$$\beta_1\omega^6 + \beta_2\omega^4 + \beta_3\omega^2 + \beta_4 = 0 \qquad (20)$$

For the first order shear deformation theory;

$$\delta_1\omega^{10} + \delta_2\omega^8 + \delta_3\omega^6 + \delta_4\omega^4 + \delta_5\omega^2 + \delta_6 = 0 \qquad (21)$$

Which β_i and δ_i are the constant coefficient. By solving two above equations, the frequencies will be achieved. Substituting the frequencies in the (18) and (19) the constant coefficient and modes shape will be acquired.

2. RESULTS AND DISCUSSIONS

In order to check the accuracy of the obtained results from the presented analysis, the values of the buckling load obtained from the two analytical methods (i.e. classical shell theory and first order shear deformation theory of the shell) and finite element methodresults are compared.As it is shown, the results admit remarkable accuracy. Also in order to check the accuracy of the analysis, the natural frequencies for the shell are given in the table 2. And the mechanical properties for the table 1 and table 2 and figures are as follows;

$$E_{11} = 19GPa \ , \ E_{22} = 7.6GPa \ , \ G_{12} = 4.1GPa \ , \ G_{13} = 4.1GPa \ , \ G_{23} = 4.1GPa$$

$$\upsilon_{12} = 0.26 \ , \quad \rho = 1643 kg/m^3$$

Note that the geometrical characteristics for table 1 and 2 and figures will be;

a. For the Figures;

$$R = 1m \ , \ L = 6m \ , \ t = 0.002m \ , \ t_1 = t_2 = t_3 = \frac{t}{3}$$

b. For the table 2

$$R = 1m \ , \ L = 20m \ , \ t = 0.002m \ , \ t_1 = t_3 = \frac{3t}{8} \ , \ t_2 = \frac{t}{4}$$

The frequency of a steel sample is obtained by modal analysis. The frequencies are determined by an available commercial cod and the results are given in the table 3. It shows that the experimental data and the analytical results are well accorded.

The mechanical and geometrical properties for the tables 3 are given as bellows;

$$E_{11} = 200GPa \ , \ E_{22} = 200GPa \ , \ G_{12} = 76.9GPa \ , \ G_{13} = 76.9GPa \ , \ G_{23} = 76.9GPa$$

$$\upsilon_{12} = 0.3 \ , \ \rho = 7800 kg/m^3 \ , \ R = 22.1mm \ , \ L = 118.1mm \ , \ t = 1mm$$

$n = 3$ $n = 3$ In Fig.3 and Fig. 4 the critical buckling load and base natural frequency regarding to t/R for two methods CST and FSDT are shown. As the ratio increases, the critical buckling load and base frequency decrease. Fig. 5 depicts the effect of L/R on the critical load for the composite cylindrical shell. As it is illustrated, the parameter L/R could be defined even as the slenderness of the cylindrical shell which has adverse relation with the buckling load. Fig. 6 shows that as the slenderness of the cylinder increases, the natural frequency decreases. In the Fig. 7 and Fig.8, the effect of the orientation angle on the buckling load and frequency for the CST are shown.The base frequencies for all fiber orientations occur at $n = 3$ .For other circumferential waves bigger than natural circumferential waves which critical loading of the buckling and base frequency occur in them, as the angle of fiber orientation increases, the frequency and buckling load increase. And for the smaller waves, as the angle of the orientation increases, the buckling load and base frequency decrease. It is also observed that the orientation of the fibers has less impact on the critical buckling load and base frequency. Fig. 9 illustrates the impact of the axial loading on the shell natural frequency. As the axial compressive loading increases, the natural frequency decreases. When the axial loading is equal to the critical loading of the buckling, the base frequency will be zero. It is noted that the base frequency and critical loading occur at$m = 1, n = 3$.

Table.1 Comparison of critical buckling load (N/m) for clamp-free shell, [90/0/90]

	FSDT	CST	Commercial cod
m=1 , n=3	15500.8	15560.5	14950
error	3.68	41.1	-

Table.2 Comparison of natural frequencies of the clamp-free shell [90/0/90] for m=1

N	FSDT	CST	Ref[5]	Ansys
1	2.454	2.454	2.55	2.25
2	0.977	0.978	1.23	1.17
3	2.403	2.403	2.52	2.39
4	4.588	4.589	4.86	4.55
5	7.419	7.420	7.57	7.42
6	10.88	10.88	10.56	10.95
7	14.98	14.98	15.3	15
8	19.71	19.71	19.45	19.91
9	25.06	25.07	25	25.6
10	31.05	31.06	31.11	31.11

Table.3 comparison of natural frequencies of the clamped-free shell with experimental test, [0]

	Present Method	**Practical Test**	**%error**
m=1 , n=1	1947.96	1904	2.3
m=1 , n=2	1497	1507	0.66
m=1 , n=2	3945	3980	0.88
m=2 , n=3	4935.95	4960	0.48
m=3 , n=1	3233.53	3128	3.37

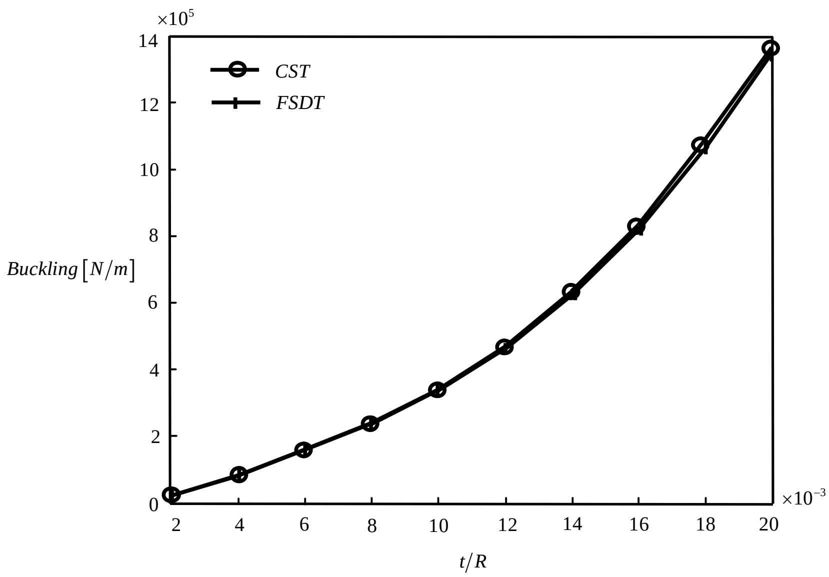

Fig.3 Effect of the $\frac{t}{R}$ ratio on the critical buckling load for a composite cylindrical shell

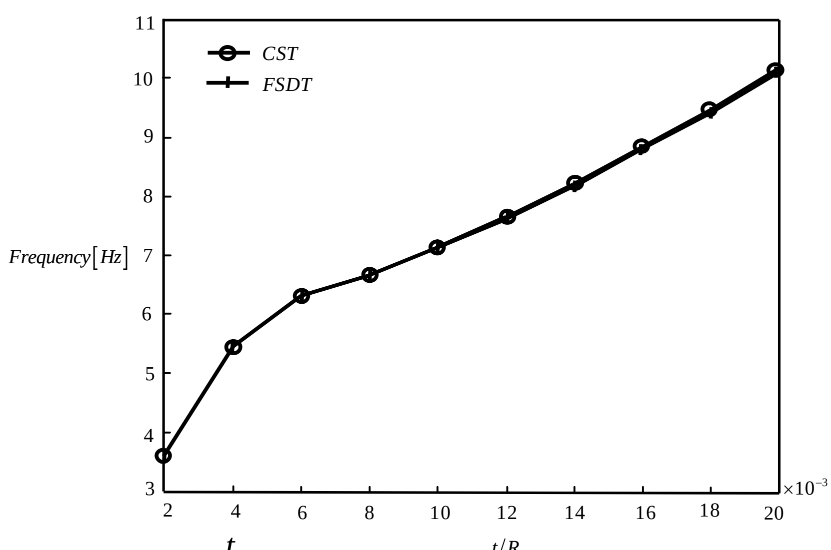

Fig.4 Effect of the $\frac{t}{R}$ ratio on base natural frequency of the composite cylindrical shell

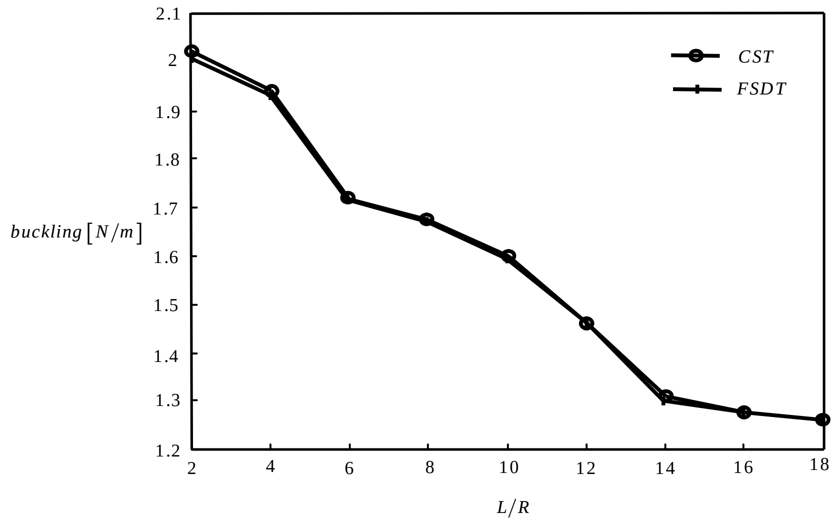

Fig.5 Effect of the $\frac{L}{R}$ ratio on the critical buckling load for a composite cylindrical shell

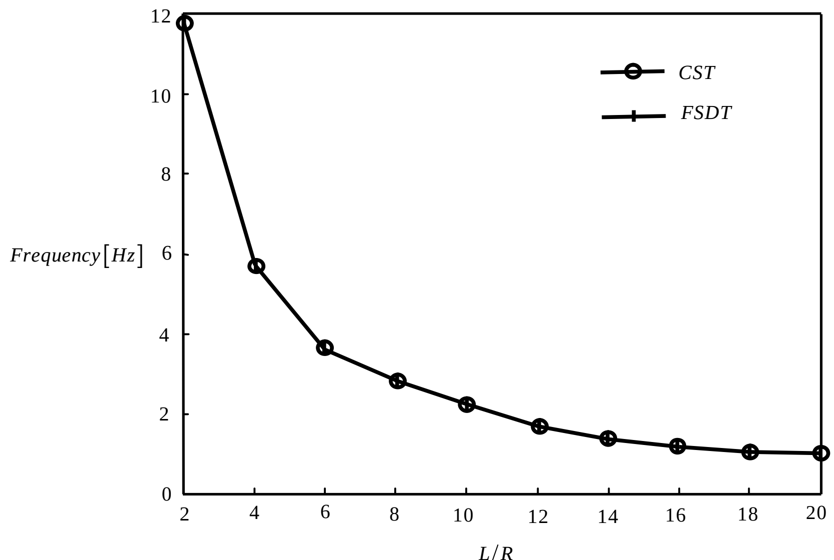

Fig.6 Effect of the $\frac{L}{R}$ ratio on base natural frequency of the composite cylindrical shell

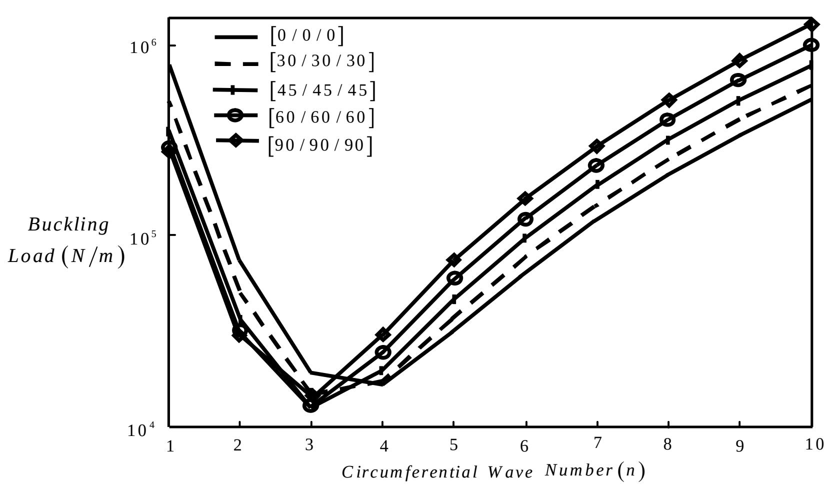

Fig.7 Effect of the fiber orientation on critical buckling load for a composite cylindrical shell

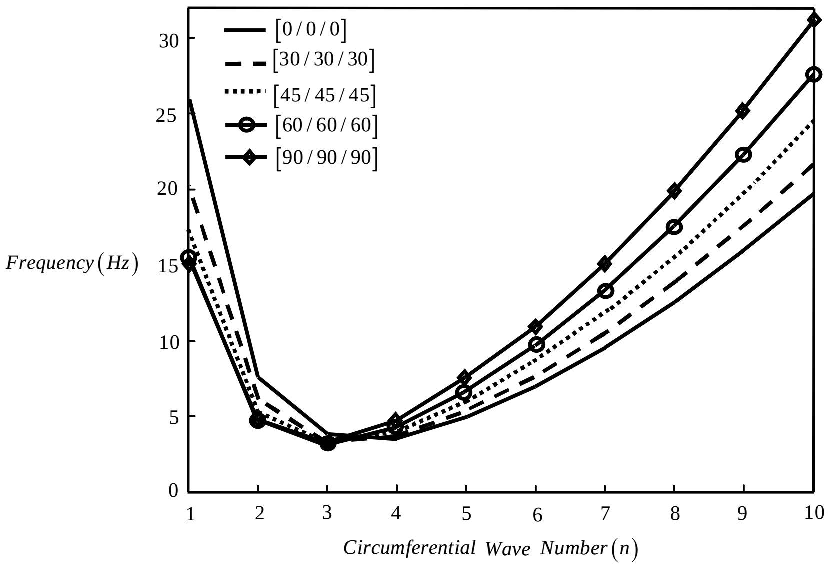

Fig.8 Effect of the fiber orientation on base natural frequency of the composite cylindrical shell

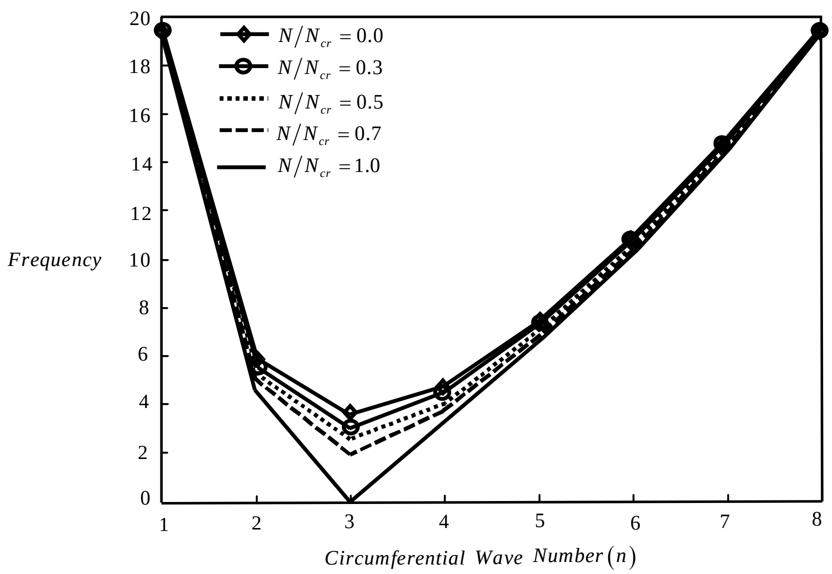

Fig.9 Effect of axial compressive load on base natural frequency of the composite cylindrical shell

3. CONCLUSION

In this paper, the effect of compressive axial loading on natural frequency of the shell and the influences of geometrical parameter and the fiber orientation on the buckling load and natural frequency are studied. Bucking and free vibration of cylindrical laminate composite shell are examined regarding to three method; classical theory of laminate, first order shear deformation and using finite element. Regarding to the obtained data prediction of response of the system is possible.$\frac{l}{R}$and$\frac{t}{R}$ are two essential parameter in buckling analysis of composite shell. According to these parameters as well as fiber orientation and magnitude of axial loading, a good vision of system response could be predicated.

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5. APPENDIX

a. L_{ij} in classical shell theory will be;

$$L_{11}=\left[\left(A_{11}R\right)\frac{\partial^2}{\partial x^2}+\left(2A_{16}\right)\frac{\partial^2}{\partial x\partial\varphi}+\left(\frac{A_{66}}{R}\right)\frac{\partial^2}{\partial\varphi^2}\right]$$

$$L_{12}=\left[\left(A_{16}R+B_{16}\right)\frac{\partial^2}{\partial x^2}+\left(A_{12}+\frac{B_{12}}{R}+A_{66}+\frac{B_{66}}{R}\right)\frac{\partial^2}{\partial x\partial\varphi}+\left(\frac{A_{26}}{R}+\frac{B_{26}}{R^2}\right)\frac{\partial^2}{\partial\varphi^2}\right]$$

$$L_{13}=\left[\left(B_{11}R\right)\frac{\partial^3}{\partial x^3}+\left(2B_{16}\right)\frac{\partial^3}{\partial x^2\partial\varphi}+\left(\frac{B_{66}}{R}+\frac{B_{12}}{R}\right)\frac{\partial^3}{\partial x\partial\varphi^2}+\left(-A_{12}\right)\frac{\partial}{\partial x}+\left(-\frac{A_{26}}{R}\right)\frac{\partial}{\partial\varphi}+\left(\frac{B_{26}}{R^2}\right)\frac{\partial^3}{\partial\varphi^3}\right]$$

$$L_{21}=\left[\left(A_{16}R-B_{16}\right)\frac{\partial^2}{\partial x^2}+\left(A_{12}+A_{66}-\frac{B_{12}}{R}-\frac{B_{66}}{R}\right)\frac{\partial^2}{\partial x\partial\varphi}+\left(\frac{A_{26}}{R}-\frac{B_{26}}{R^2}\right)\frac{\partial^2}{\partial\varphi^2}\right]$$

$$L_{22}=\left[\left(A_{66}R-\frac{D_{66}}{R}+N_aR\right)\frac{\partial^2}{\partial x^2}+\left(2A_{26}-2\frac{D_{26}}{R^2}\right)\frac{\partial^2}{\partial x\partial\varphi}+\left(\frac{A_{22}}{R}-\frac{D_{22}}{R^3}\right)\frac{\partial^2}{\partial\varphi^2}\right]$$

$$L_{23}=\left[\left(B_{16}R-D_{16}\right)\frac{\partial^3}{\partial x^3}+\left(B_{12}+B_{66}-\frac{D_{12}}{R}-\frac{D_{66}}{R}\right)\frac{\partial^3}{\partial x^2\partial\varphi}+\left(2\frac{B_{26}}{R}-2\frac{D_{26}}{R^2}\right)\frac{\partial^3}{\partial x\partial\varphi^2}+\left(-A_{26}+\frac{B_{26}}{R}\right)\frac{\partial}{\partial x}\right]$$
$$+\left[\left(\frac{B_{22}}{R^2}-\frac{D_{22}}{R^3}\right)\frac{\partial^3}{\partial\varphi^3}+\left(-\frac{A_{22}}{R}+\frac{B_{22}}{R^2}\right)\frac{\partial}{\partial\varphi}\right]$$

$$L_{31}=\left[\left(B_{11}R\right)\frac{\partial^3}{\partial x^3}+\left(3B_{16}\right)\frac{\partial^3}{\partial x^2\partial\varphi}+\left(\frac{B_{12}}{R}+2\frac{B_{66}}{R}\right)\frac{\partial^3}{\partial x\partial\varphi^2}+\left(A_{12}\right)\frac{\partial}{\partial x}+\left(\frac{B_{26}}{R^2}\right)\frac{\partial^3}{\partial\varphi^3}+\left(\frac{A_{26}}{R}\right)\frac{\partial}{\partial\varphi}\right]$$

$$L_{32}=\left[\left(B_{16}R+D_{16}\right)\frac{\partial^3}{\partial x^3}+\left(2B_{66}+2\frac{D_{66}}{R}+B_{12}+\frac{D_{12}}{R}\right)\frac{\partial^3}{\partial x^2\partial\varphi}+\left(3\frac{B_{26}}{R}+3\frac{D_{26}}{R^2}\right)\frac{\partial^3}{\partial x\partial\varphi^2}+\left(A_{26}+\frac{B_{26}}{R}\right)\frac{\partial}{\partial x}\right]$$
$$+\left[\left(\frac{B_{22}}{R^2}+\frac{D_{22}}{R^3}\right)\frac{\partial^3}{\partial\varphi^3}+\left(\frac{A_{22}}{R}+\frac{B_{22}}{R^2}\right)\frac{\partial}{\partial\varphi}\right]$$

$$L_{33} = \left[\left(D_{11}R\right)\frac{\partial^4}{\partial x^4} + \left(3D_{16}\right)\frac{\partial^4}{\partial x^3\partial\varphi} + \left(2\frac{D_{12}}{R} + 2\frac{D_{66}}{R}\right)\frac{\partial^4}{\partial x^2\partial\varphi^2} + \left(N_a R\right)\frac{\partial^2}{\partial x^2} \right]$$

b. L_{ij} in first order shear deformation theory;

$$L_{11} = \left[\left(A_{11}R\right)\frac{\partial^2}{\partial x^2} + \left(2A_{16}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{A_{66}}{R}\right)\frac{\partial^2}{\partial\varphi^2} \right]$$

$$L_{12} = L_{21} = \left[\left(A_{16}R\right)\frac{\partial^2}{\partial x^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{A_{26}}{R}\right)\frac{\partial^2}{\partial\varphi^2} \right]$$

$$L_{13} = -L_{31} = \left[\left(A_{12} + RP\right)\frac{\partial}{\partial x} + \left(\frac{A_{26}}{R}\right)\frac{\partial}{\partial\varphi} \right]$$

$$L_{14} = L_{41} = \left[\left(B_{11}R\right)\frac{\partial^2}{\partial x^2} + \left(2B_{16}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{B_{66}}{R}\right)\frac{\partial^2}{\partial\varphi^2} \right]$$

$$L_{15} = L_{51} = \left[\left(B_{16}R\right)\frac{\partial^2}{\partial x^2} + \left(B_{12} + B_{66}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{B_{26}}{R}\right)\frac{\partial^2}{\partial\varphi^2} \right]$$

$$L_{22} = \left[\left(A_{66}R + N_a R\right)\frac{\partial^2}{\partial x^2} + \left(2A_{26}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{A_{22}}{R}\right)\frac{\partial^2}{\partial\varphi^2} - \frac{H_{44}}{R} \right]$$

$$L_{23} = -L_{32} = \left[\left(A_{26} + H_{45}\right)\frac{\partial}{\partial x} + \left(\frac{A_{22}}{R} + \frac{H_{44}}{R} + RP\right)\frac{\partial}{\partial\varphi} \right]$$

$$L_{24} = L_{42} = \left[\left(B_{16}R\right)\frac{\partial^2}{\partial x^2} + \left(B_{12} + B_{66}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{B_{26}}{R}\right)\frac{\partial^2}{\partial\varphi^2} + H_{45} \right]$$

$$L_{25} = L_{52} = \left[\left(B_{66}R\right)\frac{\partial^2}{\partial x^2} + \left(2B_{26}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{B_{22}}{R}\right)\frac{\partial^2}{\partial\varphi^2} + H_{44} \right]$$

$$L_{33} = \left[\left(N_a R + H_{55}R\right)\frac{\partial^2}{\partial x^2} + \left(2H_{45}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{H_{44}}{R}\right)\frac{\partial^2}{\partial\varphi^2} - \frac{A_{22}}{R} \right]$$

$$L_{34} = L_{43} = \left[\left(H_{55}R - B_{12}\right)\frac{\partial}{\partial x} + \left(H_{45} - \frac{B_{26}}{R}\right)\frac{\partial}{\partial\varphi} \right]$$

$$L_{35} = -L_{53} = \left[\left(H_{45}R - B_{26}\right)\frac{\partial}{\partial x} + \left(H_{44} - \frac{B_{22}}{R}\right)\frac{\partial}{\partial\varphi} \right]$$

$$L_{44} = \left[\left(D_{11}R\right)\frac{\partial^2}{\partial x^2} + \left(2D_{16}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{D_{66}}{R}\right)\frac{\partial^2}{\partial\varphi^2} - H_{55}R \right]$$

$$L_{45} = L_{54} = \left[\left(D_{16}R\right)\frac{\partial^2}{\partial x^2} + \left(D_{12} + D_{66}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{D_{26}}{R}\right)\frac{\partial^2}{\partial\varphi^2} - H_{45}R \right]$$

$$L_{55} = \left[\left(D_{66}R\right)\frac{\partial^2}{\partial x^2} + \left(2D_{26}\right)\frac{\partial^2}{\partial x\partial\varphi} + \left(\frac{D_{22}}{R}\right)\frac{\partial^2}{\partial\varphi^2} - H_{44}R \right]$$